Separation of Convolutive Mixtures of Cyclostationary Sources by Reference Contrasts

K. Sabri[1;2], M. Taoufiki[1], A. Adib[1;3], D. Aboutajdine[1], M. El Badaoui[2], F. Guillet[2]

[1] GSCM-Faculté des Sciences de Rabat, Rabat, Maroc.
[2] LASPI, Université Jean Monnet, I.U.T de Roanne 20 avenue de Paris 42334 Roanne. France
[3] DGC-Institut Scientifique, av. Ibn Battouta, B.P. 703, Rabat, Maroc.

Summary
This paper introduces new source separation techniques exploiting the cyclostationarity property of the source signals. Two cyclostationary-based reference contrast functions are presented for convolutive mixtures and cyclostationary signals. Contrary to the major contrasts, any para-unitarity constraint is made on the mixing system. Their maximization enjoys identifiability properties, and aims at delivering outputs satisfying specific properties, such as statistical independence or a cyclostationarity character. Simulation examples are presented to illustrate the effectiveness of this approach in terms of the estimated source variances.

Key words:
Cyclic statistics, Blind Source Separation, Convolutive mixture, Reference contrast, Cyclostationarity.

Introduction

In many real-world situations, man-made signals encountered in rotating machines, communications, telemetry, radar, and sonar systems are non-stationary and very often (quasi-) cyclostationary. Of course, the mixtures of real-world signals are usually convolutive mixtures. So that the conventional methods for the standard Blind Source Separation (BSS) problem that assume instantaneous mixtures of stationary sources [1] [2], are no longer appropriate. Increasing interest has therefore been focused on solving the problem of BSS of convolved mixtures of cyclostationary sources. A basic model of BSS is simple linear combinations (possibly noisy) of statistically independent signals. Given these observations, BSS aims to estimate both the structure of the linear combinations and the source signals. In cyclostationary context, the BSS problem is a relatively new approach. The most existing cyclostationary BSS approaches are based on second-order statistics as in reference [3] for convolutive mixture and in [4] for the instantaneous one. Furthermore, certain works [5] address the problem in frequency-domain by using spectral correlation density matrices. Indeed, the DFT allows us to have instantaneous mixtures at each frequency bin. Although, the major problem of this kind of approaches is how to correct permutation at a given frequency. Thus, at each frequency, the unmixing system is identified, thereafter the sources are recovered at the outputs. time domain channel is then formed by the inverse DFT with respect to some constraints like in [6] [7]. A very few works in the BSS literature deal with high-order statistics in the case of cyclostationary convolved mixtures as in [8] [9]. The focus of this paper is on the use of the so called reference contrasts. Recently, contrast functions have been generalized with the so-called reference signals. For the sake of clarity, we notice that reference signals can be chosen freely. The idea behind is to imply the reference signals with the estimation sources in the maximization of the contrast. Several studies have shown their good results in comparison with the classical contrasts. Indeed, this has been studied in several papers, including the first contributions dedicated to instantaneous mixtures of stationary sources [1] [2] [10]. Then, such functions have been extended in convolutive systems, under some technical hypothesis on reference system, either for MISO system in [11] or for MIMO system in [12]. In this paper, our main contribution regards the generalization of this kind of contrasts -that have been proposed to stationary input signals [11] - to cyclostationary mixture. Moreover, unlike the previous methods that need the observed signals to be orthogonal, our contribution is performed without regard to any orthogonality constraint, and it needs no prior whitening. Hereafter, a necessary and sufficient condition for BSS using a set of cyclic cumulants is given. Under this condition, two contrast functions are introduced, one for para-unitary filters and the other is for the general case. Ultimately, we test our method in the case of a MIMO system driven by periodically modulated random processes.

Manuscript received January 10, 2006.
Manuscript reviced February 28, 2006.

2. System model and new criterion

2.1 Notations

Let us consider m -dimensional sensor or measurement vector $\mathbf{x}(t)$. It's defined as the output of an unknown m inputs n outputs MIMO system with linear transfer function $\mathbf{H}[z]$. This can be written as:

$$\mathbf{x}(t)=\sum_k \mathbf{H}(k)\, s(t-k) \tag{1}$$

Here, the (Linear Time Invariant) LTI multichannel system is driven by a n -dimensional unknown vector $\mathbf{s}(t)=(s_1(t),...,s_n(t))^T$. It should be noted that the components of the source vector are cyclostationary, mutually statistically independent and non-Gaussian (at most only one can be Gaussian). As our approach is an iterative one, we will focus on the extraction of a single source. Hence, using only the observation $\mathbf{x}(t)$, the considered problem consists in estimating a $(1,m)$ LTI vector filter, called equalizer and with impulse response $\mathbf{g}(t)$ such that the scalar signal

$$y(t)=\sum_k \mathbf{g}(k)\mathbf{x}(t-k) \tag{2}$$

restores one of the components $s_i(t), i\in\{1,...,n\}$, of the source vector. Thus the global vector filter of dimension $(1,m)$ is defined by the following impulse response

$$\mathbf{w}(t)=\sum_k \mathbf{g}(k)\mathbf{H}(t-k) \tag{3}$$

and we have

$$y(t)=\sum_k \mathbf{w}(k)\mathbf{s}(t-k) \tag{4}$$

Furthermore, the following assumptions are made. The source signals $s_i(t), i\in\{1,...,n\}$ are real, cyclostationary, mutually independent, zero synchronous mean, and with unit synchronous variance. In addition, we define $r(t)$ as the output of another separating filter defined according to

$$r(t)=\sum_k \mathbf{t}(k)\mathbf{s}(t-k) \tag{5}$$

where the filter $\mathbf{t}[z]=\mathbf{tr}[z].\mathbf{H}[z]$, which corresponds to the second global system and is assumed not equal to $\mathbf{w}[z]$. The signal $r(t)$ is called the reference signal and $\mathbf{tr}$ the reference vector. It's interesting to notice that the reference signal is not identical to any original source.

In the following, if $(u(k))_{k\in\mathbb{Z}}$ is an almost periodic sequence, we define $<u(k)>$ as its temporal mean by

$$<u(k)>=\lim_{K\to\infty}\frac{1}{K}\sum_{k=0}^{K-1}u(k).$$

and as the cyclostationarity naturally conveys through the linear system to their outputs, then if $\mathrm{y(t)}=[\mathbf{g}[\mathrm{z}]]\mathbf{x}(\mathrm{t})$, both $t\to \mathbf{cum}_4(y(t))$ and $t\to \mathbf{E}(y(t)^2)$ are together periodic sequences.

2.2 New criterion

It turns out that the most appealing approach to the blind equalization problem consists in the use of an appropriate contrast function. Basically, a contrast plays the role of an objective function in the sense that its (global) maximization permits to solve the BSS problem [13]. Hence the equalization issue becomes an optimization one. Besides, identifiability conditions are provided by the definition domain of the considered contrast. To address our MISO equalization problem, we recall the definition of a contrast for i.i.d. source signals as introducing in [14]:

Definition:1. *Let $\mathcal{F}(\cdot)$ be a real function of the signal $y(t)$ as defined in 4. $\mathcal{F}(\cdot)$ is called a contrast when there exists $i_0\in\{1,...,n\}$:*

p1. $\exists\, l\in\mathbb{Z}$ such that for all possible output $y(t)$ of the equalizer $\mathcal{F}(y(t))\le\mathcal{F}(s_{i_0}(t-l))$

p2. If equality holds in p1, then there exists an index $i_0\in\{1,...,n\}$ such that the filter components in $\mathbf{w}[z]$ read: $\mathbf{w}[z]=e^{-id}(0,\dots,0,1,0,\dots,0)$

In the following, we establish a contrast dedicated to cyclostationary signals. To this end, we need to recall the technical assumption about the reference signal that has been introduced in [14]. Before announcing our result, let us denote the fourth order cross-cumulant $\mathbf{cum}\big(y_i(t),y_j(t),r_k(t),r_l(t)\big)$, of the separator outputs and the reference signals, by $\mathbf{C}_{ij}^{kl}$.

Hypothesis: 1. *$\exists l\in\mathbb{Z}$ such that $\forall\,(j,k)\in\{1,...,N\}\times\mathbb{Z}$ we have $|\,\mathbf{t}_j(k)\,|<|\,\mathbf{t}_1(l)\,|$ when $k\ne l$ or $j\ne 1$*

The main present section is devoted to propose a new contrast function for **all** signals $y_i(t)$. Furthermore, we need this intermediate step that defines two new contrasts for normalized signals.

Proposition: 1. *For a given normalized signals $y_i(t)$ and for all $\beta\ge 1$, the functions:*

$$\overline{\mathcal{J}}^{\,r}(y_i(t))=<\mathbf{C}_{ii}^{ii}> \tag{6}$$

and

$$\overset{=}{\mathcal{J}}^{r}(y_i(t)) = \begin{cases} <\mathbf{C}_{ii}^{ii}> & i=1 \\ <\mathbf{C}_{ii}^{ii}> -\beta\sum_{j<i}<\mathbf{C}_{ij}^{jj}> & 2\le i\le n \end{cases} \quad (7)$$

are contrasts. The index i references the i^{th} extracted source.

Proof: The proof of the first one is similar to the proof of the main result in [14]. In the following we furnish a short overview. Using multilinearity of the cumulant and source independence properties, we have:

$$\overline{\mathcal{J}}^{r}(y_i(t)) \le \sum_{j=1}^{n}\sum_{k}|w_j(k)|^2|t_j(k)|^2 < \mathbf{cum}_4(s_j)>$$

Assuming that: $\max_{j=1}^{n} < \mathbf{cum}_4(s_j)> = < \mathbf{cum}_4(s_1)>$. So, according to hypothesis T, we have:

$$\overline{\mathcal{J}}^{r}(y_i(t)) \le |t_1(l)|^2 < \mathbf{cum}_4(s_1)> \sum_{j=1}^{n}\sum_{k}|w_j(k)|^2$$

Now, as the $y_i(t)$ is normalized then we have

$$\sum_{j}\sum_{k}|w_j(k)|^2 = 1.$$

Thus the first property of the contrast is satisfied with equality if and only if $\mathbf{w}[z] = (0,0,w_{k_0}[z],0,..,0)$.

Using the following contrast propriety [15], for a given functions $\mathbf{I}_1$ and $\mathbf{I}_2$ such that $\mathbf{I}_2$ is a contrast, then if:

- $\mathbf{I}_1(y(t)) \le \mathbf{I}_2(y(t))$
- $\mathbf{I}_1(s(t)) = \mathbf{I}_2(s(t))$

Therefore, $\mathbf{I}_1$ is also a contrast. In the above, we have proved that $\overline{\mathcal{J}}^{r}(y_i(t))$ is a contrast. So, for $\beta \ge 1$, we have:

$$\overset{=}{\mathcal{J}}^{r}(y_i(t)) \le \overline{\mathcal{J}}^{r}(y_i(t))$$

since $y_i(t)$, $i\in\{1,...,N\}$, are independents we have:

$$\overset{=}{\mathcal{J}}^{r}(\boldsymbol{s}) \le \overline{\mathcal{J}}^{r}(\boldsymbol{s})$$

Thus, $\overset{=}{\mathcal{J}}^{r}(y_i(t))$ is a contrast.

Now, we will eliminate the normalization hypothesis, so that we will get a contrast for all vectors $y_i(t)$.

Proposition: 2. *For all $y_i(t)$, and for all $\beta \ge 1$ the function*

$$\overset{=}{\mathcal{J}}^{r}(y_i(t)) = \begin{cases} <\mathbf{C}_{ii}^{ii}> & i=1 \\ <\mathbf{C}_{ii}^{ii}> -\beta\sum_{j<i}<\mathbf{C}_{ij}^{jj}> & 2\le i\le n \end{cases}$$

$$\mathcal{L}^{r}(y_i(t)) = \begin{cases} \dfrac{<\mathbf{C}_{ii}^{ii}>}{<\mathbf{E}(y_i^2)>} & i=1 \\ \dfrac{<\mathbf{C}_{ii}^{ii}>}{<\mathbf{E}(y_i^2)>} - \beta & \\ \sum_{j<i}\dfrac{<\mathbf{C}_{ij}^{jj}>}{<\sqrt{\mathbf{E}(y_i^2)}\sqrt{\mathbf{E}(y_j^2)}>} & 2\le i\le n \end{cases} \quad (8)$$

is a contrast.

Proof: This result will be immediately reached by replacing $y_i(t)$ by replacing $\tilde{y}_i(t) = \sum_j M_{ij}\, y_j(t)$ where $\mathbf{M}$ is a diagonal matrix. This matrix's elements are the square roots of the respective powers of each component of the vector $\mathbf{y}$.

3. Cyclic statistics

3.1. Definition of cyclic statistics

The previous contrasts require the estimation of the cyclic statistics, the objective of the following section is to compute the fourth-order cyclic cumulant. In this context, we propose a new definition of this kind of statistics. To this end, we need to recall the first-order, second-order and fourth-order cyclic moments as in [16] [17]: The first-order cyclic moment is

$$M_{1x}^{\alpha} = < x(t))\exp(-j2\pi\alpha t)>,$$

The second-order cyclic moment is

$$M_{2x}^{\alpha}(k_1) = < x(t)x(t+k_1))\exp(-j2\pi\alpha t)>,$$

The third-order cyclic moment is

$$M_{3x}^{\alpha}(k_1,k_2) = < x(t)x(t+k_1)x(t+k_2)\exp(-j2\pi\alpha t)>,$$

The fourth-order cyclic moment is

$$M_{4x}^{\alpha}(k_1,k_2,k_3) = < x(t)x(t+k_1)x(t+k_2)\, x(t+k_3)\exp(-j2\pi\alpha t)>,$$

where $x(t)$ is a real cyclostationary signal of order 4, α is known as the cyclic frequency and $k_i, i=1,...,3$ is the delay. Basically, the fourth-order cyclic cumulant is defined by:

$$\mathbf{C}^{\alpha}_{4x}(k_1, k_2, k_3) = \lim_{N\to\infty} \frac{1}{N} \sum_{t=0}^{N-1} \mathbf{C}_{4x}(t; k_1, k_2, k_3) \exp(2\pi j\alpha t)$$

and is computed through the cyclic moments according to the following formula for zero-synchronous mean signal:

$$\mathbf{C}_{4x}(t) = \sum_{\alpha} (\mathrm{M}^{\alpha}_{4x} - 3(\mathrm{M}^{\alpha}_{2x})^2) \exp(2\pi j\alpha t) \quad (9)$$

from the above formula, we can see that the cyclic fourth-order cumulant can be written based on second-order and fourth-order cyclic moments.

3.2 Estimation of cyclic statistics

In fact, the contrast's input are scalars. This can dissuade its use with (almost-) periodic statistics. We use temporal average like in [9] in order to keep a scalar instead of periodic sequence.

The objective of this sub-section is to give a new estimation of the cyclic moments in order to get a consistent fourth-order cyclic cumulant. To this end, the sampling frequency is synchronized to the signal's cyclic frequency. This technique [18] and [19] is known as the synchronous sampling and it is much used in the acquisition of vibratory signals. The signal split by cyclic stochastic realization after synchronous sampling. We achieve such averages on all available cycles (period of cyclostationarity). In the case of signals with different cyclic frequencies, we look to their smallest common multiple.

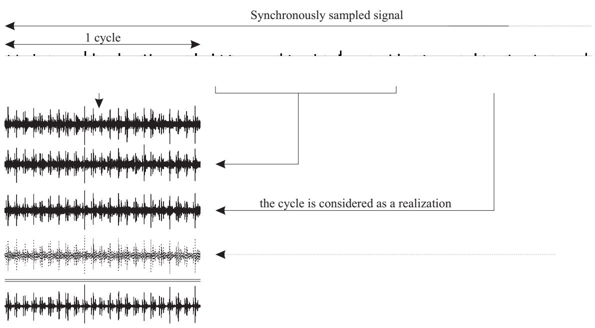


Fig. 2. Illustration of the synchronous statistics

The figure 2 illustrates the synchronous statistics as the average of stochastic process. By using the assumption of cycloergodicity. one will be able to build an estimator of the synchronous mean:

$$\hat{M}^{x}_{1}(t) = \frac{1}{K} \sum_{i=0}^{K-1} x(t + i * T_{cs} \mod N)$$

where $T_{cs} = \frac{1}{\alpha}$ is the cyclostationarity period which presents a cycle, N is the number of samples and K is the number of cycle. taking into account that $N = K * T_{cs}$. One can also define the synchronous cross correlation as follows :

$$\hat{M}^{xy}_{2}(t) = \frac{1}{K} \sum_{i=0}^{K-1} x(t + i * T_{cs} \mod N) * y(t + i * T_{cs} \mod N)$$

In the same way, one can generalize this definition to R-order synchronous moments as follows :

$$\hat{M}^{x_1 \ldots x_R}_{R}(t) = \frac{1}{K} \sum_{i=0}^{K-1} x_1(t + i * T_{cs} \mod N) * \ldots x_R(t + i * T_{cs} \mod N)$$

$$\hat{C}^{x,y,r,z}_{4}(t) = \hat{M}^{x,y,r,z}_{4} - \hat{M}^{x,y}_{2} * \hat{M}^{r,z}_{2} - \hat{M}^{x,r}_{2} * \hat{M}^{y,z}_{2} - \hat{M}^{x,z}_{2} * \hat{M}^{y,r}_{2}$$

According to the preceding parametrization, the cumulant $\mathbf{C}^{kl}_{ij}$ will take the following form:

$$\mathbf{C}^{kl}_{ij} = \hat{\mathrm{C}}^{y_i y_j r_k r_l}_{4}(t)$$

Hereafter, the above cyclic statistics will be used to assess the proposed contrast in order to achieve separation of cyclostationary sources.

4. Computer simulations

The objective of our simulations is to confirm the validity of our contrasts, proposed in the case of cyclostationary signals with some knowledge of the cyclic frequencies of the second order statistics, for any type of channel without restriction to the paraunitarity ones. In order to maximize the above contrast, we follow a deflation procedure that processes good convergence properties and each source is extracted using a gradient algorithm. In all experiments, we have taken $n = 2$ source signals, which have mixed using $(2, 2)$ FIR 2 paths channel. Our performance criteria are a simpler version of separation rate of the sources: "TSEP" used in [9] and mean square error (MSE). In each run, the source signals, the mixing system have been randomly chosen. Indeed, two different types of source signals have been considered. The first one corresponds to i.i.d circular symbol sequence randomly chosen at each trial. The reference signals have been randomly chosen. In such case, TSEP of the sources have been evaluated over 100 Monte-carlo runs when $\beta = 1$. We have plotted in figure 3 the TSEP versus number of iteration. As can been seen, there is a visible advantage of the proposed contrast.

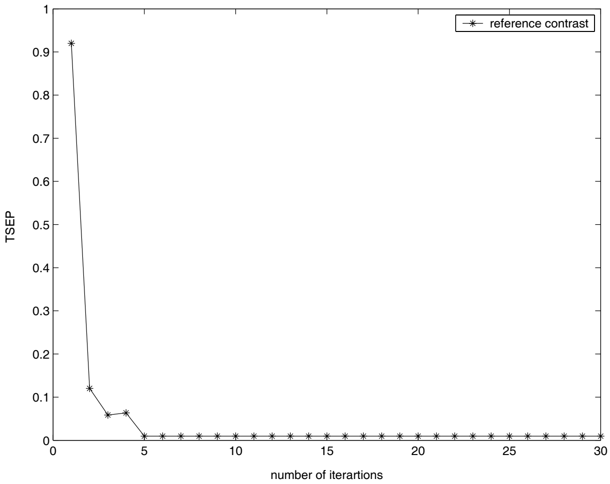
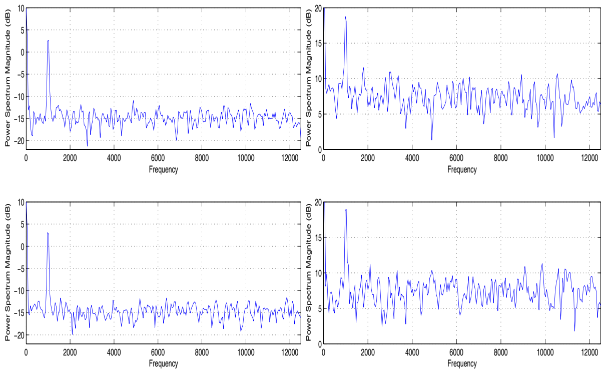


Fig. 3. Reference contrast performance

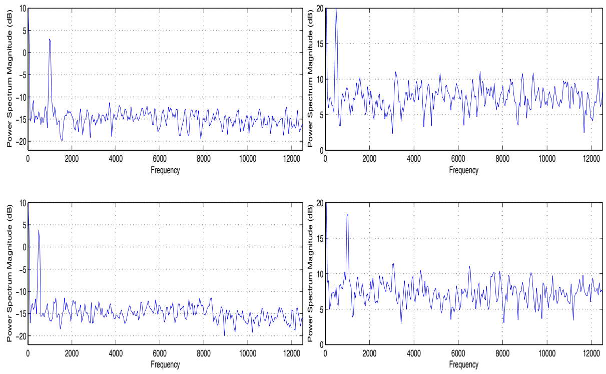


(a) Sources (b) Restored sources

Fig. 4. Sources with the same cyclic frequency

The second case of sources are randomly generated and modulated by an AM modulator of carrier 500 Hz. The sources with different cyclic frequencies are modulated at 250 and $500 Hz$ (sub-figure 5). The sample frequency is of $25000 Hz$. The sample size corresponds to the observation of 4096 symbols. In the second experiment, we suppose that the reference system is one of the sources.

On the sub-figures 4 and 5, it is seen well that we could recover the sources at their cyclic frequencies except for a scale factor and a permutation. Furthermore, Figure 6 shows the evolutions of the averaged values of MSE for various SNR over 100 Monte Carlo trials. There is a visible advantage of the the $\mathcal{L}^r$ contrast especially when we introduce the cross cumulants in the general form of contrast i.e. $\beta \geq 1$.



(a) Sources (b) Restored sources

Fig. 5. Sources with the different cyclic frequency

5. Conclusion

This paper is devoted to the separation of the cyclostationary signals via the reference contrast. In this article, we have essentially proposed a generalization of a contrast that have proposed in [12] from the stationary signals to cyclostationary ones without constraint of orthogonality. As shown by the computer simulations, the maximization of the proposed contrast yields better results.

Acknowledgments

The authors gratefully thank the financial support from the Region Rhone Alpes (France).

References

[1] A. Adib, E. Moreau, and D. Aboutajdine, “Sources separation using a reference signal,” IEEE Signal Processing Letters, vol. 11, no. 3, pp. 312–315, March 2004.

[2] A. Jbari, A. Adib, and D. Aboutajdine, “Solution analytique et généralisation d’un contraste à référence,” in accepted for publication in GRETSI’05, 2005.

[3] W. Wang, M. G. Jafari, S. Sanei, and J. A. Chambers, “Blind separation of convolutive mixtures of cyclostationary signals,” International Journal Of Adaptive Control And Signal Processing, vol. 18, no. 3, pp. 279–298, 2004.

[4] K. Abed-meraim, Y. Xiang, J. H. Manton, and Y. Hua, “Blind source separation using second-order cyclostationary statistics,” IEEE Trans. On Sig. Proc, vol. 49,no. 4, pp. 694–701, Apr 2001.

[5] J. Antoni, F. Guillet, M. El Badaoui, and F. Bonnardot, “Blind separation of convolved cyclostationary processes,” Signal processing, vol. 85, pp. 51–66, 2005.

[6] L. Parra and C. Spence, “Convolutive blind separation of non-stationary sources,” IEEE Trans On Speech And Audio Proc, vol. 8, no. 3, pp. 320–327, May 2000.

[7] K. Rahbar and J. P. Reilly, “A frequency domain method for blind source separation of convolutive audio mixtures,” IEEE Trans On Speech And Audio Proc, vol. 13, no. 5, pp. 832–844, Sept 2005.

[8] A. Ferreol and P. Chevalier, “On the behavior of current second and higher order blind source separation methods for cyclostationary sources.,” IEEE Trans. on Signal Processing, vol. 48, no. 6, pp. 1712–1725, 2000.

[9] P. Jallon, A. Chevreuil, P. Loubaton, and P. Chevalier, “Separation of convolutive mixtures of cyclostationary sources: a contrast function based approach,” in ICA, Spain, Granada, september 2004, pp. 508–515.

[10] A. Adib, E. Moreau, and D.Aboutajdine, “Blind sources separation by simultaneous generalized referenced contrasts diagonalization,” in ICA, Nara, Japan, 2003, pp. 657–661.

[11] M. Castella, S. Rhioui, E. Moreau, and J.-C. Pesquet, “Source separation by quadratic contrast functions:a blind approach based on any higher-order statistics,” in ICASSP, Philadelphia, USA, 2005, vol. 3, pp. 569–572.

[12] M. Taoufiki, A. Adib, and D. Aboutajdine, “Contrast with reference a new approach to the blind equalization

[13] P. Comon, “Independent component analysis, a new concept?,” Signal Processing, vol. 36, no. 3, pp. 287– 314, Apr 1994.

[14] M. Castella, E. Moreau, and J.C. Pesquet, “A quadraticmiso contrast function for blind equalization,” in ICASSP, 2004, vol. 2, pp. 681–684.

[15] B. Stoll, Optimisation de Fonctions de Contraste en Séparation de Sources, Ph.D. thesis, Université de Toulon et du Var, 2000.

[16] W. A. Gardner, Cyclostationarity in communicationsand signal processing, IEEE PRESS, 1993.

[17] A. V. Dandawate and G. B. Giannakis, “Nonparametric polyspectral estimators for kth-order (almost) cyclostationary process,” IEEE Trans. on Inforamation Theory, vol. 40, no. 1, pp. 67–84, Jan 1994.

[18] P.D. McFadden, “Detecting fatigue cracks in gears by amplitude and phase demodulation of the meshing vibration,” Journal of Vibration, Acoustics, Stress, and Reliability in Design, vol. 108, pp. 165–170, 1986.

[19] F. Bonnardot, M. El Badaoui, R.B. Randall, J. Danière, and F. Guille, “Use of the acceleration signal of a gearbox in order to perform angular resampling (with limited speed fluctuation),” Mechanical Systems and Signal Processing, vol. 19, pp. 766–785, 2005.