Processing with networks and group selection

Zajac Jaroslav

Department of Economics and Management, FEI, Slovak University of Technology, Ilkovicova 3, 81219 Bratislava, Slovakia

Summary
We analyze how the decisions of agents contribute to network formation. Incentives promote effort and performance, and there is a lot of evidence that they do. Given that incentives work quite effectively in many instances, and the agent who faces uncertainty about his payoff from taking a particular action. The agent will undertake the task only if he has sufficient confidence in his own ability to succeed, and in the project's net return. Agents with a stake in his performance have incentives to manipulate signals relevant to his self-knowledge. We model the agent as an information processing network that is capable of learning a data set of environmental variables.

Key words:
hierarchy, organization and learning, random walk

1. Introduction

We model agent of different organizational structures competing given that they have to learn the effect of changing environmental states on the demand parameters. We model the structure of the agent as the size of the network, given by the number of processing units, that agents face a trade-off between speed and accuracy. Over time, agents learn to perform the mapping between environmental characteristics and optimal decisions. Given competition between agents, small firms networks reach relatively quickly a satisfactory knowledge of the function linking environmental factors and demand. And larger firms, initially slower to learn, tend in the long run to outperform the small ones by becoming more accurate in their mapping. We shows that an equilibrium configuration may be found and that it is also related to the complexity of the environment. The agents have to learn how to react to their opponents' behavior. The convergence to the Walrasian prices and quantities is more probable when learning takes place, and agents are bounded-ly rational. Learning is in regards to the opponent's strategy, or the firm's own influence on prices, information about the environment is unnecessary for learning to take place. If demand is mis-specified, then even best reply dynamics may converge to steady states different from the unique Nash outcome. The directed networks end up having different incentive properties as agents can unilaterally form new links, whereas here we need to consider the incentives. Focus as a way to identify equilibrium which is quite different from the stochastic dynamic, and examines which strategies players play in a game when the set of opponents that a player might face depends on the structure. The agents may have such limited information in large network settings. Although a number of networks may be pair-wise stable, they can differ in how resilient they are to random mutations.

Information processing has to be decentralized among many agents. It can depend on broader information about the organization's strategy. Organizational decision making, in which information processing is communicated along hierarchical lines. Aggregation entails a loss of useful information, in the sense that when agents summarize their information to their hierarchical superiors. Agents' decision problems interact, in the sense that an agent's optimal decision should depend on information held by agents in other parts of the organization. We determine what hierarchical structure an organization should adopt, and to what hierarchical level the returns to employing agents who are better able to process information are the highest. They follow a multi-stage hierarchical procedure, to the security selection, combinations os assets in each of the several groups, and involves the determination of an appropriate combination of the group portfolios and an appropriate combination of the group portfolios. The final stage is devoted to asset allocation, using the asset portfolios as asset-class portfolios. Decisions are made myopically, considering a subset of the available assets. In this hierarchical procedure, agent's decision problems obviously interact. To solve the organization design problem, we must make an assumption on the probability distribution of factor as of the design stage. Given the optimal decision rules, we can evaluate the performance of different hierarchical structure, and all agents have subordinate. With missing markets, earnings differentials in steady state must overcompensate for training cost margin, yielding higher levels of earnings net of costs in occupations with higher gross earnings. We provide a broad set of conditions under which there is a unique steady state distribution. This

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condition characterizes efficient steady states in a rate of return to all occupations.
The long-run rate of an individual is weakly increasing in the initial status, number of social ties and human capital of any other member of his of her group and strictly increasing in her own initial status. The group-level relationship between human capital and long-run rates. We described the relationship between network itself and the long-run behavior of the rate. Small groups without ties to the rest of society are at risk. The structure of the ties within the group matter.
The structure of the paper is as follows. Section 2 shows that in these problems, the set of stochastically stable network links coincides with the set of core stable networks, which are necessarily Pareto efficient. Section 3 characterizes informational connections. Section 4 builds a framework of hierarchical group selection.. Section 5 presents information strategies.

2. Network links

The network relations among the players are represented by graphs. Jackson and Watts (2002) focus in non-directed networks where links are reciprocal. Consider a population of agents, each of them having a certain level a_i. The utility π_i obtained by the agent in the interaction with market The agents live in dimensional chain with periodic boundary conditions and they can interact with their nearest-neighbors only. Each network in the sequence differs by one link from the previous network. If a link is deleted, then it must be that at least one of the two agents involved in the link strictly benefit from its deletion. An improving path from a network *g* to a network g' are a finite sequence of networks $g_1,.....,g_K$ with $g_1 = g$ and $g_K = g'$ such that for any $k \in \{1,....,K-1\}$ either:
$g_{k+1} = g_k - ij$ for some *i* *j* such that $Y_i(g_k - ij) > Y_i(g_k)$, or
$g_{k+1} = g_k + ij$ for some *i* *j* such that $Y_i(g_k + ij) > Y_i(g_k)$ and $Y_j(g_k + ij) \geq Y_j(g_k)$.
The behavior implicit in an improving path may be m turn leaves the first agent worse off relative to the starting position. However, in larger networks and networks where agent's information might be limited, myopic behavior in our setting all a agent needs to know is whether adding or deleting a given link is directly beneficial to him in the current circumstances. The improving paths from any starting network lead either to a stable network or to a cycle.
A set of networks *C*, form a cycle if for any $g \in C$ and $g' \in C$ there exists an path connecting *g* to g'. A cycle *C* is a closed cycle if no network in *C* lies on an improving path leading to a network that is not in *C*. A closed cycle is necessarily a maximal cycle.
For any *v* and *Y* there exists at least one pair-wise stable network or closed cycle of networks.
A network is pair-wise stable if it does not lie on an improving path to any other network. Given the finite number of possible networks, either the improving path ends at some network which has no improving paths leaving it, which then must be stable, or it can be continued through each network is hits. The improving path must form a cycle. Since there must exist a cycle, given the finite number of networks there must exists a maximal cycle, then there would be a larger cycle, there exists a closed cycle. Agents benefit from trading with other agents with whom they are linked, and trade can only flow along links. Agents begin by forming a network, they receive random endowments and trade along chains of the network. The expected utility for a agent of being in a given network is calculated by expecting over the Walrasian equilibrium that result in the agent's connected component as a function of realized endowments. Each agent has a random endowment, which is independently and identically distributed. A agent's' endowment is either $(1,0)$ and $(0,1)$, each with probability $1/2$, realized after the network is in place. For a given network, Walrasian equilibrium occur on each connected component, regardless of the configuration of links. The expected utility of a agent is strongly increasing and a concave in the number of other agents that she is directly or indirectly connected to, ignoring the cost of links. Accounting for the cost of a link, if *k* agents are in connected component of a pair-wise stable network, then thee must be exactly $k-1$ links, then there is at least one link that can be severed without changing the component structure of the network. So some agent can sever a link and save the cost of the link without losing any everyone else, loses in expected utility by severing the link.
At s set of times $\{1,2,3,.......\}$ decisions to add or sever a link are made. At each time a pair of agents *ij* is randomly identified with probability $p(ij) > 0$. The potential link between these agents is only link that can be altered at that time. One may think a random meeting process where agents randomly bump into each other and time id identified. If the link is already in the network, then the decision is whether to sever it, and otherwise the decision is whether to add the link. The agents involved act myopically, adding the link if it makes at least as well off and one strictly better off. The above process defines a Markov chain with states being the network in place at the end of period. The stationary distribution converges to a unique limiting stationary distribution, after the action is

taken, there is some small probability $\varepsilon > 0$ that a mutation occurs and the link is deleted if it is present, and added if it is absent.
A network that is in support of the limiting stationary distribution of the above –described Markov process is stochastically stable.
A path $p = \{g_1,....., g_K\}$ is a sequence of adjacent networks. The resistance of a path $p = \{g_1,......, g_K\}$ from g' *to* g, denoted $r(p)$, is computed by $r(p) = \sum_1^{K-1} I(g_i, g_{i+1})$, where $I(g_i, g_{i+1}) = 0$.
A mutation is necessary to move from one network to an adjacent one whenever it is not in the relevant agent's interests to sever or add the link .Let $r(g', g) = \min\{r(p) | p \text{ is path from } g' \text{ to } g\}$. Thus, if g' and *g* are in the same cycle, then $r(g', g) = 0$.
Given a network *g*, a *g*-tree is a graph which has as vertices all networks and has a unique path leading from each g' to *g*. Let $T(g)$ denote all the *g*-trees, so that $g'g'' \in t$ if there is a edge g,C
The resistance of a network g is computed as $r(g) = \min_{t \in T(g)} \sum_{g'g'' \in t} r(g', g'')$.
The stochastically stable network is the set $\{g | r(g) \le r(g') \text{ for all } g'\}$, and the set of stochastically stable networks is always nonempty as we are taking a minimum over a finite set. Thus , if *g* is stochastically stable, then either *g* is pair-wise stable or part a closed cycle. If one network in a closed cycle is stochastically stable then all networks in the closed cycle are stochastically stable. A closed cycle *C* and a network *g*, let $r(C, g) = r(g', g)$ is any network in *C* and $r(g, C) = r(g, g')$ where g' in *C*. A network *g*, restricted *g*-tree is a graph. A network *g* is core stable if there is no group of agents who each prefer network g' to *g* and who can change the network from *g* to g' without the cooperation of the remaining agents. A network $g \in G$ is core stable if there does not exist any set of agents *A* and $g' \in G$ such that:

$Y_i(g') \ge Y_i(g)$ for all $i \in A$, (1)

if $ij \in g'$ but $ij \notin g$, then $i \in A$ and $j \in A$, and (2)

if $ij \notin g'$, but $ij \in g$, then either $i \in A$, and/or $j \in A$. (3)

A simultaneous path, is a sequence of networks $g_0,....., g_K$ in *G* such that if g' follows *g* in the sequence then either:

$g' = g - ij$ and either $Y_i(g') > Y_i(g)$ or $Y_j(g') > Y_j(g)$, or (4)

$g' \in G$ and $g' \in \{g + ij - ik, g + ij - ik - jm, g + ij, g + ij - jm\}$ (5)

where $ij \notin g$ and $Y_i(g') \ge Y_i(g)$ and $Y_j(g') \ge Y_j(g)$. (6)

Improving paths are a subset of simultaneous improving paths. Here the simultaneity refers to the fact that a agent may both sever an existing link and add a new one. Cycles can exist with the notion of simultaneous improving path.
At each time a agents is randomly identified. If the link is already in the network, then the decision is whether to sever it. Their actions are constrained to lead to a feasible *g* in *G*. The agents involved act myopically, adding the link if it makes each at least as well off, and severing the link of its deletion makes either agent better off. After the action is taken, there is some small probability that a tremble occurs and the link is deleted.
Let $N = \{1,2,...., n\}$ be the set of agents. For any $S \subset N$, let $g^S \{T | T \subset S\}$be the set of all subsets of *S*. A network , Mutuswami and Winter (2002) denoted generically by *g*in some subset of g^N. The element $\{i, j\}$ of *g* is called the link between *i* and *j* and denoted as (ij). For every $S \subset N$, $G_S = \{g | g \subset g^S\}$ denotes the set of links only between agents of *S*., if there exists a sequence of agents $i = i_0, i_1,....., i_K = j$ such that $(i_k i_{K1}) \in g$ for all $k = 0,...., K-1$. All agents in $N \setminus N(g)$ are said to be isolated. The graph $h \subset g$ is a connected component of *g* if all agents in $N(h)$ are connected to each other in *h*. The set of all connected components of *g* is denoted as $C(g)$.
An agent's benefit from being path of a network is given by a utility function $U_i(g, x_i) = v_i(g) - x_i$, where x_i is as the cost share imputed to agent *i*. For all $i \in G_N$, and all $g, g' \in G_N$, $g \subset g'$, $v_i(g') \ge v_i(g) \ge 0$. (7)
Where the gross benefit to an agent is non-decreasing in the set of links. Thus, if $i \in N(h)$, $h \in C(g)$, then her utility can be affected by the formation of a link between

agents in $N \setminus N(h)$.

The marginal contribution of agent $k, k = 1,...,n$ is $u_k^* = sa(S_k) - sa(S_{k+1})$.

If there are externalities across components, then there is way to measure the worth of a coalition. We therefore have $sa(T) \geq \sum_{i \in T} v_i(g_S^*) \geq sa(S)$, where inequality follows g_S^* will not achieve the stand payoff for *T*. The net payoffs to the agents are given by the marginal vector $(u_1^*,.....,u_n^*)$.

If there are number of possible efficient graphs, then, they are able to pin down the equilibrium net payoffs uniquely. Suppose that agents in $\{1,....,k\}$ have announced $\{(g_i, x_i)\}_{i=1}^k$. Let $i_k \in \{1,...,k\}$ be the largest such that there exists a graph $g \in G_{S_{i_k}}$ satisfying

$$g_i \subset g \quad \text{for all } i \in \{i_k,...,k\}, \tag{8}$$

$$\sum_{j \in S_{k+1}} v_j(g) - c(g) + \sum_{j=i_k}^{k} x_j > sa(S_{k+1}), \tag{9}$$

where says that by utilizing by contributions of agents in $\{i_k,....,k\}$ the agents in S_{k1} can obtain a payoff greater than their stand-alone payoff.. Let $(u_{K+1},....,u_n)$ be the resulting net utilities of the agents following *K*. Since the maximal coalition must be a set of S_{i_K} if agents in S_{K+1} are receive a collective payoff greater than $sa(S_{K+1})$, we must have $\sum_{j=K+1}^{n} u_j \leq sa(S_{K+1})$.

The maximal compatible coalition resulting from any *j* has a profitable deviation.

Let $u_j = u_j^*$ for all $j \in \{K+1,....,n\}$.

Choose some $g_K^* \in G_K^*$. Let agent $K+1$ deviate by announcing $\left(g_K^*, x_{K+1}{}'\right)$, where

$$sa(S_{K+2}) - \sum_{i=K+2}^{n} v_i(g_K^*) + c(g_K^*) - \sum_{i=i_K}^{K} x_i < x_{K+1}{}' < v_{K+1}(g_K^*) - u_{K+1}^* \tag{10}$$

.. . The hypothesis implies that the compatible coalition must be a set of $S_{i_{K+1}}$

Since $i_{K+1} \leq K+1$, it follows that $K+1$ is a member of the compatible coalition. Let *k* deviate by announcing $\left(g_k{}', x_k{}'\right)$ where g_k' is efficient for S_k and x_k' such that

$$sa(S_{k+1}) - \sum_{k=k+1}^{n} v_j(g_k') + v(g_k') < x_k' < v_k(g_k') - u_k \quad . \tag{11}$$

It follows that the compatible coalition after *k's* deviation in a set of S_k, and will therefore, always contain *k*.

Let $(u_1,.....,u_n)$ be the net payoffs to the agents of Γ_n. The fact that an efficient forms follows from the observation that $\sum_{i=1}^{n} u_i = sa(N)$.

For any $S \subset N$, let $\bar{g}^S = \{(i,j) \in S \times S \mid i \neq j\}$ be the collection of all elements in $S \times S$ excluding those of the form (i,i). We can think of g^N as the complete directed graph on *A*, which is some subset of $\bar{g}^N$. The element (i,j) of *g* is referred to as the link from *i* to *j* and denoted by (ij). Let $\overline{G_S} = \{g \mid g \subset \bar{g}_S\}$ be the set of all graphs involving links only between members of *S*. A path from *i* to *j* in *g* is a sequence of distinct agents $\{i_0,.....,i_K\}$ such that $i_0 = i,\ i_K = j$ and $(i_k i_{k+1}) \in g$ for all $k = 0,....,K-1$. The utility function of *i* is given by $U_i(g) = v_i(g) - x_i$, where $v_i(g)$ is *i*'s gross benefit from *g* and x_i is his cost share. The coalition S is compatible with the announcement $w = \{(g_i, x_i)\}_{i=1}^n$, if for all $i \in S$, $g_i \in \overline{G_S}$.

If $\sum_{i \in S} x_i \geq c(g)$, where $g = \bigcup_{i \in S} g_i$.

The compatible coalition is in the set $\{\otimes, S_1,.....,S_n\}$.

Immunity to deviations by coalition is a desirable property of any mechanism. For all $i \in N$, for all $g, g' \in G_N$, if $h \in C(g) \cap (g')$ and $i \in N(h)$, then $v_i(g) = v_i(g')$, where says that an agent's gross utility depends on the connected component to which she belongs and is not affected by link formation outside this component.

For all $g \in G_N$, $c(g) = \sum_{h \in C(g)} c(h)$, where rules out cost externalities among connected components. The stand-alone value of a coalition is true reflection of the power of that coalition. In such a situation, the core (N, sa) represents allocations that cannot be improved upon by any coalition. Value function was the natural analogue of the characteristic function. An allocation rule could be designed so, that agents acting in their own interest would form an efficient network. Explicitly costs of link formation and also the payoffs but the approach for the most part has been to analyze games which are regarded as stylized descriptions of actual network formation processes.

3. Informational connections

Productivity varies across agents worker-firm matches, and firms have limited information on the productivity of a agents. Krauth (2004) suppose small change in neighborhood composition led to a large deterioration on employment conditions through a vicious cycle of increased unemployment and further weakening of job networks. Individuals randomly discover job vacancies, and currently networks facilitate the transmission of information on productivity.

Actual networks are unlikely to be simple, so it is important to know the impact of network structures. A worker's productivity is determined by his capital as well as a match-specific component, which is differs across worker-firm matching. Let a_i be the human capital, and $m_i^f(t)$ be the quality of the match between worker *i* and firm *f* . The worker's output at the firm is

$$y_i^f(t) = a_i m_i^f(t) n_i^f(t), \tag{12}$$

where $n_i^f(t)$ is the amount of labor supplied to firm *f*. The quality of a match between a particular worker and firm changes over time, and the random variable $m_i^f(t)$ is across workers, firms, and time, with continuous F_m :

$$F_m(x) = \Pr\left(m_i^f(t) \le x\right). \tag{13}$$

The firm can acquire more information on a worker by either observation, or referrals, and the information received is complete and cost-less. Let $E_f\left(m_i^f(t)\right)$ be the expected value of $m_i^f(t)$ based on firm *f*' s information at the beginning of period *t*.

Agents are risk and maximize current expected utility in each period. This allows analysis of the behavior of the economy for a wide class of networks, if agents do not discount the future, each job match provides current output and future matches to the firm. The number of these future matches depends on the state of the economy, and the state of the network and the identity of each agent. The efficient allocation of labor $\left\{n_i^f(t)\right\}_{f\in F}$ is defined as follows

$$y_i(t) = \max_{\left\{n_i^f(t)\ge 0\right\}}\left(\sum_f a_i E_f\left(m_i^f(t)\right)n_i^f(t)\right) + h_i\left(1-\sum_f n_i^f(t)\right). \tag{14}$$

A competitive equilibrium is a set of match –specific wages $\left\{w_i^f\right\}$ and allocations $\left\{n_i^f\right\}$ such that, the allocations solve each firm's utility maximizing problem

$$\max_{\left\{n_i^f(t)\ge 0\right\}}\left(\sum i w_i^f(t)n_i^f(t)\right) - \left(\sum_i a_i E_f\left(m_i^f(t)\right)n_i^f(t)\right), \tag{15}$$

and the worker's utility maximization problem:

$$\max_{\left\{n_i^f(t)\ge 0,\sum_f n_i^f(t)\le 1\right\}}\left(\sum_f w_i^f(t)n_i^f(t)\right) + h_i\left(1-\sum_f n_i^f(t)\right) \tag{16}$$

This implies that the worker's utility maximization problem corresponds any competitive equilibrium allocation is also an efficient allocation.

There are economic rents associated with a worker's highest-productivity match because its expected output can be higher than any alternative. The Nash bargaining solution has basic properties a match occurs if it is efficient , and each participant receives utility from a match at least to his opportunity cost, and the worker receives an fraction β of the surplus and the firm receives $1-\beta$, where β is interpreted as the worker's bargaining power. Nash bargaining corresponds to the equilibrium outcome of a wide variety of bargaining games, any bargaining process corresponding to the Nash solution will have each efficient match occurring, so the equilibrium allocation will correspond to the efficient allocation.

If binding offers and the labor market corresponds to a set of independent sealed-bid first-price, and firm's bid is increasing in $E_f\left(m_i^f\right)$ so the equilibrium allocation will correspond to the efficient allocation.

4. Group selection

We start with a population of *N* agents subdivided into groups *j*, each with n_j members. Henrich (2004) suppose variable x_j gives the frequency of the trait/allele in sub-population *j*, x_j' represents the same frequency in the next period. The average change in the frequency of the trait expresses $\Delta\bar{x}$, and $q_j = n_j / N$. The proportion of the population accounted for in group *j*, and the proportion in the next time step, q_j', means

$$\Delta\bar{x} = \sum_j q'_j x'_j - \sum q_j x_j \,. \tag{17}$$

Noting that $\Delta x_j = x'_j - x_j$ gives us

$$\Delta\bar{x} = \sum_j q'_j \left(x_j + \Delta x_j\right) - \sum_j q_j x_j \,. \tag{18}$$

We can comparing the fitness of group *j*, w_j, to the mean fitness across all groups:

$q'_j = \frac{q_j w_j}{\bar{w}}$, and we arrive at the following expression:

$$\Delta\bar{x} = \sum_j \frac{q_j w_j}{\bar{w}}\left(x_j + \Delta x_j\right) - \sum_j q_j x_j = \sum_j q_j x_j \left(\frac{w_j}{\bar{w}} - 1\right) + \sum_j q_j \left(\frac{w_j}{\bar{w}}\right)\Delta x_j \,. \tag{19}$$

If w_j, the fitness of group *j*, positively covariance $Cov\left(w_j, x_j\right)$ with the frequency of the trait in group $j\left(x_j\right)$, then will increase the average value of the trait in the population. Expectations $E\left(w_j \Delta x_j\right)$ show selection within groups. The frequency of an allele is x_{ij} in agent *i* and takes on the values of 1 (present) or 0 (absent). We have

$$\bar{w}\Delta\bar{x} = Cov\left(w_j, x_j\right) + E_j\left(Cov\left(w_{ij}, x_{ij}\right) + E\left(w_{ij}\Delta x_{ij}\right)\right), \tag{20}$$

where the first term present selection between groups. The second term present selection and transmission within groups.

Individuals live in different area, θ is a distribution of quality across individuals. Aruka (2004) suppose variance σ_θ^2, θ_i is the individual *i*' s quality, assumed to be constant across areas. X_i represents individual-level characteristics. The idiosyncratic tastes of individual *i* may then be written:

$$\Theta_i = \theta_i + f\left(X_i\right), \tag{21}$$

the marginal utility of the individual's *i* action, and his neighbor's action A_{i-1}:

$$U\left(A_i, A_{i-1}, \Theta_i\right) = \Theta_i A_i - \frac{1}{2}(1-\alpha)A_i^2 - \frac{1}{2}\alpha\left(A_i - A_{i-1}\right)^2 \,. \tag{22}$$

Agents may live in their areas *j* on the real line. The choice of action by individual *i* is based either on his taste for the action his predecessor $i-1$ on the domain. Agent *i* chooses an action A_i based on their idiosyncratic tastes Θ_i, and their predecessor's action level A_{i-1} is defined:

$$A_i = \theta_i + f\left(X_i\right) + \alpha A_{i-1} \,. \tag{23}$$

Let $\bar{A}_j$ be the mean action level in area *j*, and

$$A_i - \bar{A}_j = \theta_i + f\left(X_i\right) - f\left(\bar{X}\right)_j + \alpha\left(A_{i-1} - \bar{A}_j\right), \tag{24}$$

where α is regarded as the coefficient of A_i with respect to A_{i-1}. We can then calculate the variance of $A_i - \bar{A}_j$, in equilibrium it is:

$$Var\left(A_i - \bar{A}_j\right) = Var\left(A_{i-1} - \bar{A}_j\right). \tag{25}$$

$Var_j^{f(X)}$ represents the variance of $f\left(X\right)$ within area *j*, and

$$Var\left(A_i - \bar{A}_j\right) = \sigma_\theta + Var_j^{f(X)} + \alpha^2 Var\left(A_{i-1} - \bar{A}_j\right) = \frac{\sigma_\theta^2 + Var_j^{f(X)}}{1-\alpha^2} \,. \tag{26}$$

It seems us difficult to find a condition on group selection towards a cooperative system.

The market-maker's it involves optimization over a large set of hierarchical structures and assignments of assets to agents, and also over an infinite set of agents'.

The small factor holdings assumption $\lambda_n = \lambda \ell_n$, where λ goes to zero and the probability distributions of $\left\{\ell_n\right\}_{n=1,\ldots\ldots,N}$ preserving many of the economic insights.

We determine agents' optimal decision rules for a hierarchical structure, and then determine the optimal hierarchical structure.

There exists a selection of optimal decision rules, with parameter λ, such that for each agent *j*:

$$x_n\left(j\right) = \frac{\mu}{a\sigma^2}\left[1 + f_n\left(j\right) + \sigma\left(\lambda^2\right)\right], \tag{27}$$

for $n \in A_M\left(j\right)$, and

$$y_i\left(j\right) = 1 + g_i\left(j\right) + \sigma\left(\lambda^2\right), \tag{28}$$

for $i = 1,\ldots.,S\left(j\right)$, where $f_n\left(j\right)$ and $g_i\left(j\right)$ contain first and second-degree terms in $\gamma\left(j\right)$, we set

$$\Lambda\left(j\right) = \sum_{n \in A_N(j)} \lambda_n \,. \tag{29}$$

$\Lambda\left(j\right)$ represents the factor loading of agents *j*' portfolio for small λ, and receive the investment $\mu / a\sigma^2$. The portfolio's factor loading is

$$\lambda\left(j\right) = \frac{\mu}{a\sigma^2}\sum_{n \in A_n(j)} \lambda_n + \sigma\left(1\right) = \frac{\mu}{a\sigma^2}\Lambda\left(j\right) + \sigma\left(1\right). \tag{30}$$

Suppose there exists a selection of optimal decision rules such that for each agent *j*, $x_n\left(j\right)$ and $y_i\left(j\right)$ with

$$f_n\left(j\right) = -\lambda_n \Lambda\left(j\right) + \sigma\left(\lambda^2\right), \tag{31}$$

and

$$g_i(j) = -\frac{\Lambda(j,i)}{N(j,i)}\left[\Lambda(j) - \Lambda(j,i)\right] + \sigma\left(\lambda^2\right). \qquad (32)$$

The investment of agent 1 is

$$x_1(1) = y_1(1)x_1(1,1) = \frac{\mu}{a\sigma^2}\left[1 - \lambda_1(\lambda_1+\lambda_2) - \frac{\lambda_1+\lambda_2}{2}\lambda_3 + \sigma\left(\lambda^2\right)\right]$$

, (33)

where agent 1 adjusts agent 1.1 investment to take into account the factor loading λ_3 . Agent 1 does not know λ_1, which is $(\lambda_1+\lambda_2)/2$.

Proceedings as for agent 1we find that the investment of the top agent is

$$x_1 = \frac{\mu}{a\sigma^2}\left[1 - \lambda_1(\lambda_1+\lambda_2) - \frac{\lambda_1+\lambda_2}{2}\lambda_3 - \frac{\lambda_1+\lambda_2+\lambda_3}{3}\lambda_4 + \sigma\left(\lambda^2\right)\right].$$

(34)

The difference between the organization's investment in an asset n, x_n , and the first-best investment , x_n^* reflects the organization's decision-making error.

A hierarchical structure that maximizes expected utility for small λ, must minimize

$$\sum_{n=1}^{N} E\left(e_n^2\right),$$

where is a sum over assets of the expected error for each asset. This measures the distance between the firm's decision rule and the first-best decision rule. An optimal hierarchical structure must minimize this distance. Then

$$\sum_{n=1}^{N} E\left(e_n^2\right) = \sigma_\lambda^4 \sum_{j\in J} \sum_{i=1}^{S(j)} \left[N(j,i)-1\right]\left[N(j) - N(j,i)\right]$$

. (35)

Consider an agent *j*, and an asset *a* to the portfolio of *j' s i* –th subordinate agent *j ,i*.

The interaction term is the sum of the factor loading of the $N(j) - N(j,i)$ asset that are under the control of *j* but not of *j, i.* Since for random variables $\{\lambda_n\}_{n=1,\dots,N}$, we have

$$E\left(\lambda_1 - \frac{\sum_{n=1}^{N}\lambda_n}{N}\right)^2 = \sigma_\lambda^2 \frac{N-1}{N} \quad , \qquad (36)$$

the expected aggregation loss terms by the $N(j,i)$ assets in *i' s* portfolio and summing over *i* and *j*.

In any optimal hierarchical structure: agents must have subordinate. Agents must handle *K* assets or portfolios, except at most agent.

In optimal hierarchical structure, and all agents handling *K* assets. The intuition which arises from the processing constraint reduces decision quality.. The benefit of placing a high-ability agent at the top of the hierarchy and the agent process more dis-aggregated information equal to the benefit of placing the agent at the bottom. The characterization of agent's optimal decision rules, we show that these can be computed when the hierarchical structure and probability distribution of factor loading.

5. Informational strategies

The interactions between an agent with imperfect self-knowledge and an informed market-maker who chooses an incentive structure. There are an agent and market maker. The agent selects a continuous action or effort level *e* that impacts his and market maker's utilities. Bénabou and Tirole (2003) suppose that market maker knows a parameter β, such as the difficulty of the task or the agent's ability to perform it, that affects the agent's payoffs from *e*. Thus informed marker maker selects a policy *p* prior to the agent's choice of action, this may be a disclosure of information, the agent behavior. The agent's and the market maker's payoffs are denoted $U_A(\beta, e, p)$ and $U_P(\beta, e, p)$. Prior to his decision, the agent may receive a signal σ that is informative about β. The market maker has information relevant to the agent's perception of himself, that the market maker be uncertain about the agent's motivation.

The market maker learns the parameter β and selects a policy *p*. After observing the policy chosen by the market maker and learning σ, the agent chooses an action *e*. The agent 's participation in the relationship and the market maker's expected payoff from choosing a policy *p* when she has information β is

$$E_\sigma\left[U_P\left(\beta, e^*\left(p, \hat{\beta}(\sigma, p)\right), p\right)\middle|\beta\right]. \qquad (38)$$

The market maker's choice of policy takes effects into consideration:

$$E_\sigma\left[\frac{\partial U_P}{\partial p} + \frac{\partial U_P}{\partial e}\frac{\partial e^*}{\partial p} + \frac{\partial U_P}{\partial e}\frac{\partial e^*}{\partial \hat{\beta}}\frac{\partial \ddot{\beta}}{\partial p}\middle|\beta\right] = 0, \quad (39)$$

If the policy is the direct cost of this compensation, keeping the agent's behavior. The second term corresponds to the direct impact of *p* on the agent's behavior. The market maker's choice of policy is guided by private information, the agent will update his beliefs in reaction to the choice of *p*. The market maker must then take into account how the agent's interpretation of her choice will affect his self-confidence affects the agent's decision making. . His perceived prospects from undertaking the task. When the agent's type, on which the market maker has private information, enters the market maker's objective function in a way that would lead her to offer different policies to different agents. The market maker will then, delegate more to agents. The market maker's private knowledge concerns the cost of accomplishing the task, and not on her own payoff. The trust effect, arises when the market maker's private information such as the cost of accomplishing the task. A market maker who has bad information about the agent's parameter β will be pessimistic about the agent's own signal.

Providing be will not be motivated enough to exert effort on the absence of added incentives. Providing stronger incentives, will at least reveal the market market's damaging information signal σ There must be some uncertainty σ on the part of the market maker about the incentives perceived by the agent. The latter's response $\hat{\beta}(p)$ to any policy *p* would be predictable, and the market maker would maximize $U_P\left(\beta, e^*\left(p, \hat{\beta}(p)\right), p\right)$. However, that the agent does receive a private signal, causing the market maker to worry about his motivation. The agent's own initial perception as he starts performing the task. The market maker's payoff function can be written as $U_P(\beta, e, p) = e\Lambda(\beta, p)$. The agent's equilibrium effort *e*, and the function Λ is the market maker 's expected payoff when $e = 1$. The utility effect is then governed by the sorting condition

$$\frac{\partial}{\partial\beta}\left(\frac{\partial U_P / \partial p}{\partial U_P / \partial e}\right) = \frac{\partial}{\partial\beta}\left(\frac{e\Lambda_P(\beta, p)}{\Lambda(\beta, p)}\right), \qquad (40)$$

which is a form of complement-arity. Under conditions on $U_A(\beta, e, p)$ and the distribution $G(\sigma|\beta)$, the agent receives a signal σ better than some threshold $\sigma^*(p)$ which depends on the policy *p*. The market maker's expected utility can then be written as $\left[1 - G\left(\sigma^*(p)\middle|\beta\right)\right]\Lambda(p)$.

The agent's action threshold σ^* as the effort variable, that the market maker is trying to influence through her policy *p*. This yields

$$\frac{\partial}{\partial\beta}\left(\frac{\partial U_P / \partial p}{\partial U_P / \partial\sigma^*}\right) = \frac{\partial}{\partial\beta}\left(\frac{1 - G\left(\sigma^*|\beta\right)}{g\left(\sigma^*|\beta\right)}\right) \times \frac{\Lambda'(p)}{\Lambda(p)}. \qquad (41)$$

A market maker who observes a bad β is worried that the agent will receive a bad signal σ, so she feels compelled to offer him a higher *p*.

The agent dis-utility of effort is denoted *c*. If the task is successful it yields payoffs *V* to the agent and *W* to the market maker. Let $\theta \in (0,1]$ denote the probability of success when the agent exerts effort. The asymmetry concerns the cost that the agent will bear if he decides to undertake the task. The agent knows that *c* is drawn from a cumulative distribution function $F(c)$ with a density $f(c)$ that has support. He learns a signal $\sigma \in [0,1]$ with conditional distribution $G(\sigma|c)$ and density $g(\sigma|c)$. For all σ_1 and σ_2 with $\sigma_1 > \sigma_2$, $\frac{g(\sigma_1|c)}{g(\sigma_2|c)}$ is decreasing in *c*. The agent to perform the task, the market maker can offer a reward that on effort if she observes it. The probability of success θ is common knowledge, we shall therefore focus the exposition on contract where the market maker a reward $b \leq W$.

The agent's net benefit is thus $V + b$ and the market maker's is $W - b$, and policy decision for the market maker thus the choice of a *b* could with slight modifications. Were the agent to know his cost *c*, he would choose to exert effort if

$$\theta(V + b) \geq c . \qquad (42)$$

When the agent has same information as the market maker observes *c*. The agent receives a signal σ about *c*. When offered a reward *b*, the agent updates his beliefs about *c* using the market maker's equilibrium strategy. Let $\hat{c}(\sigma, b) = \mathrm{E}\{c|\sigma, b\}$ denote the agent's assessment of the task's difficulty. This expectations is a weakly decreasing function of the signal σ. Letting $e \in [0,1]$ denote the agent's effort, his utility is $U_A = [\theta(V + b) - \hat{c}(\sigma, b)]$, and there exists a threshold signal $\sigma^*(b)$ in $[0,1]$ such that:

$$\hat{c}(\sigma, b) \leq \theta(V + b) \ if \ \sigma \geq \sigma^*(b). \qquad (43)$$

The market maker's payoff if she offers the performance *b* when her information is *c* is thus

$$E_\sigma[U_P] = \theta\left[1 - G\left(\sigma^*(b)\middle|c\right)\right][W - b], \qquad (44)$$

which she maximizes over *b*. Delegate equilibrium are ruled out when $(V + W) > \bar{c}$ by offering a below *W*., the market maker can ensure that the agent works.

In equilibrium rewards are positive short-term reinforces if $b_1 < b_2$, then $\sigma^*(b_1) > \sigma^*(b_2)$. If b_1 is a reward offered when the market maker knows the task's difficulty to be c_1, and b_2 is offered when she knows it to be $c_2 > c_1$, then $b_2 \geq b_1$.

Rewards undermine the agent's assessment of the task's attractiveness, for all (σ_1, σ_2) and all equilibrium rewards $b_1 < b_2$,

$$\mathrm{E}\left[c\middle|\sigma_1, b_1\right] < \mathrm{E}\left[c\middle|\sigma_2, b_2\right]. \quad (45)$$

Assessments of task attractiveness are also reduced by an increase in the reward: the expectation of c conditional on σ, b, the action and the outcome is decreasing in b regardless of σ, the action and the outcome. Since b_i is optimal given c_i, it must be that

$$\theta\left[1 - G\left(\sigma_i \middle| c_i\right)\right]\left[W - b_i\right] \geq \theta\left[1 - G\left(\sigma_j \middle| c_i\right)\right]\left[W - b_j\right] \quad , \quad (46)$$

hence

$$\frac{1 - G\left(\sigma_1 \middle| c_1\right)}{1 - G\left(\sigma_2 \middle| c_1\right)} \geq \frac{W - b_2}{W - b_1} \geq \frac{1 - G\left(\sigma_1 \middle| c_2\right)}{1 - G\left(\sigma_2 \middle| c_2\right)}. \quad (47)$$

Hence $b_1 \leq b_2$ since $\sigma^*(.)$ is decreasing. We demonstrates the market maker's expectation of what views the agent is likely to hold, which in turn shapes the optimal contract. A higher reward is, in equilibrium, associated with a less attractive task.

The probability of effort, $1 - G\left(\sigma^*(b)\middle|c\right)$, and the probability of success, $\theta\left[1 - G\left(\sigma^*(b)\middle|c\right)\right]$, are decreasing in c, which is known to the market maker.

Let $\overline{U}$ denote his outside reservation utility, which we assume to be independent of the attractiveness of the task, if $\theta(V + b) - E\left(c\middle|\sigma^*(b), b\right) \geq \overline{U}$, the participation constraint is not binding.

The information conveyed by incentives, and rewards may have an negative impact when performance measurement is contingent. The agent learns, before making his decision, whether he is likely to be caught, and to escape detection. When the market maker has private information about the agent's ability θ rather than the cost of implementing the task, this profitability effect, when it is present depend on the type of contract allowed. The market maker observes θ exactly, the agent receives an signal σ, with conditional distribution $G(\sigma|\theta)$ and density $g(\sigma|\theta)$, but with a higher σ now signaling a higher θ. In the market maker 's function $U_P = \theta e(W - b)$, the marginal rate of substitution between b and e is independent of θ. The market maker's private information and the agent's noisy signal bear on the agent's probability of success θ rather than on the task's difficulty c. We denote by $b_k^*,\ k \in \{L, H\}$, the minimum feasible incentives that induces effort when the agent is fully informed about his ability,

$$b_k^* = \max\left\{0, \frac{c}{\theta_k} - V\right\}, \quad (49)$$

then $0 = b_H^* < b_L^* < W$. The agent's reservation utility characterizing all perfect Bayesian equilibrium and identifying a unique equilibrium.

In any equilibrium, the market maker offers a low incentives $b < b_L^*$ to a more able agent and randomizes between the incentives b and b_L^* when dealing with a less able agent.

Incentives are the same as under symmetric information, the agent's utility is now higher than under symmetric information. It still implies that the distribution of rewards across a population of agents due to the management problem. The utility effect to which it gives rise when the market maker's private information is θ, and analysis distinguishing between the trust and profitability effects makes motive will be reflected in equilibrium contracts. The agent would then just be assumed to exhibit an instinctive reaction threatened with a punishment. While agents do not really compute Bayesian equilibrium when interpreting signals from their environment. Under symmetric information motivations can be cleanly separated, under asymmetric information decreases future motivation. For all $\theta \in \{0,1\}$,

$$\frac{d}{d\theta}\left(\frac{W_1(\theta)}{W_0(\theta)}\right) > 0,$$

more over

$$\frac{W_1(0)}{W_0(0)} < 1 < \frac{W_1(1)}{W_0(1)}. \quad (50)$$

The market maker does not want to delegate the task to an agent, and prefers to delegate the task to a very talented agent. This implies that there is exists a θ^* in $(0,1)$ under symmetric information, it is efficient to delegate if $\theta > \theta^*$, and to monitor if $\theta < \theta^*$.

For all (σ_0, σ_1) with $\sigma_0 > \sigma_1$, the elasticity of the ratio $\left(1 - G(\sigma_0|\theta)\right)/\left(1 - G(\sigma_1|\theta)\right)$ with respect to θ is less than that of $W_1(\theta)/W_0(\theta)$.

The agent is unsure about his ability, suppose that the market maker offers to contribute at private cost b, and the agent decides to undertake the task. The market maker 's payoff is:

$U_P = \left[1 - G\left(\sigma^*(b)\middle|\theta\right)\right]\left[P(\theta, h)W - h\right]$, where the agent 's undertaking the task.

6. Conclusion

We have examined a model of network formation with costs where agent benefits from network formation are not known to the market-maker. The cost function is assumed to be common knowledge in the economy. The only restrictions imposed on the cost function, agents move sequentially. Each agent's announcement is consisting of the set of agents with whom he wants to form links. A contribution, interpreted as the agent's contribution towards the cost of network formation. The mechanism ensures the formation of an efficient network in all perfect Nash equilibrium, however the net payoffs to the agents are asymmetric. The agents moving earlier are better off than agents moving later. The mechanism corrects for this asymmetry and ensures not only the formation of efficient networks but also equitable net payoffs. We discuss condition under which the mechanisms we propose are immune to coalition-al deviations.
In the long run, by undermining agent's confidence in their abilities.

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Recieved M. A. in Bratislava School of Economics, Faculty of Management, PhD. In St. Petersburg University of Technology, Russia.
He is professor of economics in Slovak University of Technology Faculty of Electrical Engineering and Information Technology, Dept. of Economics and Management.