Computational Issue of Fuzzy Rule-based System

Chunshien Li

Department of Computer Science and Information Engineering, National University of Tainan
33, Sec. 2, Shu-Lin St., Tainan, 700, Taiwan, ROC.

Summary
An innovative soft-computing system is proposed in this paper to assuage the paradox of curse of dimensionality (COD) and yet to preserve the property of completeness. Although the supreme merits of a fuzzy inference system (FIS) are in its simplicity, understandability of fuzzy rules and model-free approach, it suffers with the problem of COD. The COD problem can be happened easily if the number of either input variables or partitions of each input universe increases greatly. This drawback of COD causes serious situation especially for hardware realization that computational resources will be sucked exhaustively. An event-triggering based neuro-fuzzy system (NFS) is presented to alleviate the COD problem and to reduce wasting computational resources. The proposed soft-computing system can save the computational resources effectively without missing the property of completeness. The input to the proposed NFS is considered as an event. Because the incoming input event $H(t)$ decides the position in the input space around which fuzzy sets with membership degree beyond a threshold are detected and are used to construct the fuzzy rules closely to the $H(t)$, the fuzzy rules involved in the knowledge base can be very compact. The event-triggering based knowledge base is effective, in which there are no redundant fuzzy rules and only the needed rules are in the proposed system for the $H(t)$. Moreover, the proposed soft-computing system, whose structure is time-varying and is dependent on the incoming input event, possesses the property of event-tracking structure. The knowledge base of the proposed NFS is triggered off by the event, and only few rules are fired locally around the event. It is suitable for large-scale system operation. An example is demonstrated for the proposed approach.

Key words:
Fuzzy inference system (FIS), neural-fuzzy system (NFS), curse of dimensionality (COD), COD-completeness paradox, soft-computing system.

1. Introduction

Fuzzy inference systems [8][9] have been powerful tool for real-world applications such as automatic control, data classification, decision analysis, expert systems, robotics, pattern recognition, and many others. Among the applications the most fruitful research area is in control systems [10][11][13], in which expertise, engineering experience and judgment can be integrated into the design of knowledge base for the fuzzy inference system (FIS) [17][20][21][22]. The supreme merits of an FIS [8][9] are in its simplicity, understandability of fuzzy rules, and model-free approach by which a plant is viewed as a black box, whose output is observable to the FIS, serving as a controller. The partitioning of the input space is critical to the design of an FIS no matter what application purpose. There are three main types of partitioning commonly used in the design of an FIS, which are grid-type, tree-type, and cluster-type. For most types of partitioning for an FIS such as grid-type and tree-type, the problem of curse of dimensionality (COD) [5] arises that the amount of fuzzy rules increases exponentially if input variables and fuzzy partitions of each input universe are increased. For the cluster-type partitioning [6], the COD problem is dependent on the amount of clusters in the input space of an FIS for each cluster corresponds to a fuzzy rule. Although the COD problem is not that serious with the partitions of cluster-type, it is a compromise amid the fineness of partition, the completeness of the rule base and the size of rule base in an FIS. In other words, the fewer the fuzzy rules with the cluster-type partitioning, the coarser the partitions and the worse the incompleteness to cover the input space of the FIS, and vice versa. For the tree-type partitioning, it usually does not correspond to good linguistic meanings to understand, and it is more complex than the grid-type partitioning in topological distribution of fuzzy regions and more membership functions may be needed. For design simplicity, the grid-type partitioning of the input space for an FIS is most used. The grid-type of fuzzy partition possesses the property of partition completeness and meaningful linguistic description and understanding, but suffers with the COD problem. The COD problem of an FIS will cause drawbacks in hardware realization and implementation such as in an FPGA, an ASIC or a DSP-processor.

It may be interesting to think about the paradox between the COD problem and the completeness property of knowledge base, the so-called ***COD-Completeness Paradox***, that an FIS possesses excellent property of completeness for the rule base to cover the input space and yet avoids the COD problem to speed up the necessary calculation of fuzzy inference [4][7] and to save computational resource. An event-triggering based (or called event-based) neuro-fuzzy system is proposed in the

Manuscript received January 25, 2006.
Manuscript revised February 15, 2006.

paper to overcome the COD problem and to preserve the completeness property for the system to cover the entire the input space. The neuro-fuzzy system (NFS) [1][14][18] is realized by fusing both an FIS and a neural net [15], exhibiting excellent reasoning and learning abilities to cope with complex and ill-defined systems. The rule base of the event-triggering based NFS is fictitiously algorithmed, but is not really firmly setup. When event is happened that input to the NFS is gauged, an event-triggering based rule base is visualized, in which only few fuzzy rules closely and locally related to the event are really constructed. The event-triggering based rule base is only a small fraction compared to the entire rule base of the NFS, and it becomes the real rule base with the fuzzy rules closely related to the event. Because an event to the NFS is varied with time, the event-based rule base of the NFS is varied with the event and time. In other words, the fuzzy rules in the event-based rule base always keep good track with the event and the event-based rule base possesses time-varying structure. With the time-varying structure of the event-triggering based rule base, the proposed NFS can both overcome the COD paradox and preserve the completeness of the NFS.

The paper is organized as follows. A conventional fuzzy inference system is overviewed in Section 2. The event-triggering based neuro-fuzzy system is proposed in Section 3. An example demonstration is given in Section 4 to illustrate the proposed soft computing system. Finally, discussion and conclusion are given in Sections 5 and 6, respectively.

2. Mathematical Description of a Fuzzy Inference System

The supreme merits of a fuzzy inference system (FIS) are in its simplicity, understandability of fuzzy rules, and expertise-oriented approach. An FIS can use expertise, engineering experience and judgment into the design of the knowledge base. In an FIS, there are many methods to decide the number of rules, which is determined by input space partitioning. Among the types of partitioning, the grid-type partitioning is frequently used because of its completeness, simplicity, and good linguistic meanings for understanding. Although the grid partitioning is simple and complete in design of the knowledge base of an FIS, the problem of curse of dimensionality is always occur with the partitioning for an FIS as discussed in the previous section. In this section the input-output behavior of an FIS with grid-type partitioning for input space is specified. Philosophy of fuzzy partitioning for an FIS is related with the concept of divide-and-conquer for all

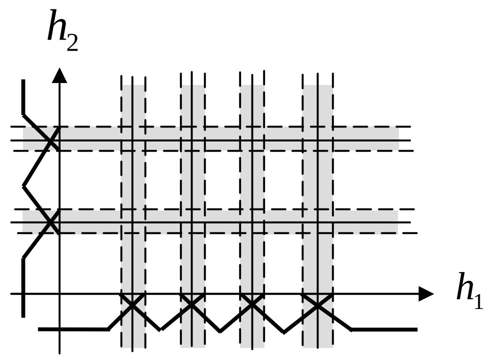

Fig.1. Input space partitioning of a grid-type fuzzy inference system.

situations of input to the FIS such that every input condition to the FIS can be reacted by the FIS. Each partition in the input space corresponds to an antecedent of a fuzzy rule, and the consequent describes the reaction behavior for the fuzzy region. Partitioning of the input space comes out with a number of fuzzy regions, interpreted as the construction of the corresponding fuzzy rules. A typical grid partitioning in a two-dimensional input space of an FIS is shown in Fig. 1, in which fuzzy regions are overlapped with transient boundaries and are lined up such that the design of fuzzy sets for the fuzzy regions is simple and easy to understand with few meaningful linguistic terms. The input space is covered everywhere with the fuzzy grid partitions so that the property of completeness is always satisfied for the FIS.

Suppose that there are M crisp input variables to an FIS and they are the base variables $h_j(t)$, j=1,2,…,M. The input variables are collected together to form an input crisp vector $H(t)$,

$$H(t)=\begin{bmatrix} h_1(t) \\ h_2(t) \\ \vdots \\ h_M(t) \end{bmatrix} \tag{1}$$

Associated with each crisp variable $h_j(t)$, there is a corresponding linguistic variable x_j. Let X denote the set of M linguistic variables, that is X=(x_1, x_2, …, x_M). Each universe of discourse of each linguistic element of X can be partitioned into several regions that overlap each other. And each partition is labeled with a linguistic term, such as “positive large”, “positive”, or “negative”. Thus the M linguistic variables have M corresponding linguistic term sets, T_j, j=1,2,…,M. In each term set, there is a collection of linguistic values. The cardinalities, c_j, j=1,2,…,M, for the M input linguistic variables are collected to form the cardinality vector C given as follows.

$$C = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_M \end{bmatrix} \quad (2)$$

The number of if-then rules in the knowledge base for an FIS with grid partitioning of its input space is determined as follows.

$$K = \prod_{j=1}^{M} c_j \quad (3)$$

The fuzzy rules in the paper use the method of Takagi and Sugeno [16] that the consequent is a linear combination of the components of $H(t)$. A fuzzy if-then rule is given as

Rule i:

IF (x_1 *is* $s_1^i(h_1(t))$ and x_2 *is* $s_2^i(h_2(t))$ and … and x_M *is* $s_M^i(h_M(t))$)

THEN

$$\sigma_k^i(t) = a_{k,0}^i + a_{k,1}^i h_1(t) + a_{k,2}^i h_2(t) + \ldots + a_{k,M}^i h_M(t) \quad (4)$$

where $s_j^i(h_j(t))$ is the fuzzy set for the j-th linguistic input variable in the i-th fuzzy rule, for i=1,2,…,K and j=1,2,…,M, and $h_j(t)$ is the j-th crisp input at time t to the FIS. Each rule of the FIS may have multiple outputs $\boldsymbol{\sigma}_k^i(t)$, k=1,2,…,Q. The coefficients, $a_{k,l}^i$, i=1,2,…,K, l=0,1,2,…,M, k=1,2,…,Q, will become appropriate values via either design or learning [3][12][19]. Let the following data types be defined.

$$A^i \equiv \begin{bmatrix} a_{1,0}^i & a_{1,1}^i & \cdots & a_{1,M}^i \\ a_{2,0}^i & a_{2,1}^i & \cdots & a_{2,M}^i \\ \vdots & \vdots & \ddots & \vdots \\ a_{Q,0}^i & a_{Q,1}^i & \cdots & a_{Q,M}^i \end{bmatrix}, \quad H_a(t) \equiv \begin{bmatrix} \varphi \\ h_1(t) \\ h_2(t) \\ \vdots \\ h_M(t) \end{bmatrix},$$

$$S^i(H(t)) \equiv (s_1^i(h_1(t)), s_2^i(h_2(t)), \ldots, s_M^i(h_M(t))) \quad (5)$$

The constant φ can be set to unity if constant terms in the consequents of fuzzy rules are involved in the inference process, otherwise it is set to zero. Let the rule actions $\boldsymbol{\sigma}_k^i(t)$, k=1,2,…,Q, from the i-th rule be collected together to form the rule action vector, given as

$$\boldsymbol{\Sigma}^i(t) = \begin{bmatrix} \boldsymbol{\sigma}_1^i(t) \\ \boldsymbol{\sigma}_2^i(t) \\ \vdots \\ \boldsymbol{\sigma}_Q^i(t) \end{bmatrix} = A^i H_a(t). \quad (6)$$

With the data types defined in Eqs.(5) and (6), a fuzzy if-then rule can be expressed in compact form, given as

IF (X *is* $S^i(H(t))$), THEN $\boldsymbol{\Sigma}^i(t) = A^i(t) H_a(t)$ (7)

for i=1,2,…,K. Let $\boldsymbol{\mu}^i(H(t))$ be the set of membership degrees of the crisp input variables in the i-th fuzzy rule, given as

$$\boldsymbol{\mu}^i(H(t)) = (\boldsymbol{\mu}_1^i(h_1(t)), \boldsymbol{\mu}_2^i(h_2(t)), \ldots, \boldsymbol{\mu}_M^i(h_M(t))) \quad (8)$$

where $\boldsymbol{\mu}_j^i(h_j(t))$ is the membership degree evaluated with $h_j(t)$ at time t for j=1,2,…,M, in the i-th fuzzy rule. The firing strength $\boldsymbol{\beta}^i(t)$ of the i-th rule is obtained with

$$\boldsymbol{\beta}^i(t) = \wedge(\boldsymbol{\mu}^i(H(t))), \quad (9)$$

where $\wedge(\boldsymbol{\mu}^i(H(t)))$ is the *fuzzy-and* operation over all elements in the set $\boldsymbol{\mu}^i(H(t))$. Usually the *fuzzy-and* operation is calculated using t-norm operator.

The fuzzy inference results, $z_k(t)$, k=1,2,…,Q, are obtained by combining all individual fired rule actions, given as

$$z_k(t) = \frac{\sum_{i=1}^{K}\left(\beta^i(t)\left(a_{k,0}^i + \sum_{j=1}^{M} a_{k,j}^i h_j(t)\right)\right)}{\sum_{i=1}^{K}\beta^i(t)} \quad (10)$$

$$= \sum_{i=1}^{K}\left(\lambda^i(t)\left(a_{k,0}^i + \sum_{j=1}^{M} a_{k,j}^i h_j(t)\right)\right)$$

and

$$\boldsymbol{\lambda}^i(t) = \frac{\boldsymbol{\beta}^i(t)}{\sum_{i=1}^{K}\boldsymbol{\beta}^i(t)}, \quad (11)$$

for k=1,2,…,Q and i=1,2,…,K. All normalized firing strengths $\boldsymbol{\lambda}^i(t)$, i=1,2,…,K, are collected together to form the normalized firing strength vector $\boldsymbol{\lambda}(t)$, given as

$$\lambda(t) = \begin{bmatrix} \lambda^1(t) \\ \lambda^2(t) \\ \vdots \\ \lambda^K(t) \end{bmatrix} = \frac{1}{\sum_{i=1}^{K}\beta^i(t)} \begin{bmatrix} \beta^1(t) \\ \beta^2(t) \\ \vdots \\ \beta^K(t) \end{bmatrix} \quad (12)$$

Let all rule action vectors defined in Eq. (16) be collected together to have the following matrix.

$$\boldsymbol{\Sigma}(t) = [\boldsymbol{\Sigma}^1(t) \quad \boldsymbol{\Sigma}^2(t) \quad \ldots \boldsymbol{\Sigma}^K(t)] \quad (13)$$

which is called the rule action matrix for the FIS. The output of the FIS at time t is expressed as the product of the rule action matrix and the normalized firing strength vector, given as

$$Z(t)=\begin{bmatrix} z_1(t) \\ z_2(t) \\ \vdots \\ z_Q(t) \end{bmatrix}=\boldsymbol{\Sigma}(t)\boldsymbol{\lambda}(t). \tag{14}$$

Alternatively, the FIS output $Z(t)$ can be expressed explicitly in terms of the crisp inputs, given as

$$Z(t)=[\,\boldsymbol{\Sigma}^1(t)\,\boldsymbol{\Sigma}^2(t)\ldots\boldsymbol{\Sigma}^K(t)\,]\;\frac{1}{\sum_{i=1}^{K}\boldsymbol{\beta}^i(t)}\begin{bmatrix} \boldsymbol{\beta}^1(t) \\ \boldsymbol{\beta}^2(t) \\ \vdots \\ \boldsymbol{\beta}^K(t) \end{bmatrix}$$

$$=\frac{\left[A^1H_a(t)\quad A^2H_a(t)\quad\cdots\quad A^KH_a(t)\right]}{\sum_{i=1}^{K}\wedge(\mu^i(H(t)))}\begin{bmatrix} \wedge(\mu^1(H(t))) \\ \wedge(\mu^2(H(t))) \\ \vdots \\ \wedge(\mu^K(H(t))) \end{bmatrix} \tag{15}$$

By the equation above, the relation between the input vector $H(t)$ and the output vector $Z(t)$ for the FIS is established, which is a highly nonlinear mapping function. Because of the highly nonlinear mapping between input and output, the FIS is easily able to handle with nonlinear real-world problems. Note that the mathematical derivation of the input-output relation given above for an FIS is for the grid-type FIS. It can also be applied to the other types of FIS, with some modification. Although the FIS is with excellent nonlinear mapping ability, it suffers with the problem of curse of dimensionality, especially for the grid-type partitioning of input space. IF the system scale of an FIS or the cardinalities of the term sets $T_j(x_j)$, j=1,2,…,M, get larger and larger, the COD problem for the FIS gets worse. For instance, a grid-type FIS with 3 input variables and 10 linguistic values for each term set will get 10^3 fuzzy if-then rules, which is unusually large for the given system scale and cardinalities of the FIS.

3. Event-Triggering Based Soft-Computing System

In the section an event-triggering based neuro-fuzzy system (NFS) is proposed to overcome the paradox of dimensionality curse for a fuzzy inference system (FIS). The approach is based on the idea that input to the NFS is regarded as an event to trigger off the knowledge base of the NFS. The input event fires on the fuzzy rules locally around the event so that only few rules are triggered at each time. This idea may be performed with the distributed structure of neural network. Not like the structure of FIS whose knowledge base is established firmly, the knowledge base of the proposed NFS is pseudo-set up, whose structure is time-varying and is dependent on the incoming input event. In the proposed approach, no matter what size of the knowledge base of an FIS, it is triggered off by event and only few rules are fired locally around the event. In such a way, the COD problem can be overcome. In the following, the proposed neuro-fuzzy approach is specified in detail. As an input event is occurred and gauged, the event is mapped to the corresponding universes of discourse so that the fuzzy sets around the event are fired.

For each of the M linguistic variables, there is a linguistic term set. Each linguistic value can be defined with a fuzzy set. Let the ***membership function set*** for the j-th input linguistic variable be denoted by

$$\boldsymbol{\mu}_j(h_j(t))=\begin{pmatrix} \boldsymbol{\mu}_{j,1}(h_j(t)) \\ \boldsymbol{\mu}_{j,2}(h_j(t)) \\ \vdots \\ \boldsymbol{\mu}_{j,c_j}(h_j(t)) \end{pmatrix} \tag{16}$$

for j=1,2,…,M, where $\boldsymbol{\mu}_{j,k}(h_j(t))$ is the k-th membership function of fuzzy set for the j-th linguistic input variable. The membership function sets of M input variables are collected together to form the ***membership function basis set*** of all linguistic variables, given as

$$\boldsymbol{\mu}(H(t))=\{\,\boldsymbol{\mu}_1(h_1(t)),\ \boldsymbol{\mu}_2(h_2(t)),\ \ldots,\ \boldsymbol{\mu}_M(h_M(t))\} \tag{17}$$

The membership function basis set provides with all necessary information needed to set up an FIS. When an input event is occurred and gauged, the membership degrees are calculated for fuzzy sets in all input universe of discourse. That is, the membership degrees of all fuzzy sets are processed for that input event at this moment in all universes of discourse. Note that up to this point the knowledge base is not set up and the rules are not participated in these calculations yet. For the input event at the moment, the fuzzy sets corresponding to the membership degrees beyond a threshold are detected and then qualified to participate in the structure setup of knowledge base and inference process of the FIS. In other words, there is no real fuzzy rules setup until the input event is occurred and measured. This is the main idea of the proposed approach to deal with the COD problem. In such a way, the structure of the proposed FIS can be very compact, and fuzzy rules involved in inference process are only tiny fraction to the original knowledge base of conventional setup.

A detector is used to detect the fuzzy sets to which the membership degrees are beyond a threshold ε for an incoming input vector $H(t)$ in each corresponding universe of discourse U_j for j=1,2,…,M. $H(t)$ is viewed as an event occurred at time t. In each input universe, these fuzzy sets detected with membership degree greater than ε are qualified to participate in the setup of the FIS.

Assume that the information of the qualified fuzzy sets by the detector for the j-th linguistic variable is included in the vector $N_j(t)$, given as

$$N_j(t) = \begin{bmatrix} n_{j,1} \\ n_{j,2} \\ \vdots \\ n_{j,c_j'(t)} \end{bmatrix} \quad (18)$$

for j=1,2,…,M, where $c_j'(t)$ denotes the cardinality of the $N_j(t)$, that is the number of qualified fuzzy sets detected in the j-th input universe, and $n_{j,k}$ is the k-th element of $N_j(t)$ whose value is the corresponding sequential number of fuzzy set in the universe. Note that the cardinality $c_j'(t)$ is dependent on the j-th component of the input vector $H(t)$ at time t.

The cardinalities $c_j'(t)$, j=1,2,…,M, are collected together to form the cardinality vector $C'(t)$, given as

$$C'(t) = \begin{bmatrix} c_1'(t) \\ c_2'(t) \\ \vdots \\ c_M'(t) \end{bmatrix} \quad (19)$$

which is called the ***event-triggering cardinality based vector*** of fuzzy sets for the proposed approach.

For a proposed approach, the number of fuzzy rules at time t in the grid-type knowledge base is determined as

$$K'(t) = \prod_{j=1}^{M} c_j'(t) \quad (20)$$

To determine the fuzzy rules triggered by an incoming event at time t, the procedure of ***rule construction*** for a rule is specified as follows. The rule constructor is define as

$$\boldsymbol{G}^i = \begin{bmatrix} n_{1,\gamma_1^i} \\ n_{2,\gamma_2^i} \\ \vdots \\ n_{M,\gamma_M^i} \end{bmatrix} \quad (21)$$

for i=1,2,…, $K'(t)$, where n_{j,γ_j^i} is from one of the components of $N_j(t)$ for j=1,2,…,M. Note that each $N_j(t)$ provides only one element to the $\boldsymbol{G}^i$. The γ_j^i, j=1,2,…,M, are calculated using the following equations.

$$\frac{i}{c_M'(t)} = q_M^i + \frac{\gamma_M^i}{c_M'(t)}, \quad 1 \le \gamma_M^i \le c_M'(t), \quad i=1,2,...,K'(t)$$

$$\frac{q_j^i}{c_{j-1}'(t)} = q_{j-1}^i + \frac{\theta_{j-1}^i}{c_{j-1}'(t)}, \quad 0 \le \theta_{j-1}^i < c_{j-1}'(t),$$

$$\gamma_{j-1}^i = \theta_{j-1}^i + 1, \quad j=M,\ M-1,\ M-2,\ ...,\ 2. \quad (22)$$

For i=1,2,…, $K'(t)$, each $\boldsymbol{G}^i$ will correspond to a fuzzy rule with a rule number denoted as $R(i)$, which is calculated using the following equations.

$$R(i) = \tau_M^i \times c_M + n_{M,\gamma_M^i}, \quad 1 \le n_{M,\gamma_M^i} \le c_M.$$

$$\tau_j^i = \tau_{j-1}^i \times c_{j-1} + n_{j-1,\gamma_{j-1}^i} - 1, \quad j=M, M-1, M-2, \ldots, 2.$$

$$\tau_1^i = 0. \quad (23)$$

By the proposed event-triggering approach, the fuzzy inference results, $z_k(t)$, k=1,2,…,Q, when the event $H(t)$ is occurred, are obtained as follows.

$$z_k(t) = \frac{\sum_{i=1}^{K'(t)} \left(\beta^{R(i)}(t) \left(a_{k,0}^{R(i)} + \sum_{j=1}^{M} a_{k,j}^{R(i)} h_j(t) \right) \right)}{\sum_{i=1}^{K'(t)} \beta^{R(i)}(t)} = \sum_{i=1}^{K'(t)} \left(\lambda^{R(i)}(t) \left(a_{k,0}^{R(i)} + \sum_{j=1}^{M} a_{k,j}^{R(i)} h_j(t) \right) \right) \quad (24)$$

and

$$\lambda^{R(i)}(t) = \frac{\beta^{R(i)}(t)}{\sum_{i=1}^{K'(t)} \beta^{R(i)}(t)}, \quad (25)$$

for k=1,2,…,Q and i=1,2,…, $K'(t)$. All normalized firing strengths $\lambda^{R(i)}(t)$, i=1,2,…, $K'(t)$, are collected together to form the normalized firing strength vector $\lambda^H(t)$, given as

$$\lambda^H(t) = \begin{bmatrix} \lambda^{R(1)}(t) \\ \lambda^{R(2)}(t) \\ \vdots \\ \lambda^{R(K'(t))}(t) \end{bmatrix} = \frac{1}{\sum_{i=1}^{K'(t)} \beta^{R(i)}(t)} \begin{bmatrix} \beta^{R(1)}(t) \\ \beta^{R(2)}(t) \\ \vdots \\ \beta^{R(K'(t))}(t) \end{bmatrix} \quad (26)$$

The rule actions $\sigma_k^{R(i)}(t)$, i=1,2,…, $K'(t)$ and k=1,2,…,Q can be collected together to form the rule action vector, given as

$$\boldsymbol{\Sigma}^{R(i)}(t)=\begin{bmatrix}\sigma_1^{R(i)}(t)\\ \sigma_2^{R(i)}(t)\\ \vdots\\ \sigma_Q^{R(i)}(t)\end{bmatrix}=A^{R(i)}H_a(t) \qquad (27)$$

where $A^{R(i)}$ and $\boldsymbol{\Sigma}^{R(i)}(t)$ are defined using Eqs.(5) and (6). All rule action vectors $\boldsymbol{\Sigma}^{R(i)}$, i=1,2,…, $K'(t)$, are collected together to have the following matrix

$$\boldsymbol{\Sigma}^H(t)=[\boldsymbol{\Sigma}^{R(1)}(t)\ \ \boldsymbol{\Sigma}^{R(2)}(t)\ \ldots \boldsymbol{\Sigma}^{R(K'(t))}(t)] \qquad (28)$$

The output of the proposed FIS at time t is expressed as the product of the rule action matrix $\boldsymbol{\Sigma}^H(t)$ and the normalized firing strength vector $\boldsymbol{\lambda}^H(t)$, given as

$$Z^H(t)=\begin{bmatrix}z_1(t)\\ z_2(t)\\ \vdots\\ z_Q(t)\end{bmatrix}=\Sigma^H(t)\lambda^H(t). \qquad (29)$$

Alternatively, the FIS output $Z(t)$ can be expressed explicitly in terms of the crisp inputs, given as

$$Z^H(t)=[\boldsymbol{\Sigma}^{R(1)}(t)\,\boldsymbol{\Sigma}^{R(2)}(t)\ldots\boldsymbol{\Sigma}^{R(K'(t))}(t)]\times\frac{1}{\sum_{i=1}^{K'(t)}\beta^{R(i)}(t)}\begin{bmatrix}\beta^{R(1)}(t)\\ \beta^{R(2)}(t)\\ \vdots\\ \beta^{R(K'(t))}(t)\end{bmatrix}$$

$$=\frac{\left[A^{R(1)}H_a(t)\quad A^{R(2)}H_a(t)\quad\cdots\quad A^{R(K'(t))}H_a(t)\right]}{\sum_{i=1}^{K'(t)}\wedge(\mu^{R(i)}(H(t)))}\begin{bmatrix}\wedge(\mu^{R(1)}(H(t)))\\ \wedge(\mu^{R(2)}(H(t)))\\ \vdots\\ \wedge(\mu^{R(K'(t))}(H(t)))\end{bmatrix} \qquad (30)$$

Note that, compared Eq.(30) to Eq.(15), the dimensions of the matrix $\boldsymbol{\Sigma}^H(t)$ and the normalized firing strength vector $\boldsymbol{\lambda}^H(t)$ are much less than those of $\boldsymbol{\Sigma}(t)$ and $\boldsymbol{\lambda}(t)$, because the $K'(t)$ is much smaller than K. This indicates that much computational resource is saved and that the size of the knowledge base of the proposed system is much less than that of the conventional FIS. The knowledge bas of the proposed system is realized around the input event $H(t)$, whose size and structure both are compact and best-fit for the incoming event $H(t)$.

Neuro-Fuzzy Structure to Implement the Proposed Event-triggering Based NFS

The layered structure of neuro-fuzzy system is suitable to implement the proposed idea of event-triggering based FIS because initially there is no rule constructed

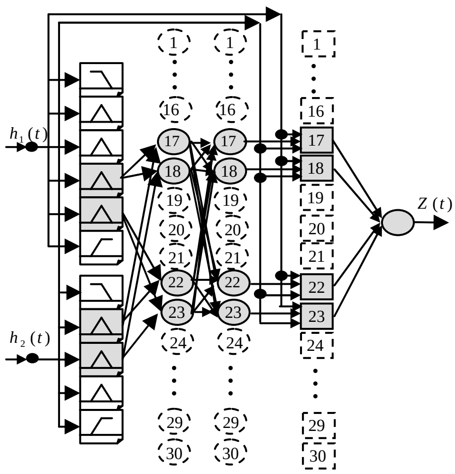

Fig. 2. Event-triggering based neuro-fuzzy system (NFS) with two inputs.

physically, and the layered structure provides excellent specification for how the proposed system is realized. There are six layers used in the neruo-fuzzy system shown in Fig.2. The explanation for each layer of the proposed event-triggering based NFS is given as follows.

Layer 0:

The nodes in the layer receive the components of the event input vector $H(t)$ and then directly send them to the corresponding nodes in layer 1. A linear function is used as activation function. The net inputs and node outputs are given as follows.

$$^0n_i = h_i(t)$$

$$^0O_i = {}^0f_i\,(^0n_i) = {}^0n_i$$

for i=1,2,…,M, where $^0f_i\,(\cdot)$ indicates the activation function for the i-th node of layer 0, ^0n_i the net input, and ^0O_i the output of the i-th node.

Layer 1:

The fuzzy matching process is performed in the layer. The nodes in the layer receive corresponding outputs of nodes from layer 0 to calculate the membership degrees of the fuzzy sets in the input space for the incoming input event $H(t)$. The node outputs are membership degrees, as given in Eqs.(16) and (17). The valuations of the membership degrees greater than a threshold ε are detected for the qualified fuzzy sets to participate in the construction of the proposed neuro-fuzzy system, as specified in Eq.(18). The event-triggering based cardinality vector, as shown in Eq.(19), for the qualified fuzzy sets are to be used to the event-triggering based knowledge base. Only the nodes of qualified fuzzy sets are allowed to send the node outputs to next layer. The net inputs and node outputs of the layer are given as follows.

$$ {}^1n_j = {}^0O_i $$

$$ {}^1O_j = {}^1f_j\,({}^1n_j) $$

for i=1,2,…,M and j= $\sum_{q=1}^{i} c_q - c_1 + s(i)$ and $s(i)$=1,2,…,c_i, where ${}^1f_j(\cdot)$ is the activation function for the i-th node of layer 0 and is a membership function.

Layer 2:

The construction of the event-triggering based knowledge base is begun in the layer. Based on the qualified fuzzy sets detected in each universe, the fuzzy rules are constructed. Each input linguistic variable provides only one qualified fuzzy set to a rule formation, by which the antecedent of the rule is set up. The process of rules creation is a permutation of the qualified fuzzy sets, as shown in Eqs. (21) to (23). The node outputs are firing strengths of fuzzy rules. The net inputs and node outputs of the layer are given as follows.

$$ {}^2n_k = ({}^2\alpha_{k,1}, {}^2\alpha_{k,2}, ..., {}^2\alpha_{k,M}) $$

$$ {}^2O_k = {}^2f_k\,({}^2n_k) = \wedge({}^2\alpha_{k,1}, {}^2\alpha_{k,2}, ..., {}^2\alpha_{k,M}) $$

for k=1,2,…, $K'(t)$, where ${}^2\alpha_{k,j}$ is the j-th input from layer 1 to the k-th nodes of layer 2 and $\wedge$ is a t-norm operator to perform "*fuzzy-and*" operation.

Layer 3:

The normalized firing strengths of the event-based fuzzy rules are performed in the layer, as shown in Eqs. (25) and (26). The number of rules involved in the fuzzy inference is dependent on the incoming input event $H(t)$, as shown in Eq.(20). The net inputs and node outputs of the layer are given as follows.

$$ {}^3n_l = ({}^3O_1, {}^3O_2, ..., {}^3O_{K'(t)}) $$

$$ {}^3O_l = {}^3f_l\,({}^3n_l) = \frac{{}^2O_l}{\sum_{k=1}^{K'(t)} {}^2O_k} $$

for l=1,2,…, $K'(t)$.

Layer 4:

The normalized consequents of the event-based fuzzy rules are performed in the layer. The Takagi and Sugeno method is used to quantify a fuzzy control rule, whose consequent is a polynomial function of the components of the incoming input event $H(t)$. Defuzzification process is not necessary. Each node in this layer receives an output from a corresponding node in layer 3 and the outputs of all nodes in layer 0. The nodes in this layer must be capable of representing M-input-Q-output fuzzy rules, as given in Eq.(4). To cope with Q outputs, Q reception sub-functions with Q corresponding activation sub-functions are integrated into nodes in the layer, called *super nodes*. Each sub-net-input of a super node given below is a set of two terms, a normalized firing strength and a linear combination of $h_j(t)$, j=1,2,…,M.

$$ {}^4n_{ki} = ([w_{ki0} + \sum_{j=1}^{M} w_{kij} h_j(t)],\ {}^3O_i\ ) $$

for i=1,2,…, $K'(t)$ and k=1,2,…,Q, where ${}^4n_{ki}$ is the sub-net-input of the k-th activation sub-function in the i-th super node of layer 4, w_{kij} the connection strength from the j-th node of layer 0 to the i-th super node of layer 4 for the k-th reception sub-function, and w_{ki0} a weight for an extra input to the k-th reception sub-function. Compared with the coefficients of the consequent $a^i_{k,j}$ in Eq.(4), the weights are given by $w_{kij} = a^i_{k,j}$. A product function is used as activation function for the super nodes. The node sub-output from the i-th activation sub-function of the j-th super node is given by

$$ {}^4O_{ki} = {}^4f_{ki}\,({}^4n_{ki}) = {}^3O_i\ [w_{ki0} + \sum_{j=1}^{M} w_{kij} h_j(t)] $$

for i=1,2,…, $K'(t)$ and k=1,2,…,Q.

Layer 5:

The outputs of the event-triggering based NFS are summarized in the layer, as shown in Eq. (30). The net-inputs and outputs of nodes in the layer are given as follows.

$$ {}^5n_i = \sum_{j=1}^{K'(t)} {}^4O_{ij} $$

$$ {}^5O_i = {}^5f_i\ ({}^5n_i) = {}^5n_i $$

for i=1,2,…,Q.

4. Example Demonstration for the Proposed Soft-Computing System

In this section, an example is used to demonstrate how the proposed event-triggering based NFS functions. A two-input-one-output event-triggering based NFS is given for demonstration purpose, although the proposed NFS can be extended to be an M-input-Q-output system. Assume that the two input universes of discourse are partitioned into six and five intervals overlapping each other, respectively. The corresponding cardinality vector for the two input linguistic variables is denoted as follows.

$$ C = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}. $$

Note that for a conventional FIS the number of fuzzy rules will be thirty in the knowledge base, but for the proposed system the rules needed will be reduced significantly.
With the definition in Eq.(16), suppose that the membership function sets of the two input variables for the incoming event $H(t)=[h_1(t)\ h_2(t)]^T=[0.89\ 1.37]^T$ at time t, shown in Fig. 3, are given as follows.

$$\mu_1(h_1(t)) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.76 \\ 0.23 \\ 0 \end{bmatrix} \text{ and } \mu_2(h_2(t)) = \begin{bmatrix} 0 \\ 0.8 \\ 0.3 \\ 0 \\ 0 \end{bmatrix}.$$

As defined in Eqs.(18) and (19), the qualified fuzzy sets detected with the threshold ε=0.1 and the event-triggering cardinality vector $C'(t)$ are given as follows.

$$N_1(t) = \begin{bmatrix} n_{1,1} \\ n_{1,2} \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix},$$

$$N_2(t) = \begin{bmatrix} n_{2,1} \\ n_{2,2} \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix},$$

and

$$C'(t) = \begin{bmatrix} c_1'(t) \\ c_2'(t) \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}.$$

What this means is that only 4 fuzzy rules are involved in construction of the grid-type NFS, $K'(t) = c_1'(t) \times c_2'(t)$ =4, and that the second and third fuzzy sets in the first input universe and the fourth and fifth fuzzy sets in the second input universe are qualified. Up to this point, the calculation corresponds to the operation of layer 1 of the event-triggering based NFS.
With the definition of rule constructor G^i defined in Eqs.(21) to (22), the four rule constructors are calculated as follows.

$$G^1 = \begin{bmatrix} n_{1,1} \\ n_{2,1} \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix},$$

$$G^2 = \begin{bmatrix} n_{1,1} \\ n_{2,2} \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix},$$

$$G^3 = \begin{bmatrix} n_{1,2} \\ n_{2,1} \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix},$$

and

$$G^4 = \begin{bmatrix} n_{1,2} \\ n_{2,2} \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}.$$

These rule constructors define the antecedents of the four event-triggering based fuzzy rules, as shown in Fig. 4. The corresponding node numbers in layer 2 are determined using Eq.(23) and they are calculated as follows.

$R(1)$=17, $R(2)$=18, $R(3)$=22, and $R(4)$=23

which are the numbers of event-triggering based rules when the event $H(t)$ is inputted at time t. The four fuzzy rules are corresponding to nodes 17, 18, 22, and 23 in layer 2. If the *min* operator is used for t-norm operation, then the outputs of the four nodes, as the firing strengths of the rules, are calculated as

β 17(t)=min(0.76, 0.8)=0.76,
β 18(t)=min(0.76, 0.3)=0.3
β 22(t)=min(0.23, 0.8)=0.23,
and
β 23(t)=*min*(0.23, 0.3)=0.23.

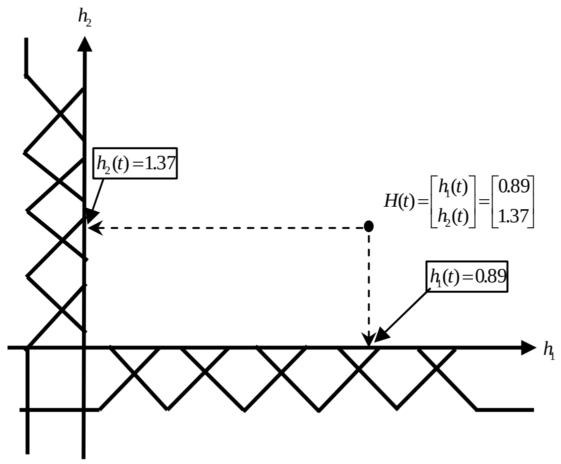

Fig. 3. Calculation of membership degree sets for the two input universe of discourse when the incoming event $H(t)=[0.89\ 1.37]^T$.

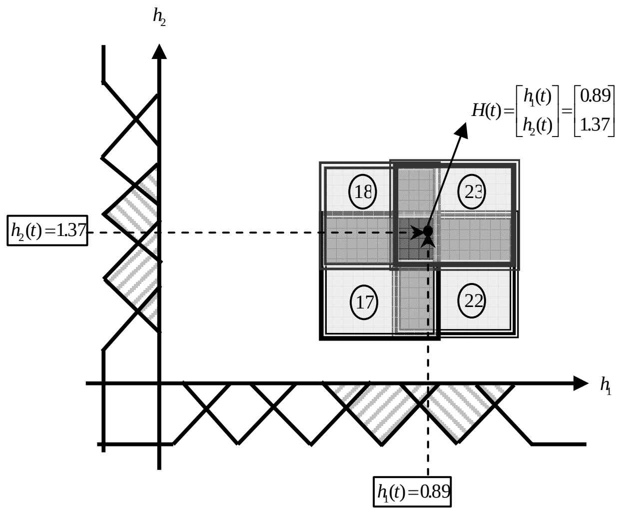

Fig. 4. Four event-triggering based fuzzy rules for the proposed soft-computing system.

The normalized firing strength vector defined in Eq.(26) $\boldsymbol{\lambda}^H(t)$ is obtained as follows.

$$\lambda^H(t) = \begin{bmatrix} \lambda^{R(1)}(t) \\ \lambda^{R(2)}(t) \\ \lambda^{R(3)}(t) \\ \lambda^{R(4)}(t) \end{bmatrix} = \begin{bmatrix} \lambda^{17}(t) \\ \lambda^{18}(t) \\ \lambda^{22}(t) \\ \lambda^{23}(t) \end{bmatrix}$$

$$= \frac{1}{\sum_{i=1}^{4} \beta^{R(i)}(t)} \begin{bmatrix} \beta^{17}(t) \\ \beta^{18}(t) \\ \beta^{22}(t) \\ \beta^{23}(t) \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.1973 \\ 0.1513 \\ 0.1513 \end{bmatrix}$$

This calculation is corresponding to the operation of layer 3 of the NFS. The outputs of nodes 17, 18, 22, and 23 are interpreted as the normalized firing strengths for the event-triggering based fuzzy rules.

Assume that the coefficients of the consequents of the four rules, defined in Eq.(5), are given as follows.

$A^{17}=\begin{bmatrix} a_0^{17} & a_1^{17} & a_2^{17} \end{bmatrix}$=[1 2 3],

$A^{18}=\begin{bmatrix} a_0^{18} & a_1^{18} & a_2^{18} \end{bmatrix}$=[4 5 6],

$A^{22}=\begin{bmatrix} a_0^{22} & a_1^{22} & a_2^{22} \end{bmatrix}$=[7 8 9],

and

$A^{23}=\begin{bmatrix} a_0^{23} & a_1^{23} & a_2^{23} \end{bmatrix}$=[10 11 12],

With Eqs.(27) and (29), the corresponding four rule actions are expressed as follows.

$$\Sigma^{17}(t) = A^{17} H_a(t) = [1\ 2\ 3]\begin{bmatrix} 1 \\ 0.89 \\ 1.37 \end{bmatrix} = 6.89,$$

$$\Sigma^{18}(t) = A^{18} H_a(t) = [4\ 5\ 6]\begin{bmatrix} 1 \\ 0.89 \\ 1.37 \end{bmatrix} = 18.829,$$

$$\Sigma^{22}(t) = A^{22} H_a(t) = [7\ 8\ 9]\begin{bmatrix} 1 \\ 0.89 \\ 1.37 \end{bmatrix} = 29.689,$$

and

$$\Sigma^{23}(t) = A^{23} H_a(t) = [10\ 11\ 12]\begin{bmatrix} 1 \\ 0.89 \\ 1.37 \end{bmatrix} = 40.546.$$

The outputs of the four rules are corresponding to the operation of layer 4 of the NFS. Each rule output is calculated by its corresponding node in layer 4, in which a normalized firing strength from layer 3 and a rule action from its net input information are multiplied together. The four rule outputs are obtained as follows.

$u^{17}(t) = \Sigma^{17}(t)\lambda^{17}(t) = 6.89 \times 0.5 = 3.445$,

$u^{18}(t) = \Sigma^{18}(t)\lambda^{18}(t) = 18.829 \times 0.1973 = 3.715$,

$u^{22}(t) = \Sigma^{22}(t)\lambda^{22}(t) = \times 0.1513 = 4.492$,

and

$u^{23}(t) = \Sigma^{23}(t)\lambda^{23}(t) = 40.546 \times 0.1513 = 6.135$.

The output of event-triggering based NFS, defined in Eq.(30), is calculated as follows.

$$Z^H(t) = [\Sigma^{17}(t)\ \Sigma^{18}(t)\ \Sigma^{22}(t)\ \Sigma^{23}(t)]\begin{bmatrix} \lambda^{17}(t) \\ \lambda^{18}(t) \\ \lambda^{22}(t) \\ \lambda^{23}(t) \end{bmatrix}$$

$$= [6.89\ 18.829\ 29.689\ 40.546]\begin{bmatrix} 0.5 \\ 0.1973 \\ 0.1513 \\ 0.1513 \end{bmatrix}$$

$$= 17.787$$

This calculation for the output of the proposed two-input-one-output soft-computing system is summarized in layer 5 of the NFS.

The event-triggering based NFS of the example demonstration is shown in Fig. 2, with which the idea of the proposed event-triggering based soft-computing system is illustrated very clearly. As one can observe in Fig. 4, the 4 rules constructed for the proposed NFS are closely around the incoming input event $H(t)$. The knowledge base of the event-triggering based NFS is much smaller than that of the corresponding conventional grid-type FIS, and is always related to the event $H(t)$.

5. Discussions

In this section, several properties for the proposed soft-computing system are discussed, which are the reduction of computational resource, the maximal size of knowledge base, the event-tracking and compact structure of the system, the preservation of completeness property, the suitability for large-scale system operation, and the overcoming of curse of dimensionality. These properties are coupling together.

As discussed in the section of example demonstration, the proposed soft-computing system needs much less computational resource than the corresponding conventional FIS. In the example given, the conventional FIS needs the computation operations of 209 additions, 150 multiplications, 61 divisions, and 210 comparisons to finish the fuzzy inference, while the proposed soft-computing system with 4 rules fired needs only 30 additions, 12 multiplications, 12 divisions, and 32 comparisons. More comparisons of computation operations are summarized in Tables 1 and 2.

The size of knowledge base of the proposed soft-computing system is dependent on both the incoming input event $H(t)$ and the overlapping of fuzzy sets. The overlapping of fuzzy sets in an input universe may decide the maximum number of qualified fuzzy sets for that corresponding linguistic variable to participate in construction of knowledge base, while the incoming event $H(t)$ decides the position at which membership degrees are calculated in the corresponding input universe. For a grid type partitioning of the input space, the number of event-triggering based fuzzy rules are determined using Eq.(20). Because the incoming input event $H(t)$ decides the position in the input space around which fuzzy sets with membership degree beyond a threshold are detected and are used to construct the fuzzy rules closely to the $H(t)$, the fuzzy rules involved in the knowledge base can be very compact. In other words, the event-triggering based knowledge base is effective, in which there are no redundant fuzzy rules and only the needed rules are in the proposed system for the $H(t)$. Moreover, the proposed soft-computing system possesses the property of event-tracking structure. Thanks to the properties specified, the proposed soft-computing system is suitable for large-scale system operation.

The property of completeness for the proposed soft-computing system is always preserved as long as the corresponding conventional FIS possesses the property that the partitions cover everywhere in all input universes and all input conditions are cared with. The property of completeness is inherently dependent on the partitions of input space. The proposed soft-computing system preserves the excellent property of completeness, and yet avoids the curse of dimensionality, for the knowledge base is compact and is constructed around the event $H(t)$ only. The proposed soft-computing system has the structure of event-tracking knowledge base and time-varying property.

Table 1: Comparison of computation for the proposed soft-computing system and the corresponding conventional FIS (2 inputs and 1 output).

	FIS	Proposed event-triggering based NFS		
Fuzzy Rules in Operation	30	4 ($c_1'=2$ and $c_2'=2$)	2 ($c_1'=2$ and $c_2'=1$)	1 ($c_1'=1$ and $c_2'=1$)
Comparison	210	32	30	29
Addition	209	30	23	18
Multiplication	150	12	6	3
Division	61	12	10	8
* System with input cardinalities of fuzzy sets $c_1=6$ and $c_2=5$.				

Table 2: Comparison of computation for the proposed soft-computing system and the corresponding conventional FIS (3 inputs and 1 output).

	FIS	Proposed event-triggering based NFS		
Fuzzy Rules in Operation	150	8 ($c_1'=2$, $c_2'=2$ and $c_3'=2$)	4 ($c_1'=2$, $c_2'=1$ and $c_3'=2$)	2 ($c_1'=2$, $c_2'=1$ and $c_3'=1$)
Comparison	1500	46	42	40
Addition	1499	52	36	28
Multiplication	1050	24	12	6
Division	451	19	15	13
* System with input cardinalities of fuzzy sets $c_1=6$, $c_2=5$ and $c_3=5$.				

6. Conclusions

An event-triggering based soft computing system is proposed in the paper to alleviate the paradox of the problem of curse of dimensionality (COD) and to preserve the completeness property. The proposed soft computing system has been specified in detail. An example is given for demonstration of the proposed approach. The proposed soft-computing system possesses the properties of the reduction of computational resource, the event-tracking and compact structure of knowledge base, the preservation of completeness property, the suitability for large-scale system operation, and alleviation of COD paradox, although these properties are coupling together.

Acknowledgments

The author is grateful to Ms Y.C. Chen for the help in the preparation of the paper.

References

[1] Debrup Chakraborty and Nikhil R. Pal, "Integrated feature analysis and fuzzy rule-based system identification in a neuro-fuzzy paradigm," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 31, no.3, pp. 391-400, June 2001.

[2] J-S R. Jang, "ANFIS: adaptive-network-based fuzzy inference system," *IEEE Transactions on Systems, Man and Cybernetics*, vol. 23, no.3, pp. 665-685, May/June 1993.

[3] J-S R. Jang, "Self-learning fuzzy controllers based on temporal back propagation," *IEEE Transactions on Systems, Man, and Cybernetics,* vol. 3, no.5, pp. 714-723, September 1992.

[4] J-S R. Jang and C. -T. Sun, "Functional equivalence between radial basis function networks and fuzzy inference systems," *IEEE Transactions on Neural Networks*, vol. 4, no. 1, pp. 156-159, January 1993.

[5] J-S R. Jang, C. -T. Sun and E. Mizutani, *Neuro-Fuzzy and Soft-Computing*, Prentice Hall, pp. 86-89, 1997.
[6] Chia-Feng Juang and Chin-Teng Lin, "An on-line self-constructing neural fuzzy inference network and its applications," *IEEE Transactions on Fuzzy Systems*, vol.6, no. 1, pp. 12-31, February 1998.
[7] N.K. Kasabov and Qun Song, "DENFIS: dynamic evolving neural-fuzzy inference system and its application for time-series prediction," *IEEE Transactions on Fuzzy Systems*, vol. 10, no.2, pp. 144 –154, Apr., 2002.
[8] C. C. Lee, "Fuzzy logic in control systems: fuzzy logic controller—Part I," *IEEE Transactions on Systems, Man and Cybernetics*, vol. 20, no. 2, pp. 404–418, Mar./Apr., 1990.
[9] C. C. Lee, "Fuzzy logic in control systems: fuzzy logic controller—Part II," *IEEE Transactions on Systems, Man and Cybernetics*, vol. 20, no. 2, pp. 419–435, Mar./Apr., 1990.
[10] Chunshien Li and R. Priemer, "Fuzzy control of unknown multiple-input-multiple-output plants," *Fuzzy Sets and Systems*, vol. 104, no. 2, pp. 245–267, 1999.
[11] Chunshien Li and Chun-Yi Lee, "Fuzzy motion control of and auto-warehousing crane system," *IEEE Transactions on Industrial Electronics*, vol.48, no.5, pp.983-944, October 2001.
[12] Chunshien Li and R. Priemer, "Self-learning general purpose PID controller," *Journal of the Franklin Institute*, vol. 334B, no.2, pp. 167-189, 1997.
[13] Chunshien Li and Chun-Yi Lee, "Soft computing system for motion control," *IEEE Intelligent Transportation Systems Conference Proceedings*, Oakland (CA), USA, pp. 364-369, August 2001.
[14] C. T. Lin and C.S.G. Lee, "Neural-network-based fuzzy logic control and decision system," *IEEE Transaction on Computers*, vol. 40, no. 12, pp. 1320-1336, Dec., 1991.
[15] R. P. Lippmann, "An introduction to computing with neural nets," *IEEE ASSP Magazine*, pp. 4-22, 1987.
[16] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its application to modeling and control," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. SMC-15, no. 1, pp. 116-132, January/February 1985.
[17] L. X. Wang, "Fuzzy systems are universal approximators," *IEEE International Conference on Fuzzy Systems*, pp. 1163-1169, Mar., 1992.
[18] Shiqian Wu and Meng Joo Er, "Dynamic fuzzy neural networks- a novel approach to function approximation," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 30,no. 2, pp. 358-364, April 2000.
[19] Jeen-Shing Wang; C.S.G. Lee, "Self-adaptive neuro-fuzzy inference systems for classification applications," *IEEE Transactions on Fuzzy Systems*, vol. 10, no.6, pp. 790 –802, Dec., 2002.
[20] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, pp. 338-353, 1965.
[21] L. A. Zadeh, "Fuzzy algorithm," *Information and Control*, vol. 12, pp. 94-102, 1968.
[22] L. A. Zadeh, "Fuzzy logic and approximate reasoning," *Syntheses*, vol. 30, pp. 407-428, 1975.

Chunshien Li received the Ph.D. degree in Electrical Engineering and Computer Science in 1996 from University of Illinois at Chicago, USA. He is currently with Department of Computer Science and Information Engineering, National University of Tainan, Taiwan. He was with the Department of Electrical Engineering, Chang Gung University, Tao-Yuan, Taiwan. He received the Research Award from the National Science Council, Taiwan, in 1999 and the Research Award of Excellent Teacher from Chang Gung University, Taiwan, in 2000. He has been the first author of more than 60 technical papers. He is biographized in Marquis Who'sWho in Science and Engineering, in Medicine and Healthcare, and in the World. He has served as a referee of paper review for the journals of *IEEE Transactions on Industrial Electronics*, *IEEE Transactions on Systems, Man and Cybernetics-Part B: Cybernetic*, *IEEE Transactions on Fuzzy Systems*, *Fuzzy Sets and Systems*, and other international journals. His current research interests include soft computing, computational intelligence, neuro-fuzzy systems, pattern recognition, intelligent signal processing, intelligent systems and control, machine learning, and chip/processor-based real-time systems.