

Cooperation Evolution with Signal Quality

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Summary

In a group, each kind of individuals appears signal quality. This paper attempts to probe into the relation between cooperation evolution and subjective probability caused by signal quality. We show that cooperation mechanism is easier to establish when the signal quality is greater. We also find that cooperation mechanism becomes more difficult to establish if the interaction is more than one round in a generation.

Key words:

Cooperation, Evolution, Signal quality, Best response

Introduction

Cooperation¹ is a paradox in Evolutionary Theory [1]. Cooperators help others at certain cost to themselves, while defectors receive the benefits of altruism without providing any help in return. Even though cooperation behavior is beneficial to the collectivity or society, it is hard to survive for cooperator, for the fitness of cooperator is often less than the fitness of defector. But it is not unusual during the human history, as well as in some behavior experiments [2-5].

Explanation of human cooperation is often based on genetic relatedness [6-7] and repeated game [8-9]. Kin selection is an important explanation for human cooperation as well as for other animals. However, it seems to be implausible to explain human cooperation among large group of unrelated individuals in this way. Thus, repeated game and indirect reciprocity based on repeated game can widely explain the cooperation behavior. Indirect reciprocity involves reputation and status, and results in every group member being continually assessed and re-assessed [10]. In the indirect reciprocity, cooperation is channeled towards the valuable members of the community.

The standard game dynamical formulation is the "Prisoner's Dilemma", in which two players have a choice between cooperation and defection. In the no repeated Prisoner's Dilemma, defectors dominate cooperators. In general, defectors are stable against invasion by cooperators. In the repeated Prisoner's Dilemma, the same two players meet more than once, and there are many conceivable strategies that allow cooperative behavior which cannot be defeated by defectors. The most famous

such strategy is tit-for-tat (TFT), in which the player cooperates in the first round and then does whatever the opponent did in the previous round. If the number of rounds is sufficiently large, then ALLD (always to defect) and TFT resist invasion attempts by the other strategy [8]. Cooperation flourishes if altruistic punishment is possible, and breaks down if it is ruled out [11]. Discrimination and punishment mechanism are hence prerequisite [12]. In past models, TFT is the predominant strategy to discriminate, but it isn't ideal. On the one hand, it spends the cost of cooperation in the first round. On the other hand, it depends on repeated game or reputation system based on indirect reciprocity. In real word, it is too demanding to meet the repeated game and reputation system is often absent, and it loses much for the less interaction iterations. Most of the existents of literature of cooperation evolution by discrimination neglect a simple discrimination mode, which based on the signal quality in a group. In the group, there are some kinds of individuals, such as cooperators, defectors and so on. Each kind must have some traits which differ from other kinds. The signal quality is determined by those traits. The signal quality supports the individual to identify the type of co-player. Some players have a subjective probability to distinguish his potential co-player according as the signal quality.

There are only three papers of correlative topic considering such incomplete information, [13-14] and [15]. The former assumed that with a probability a given individual knows the score of a randomly chosen opponent. The subjective probability in the latter is the just one in this paper, but it focuses on the trade within not only the social network but also anonymous market. It also ignores the symmetry in the interaction and the impact induced by false in judgment. In this paper, we attempt to probe into the relation between cooperation evolution and subjective probability caused by signal quality.

The paper is organized as follows. In Section 2, we develop a basic model showing the process of cooperation evolution by discrimination, analyzing the payoff of each kind of individuals and digging out the plausible conditions under which the equilibrium is dynamically stable. In Section 3, we explore dynamic stable equilibrium with two rounds. Section 4 draws some conclusions and implications for further research.

2. Cooperation Evolution Model

Let us consider a Prisoner’s Dilemma game between two strategies, cooperation and defection, with payoff matrix in Table 1.

Table 1: Payoff matrix in PD Game($b > c > 0$)

Posture person	Cooperation	Defection
Cooperation	$b-c, b-c$	$-c, b$
Defection	$b, -c$	$0, 0$

In this game, two players are offered a certain payoff, $b - c$, for mutual cooperation and a lower payoff, 0 , for mutual defection. If one player cooperates while the other defects, then the cooperator gets the lowest payoff, $-c$, while the defector gains the highest payoff, b .

For the sake of simpleness, assume in a group that there are two kinds of individuals, discriminators and defectors.

We shall denote the frequency of the discriminators by x_1 , and that of the defectors by x_2 and assume these are continuous variables. In this group, each kind shares several traits in common. These traits are represented as signal quality. So each one has a subjective probability to estimate the identity of the latent co-player. The discriminator would play the cooperate strategy if he meets another “discriminator” by his own judgment, otherwise he would play the defect strategy. The defector would always defect whatever he meet.

We also assume that the signal correctly identifies a defector with probability p and correctly identifies a discriminator with the same probability p . Here $0 < p \leq 1$.

Let P_{1t} and P_{2t} be the payoffs of the discriminator and the defector at time t respectively. We find that

$$P_{1t} = x_1 p(b-c) - (1-p)cx_2 \tag{1}$$

$$P_{2t} = (1-p)x_1 b \tag{2}$$

We can use replication dynamics equation [16-17]

$$\dot{x}_i = x_i(P_i - \bar{P}), \bar{P} = \sum x_i P_i \tag{3}$$

to investigate the evolution of the frequencies of the two types of players under the influence of selection. We only pay attention to the discriminator for the group consists of two kinds. We find that the Nash equilibrium frequencies x_1^* satisfy

$$x_1^* = \frac{c(1-p)}{(b-c)(2p-1)} \tag{4}$$

From (1-4), we have the following findings.

When $p = 1$, $P_1^t > P_2^t$, thus $x_1^* = 1$. The discriminator is intelligent and can prevent invasion from the defector.

When $p \leq 1/2$, $P_1^t < P_2^t$, thus $x_1^* = 0$. The defector would win.

When $1/2 < p < 1$, if $x_1 > \frac{c(1-p)}{(b-c)(2p-1)}$, the system

converges to be full discriminator. At this time the discriminator dominates the defector. It can prevent

invasion from the defector. If $x_1 < \frac{c(1-p)}{(b-c)(2p-1)}$, the

system reverses. It is a fixed point if

$$x_1 = \frac{c(1-p)}{(b-c)(2p-1)}, \text{ both types do equally well.}$$

3. Discussion

From above, it is more likely to establish cooperation if p increases. Moreover, cooperation evolution relates to

the value of b and c . If $\frac{b}{c}$ is great enough, it is more easily to obtain the threshold. See Figure 1 and Figure 2.

According to the effect of discrimination [12], the threshold would decrease if add some cooperators in the group.

In a large population, we assume that the discriminator would cooperate or defect according to the signal quality in the first round and then does whatever the opponent did in the previous round. It is similarly to the best response learning only with small memory capability. For the co-player of discriminators, there are $x_1 p$ cooperate and $1 - x_1 p$ defect. Thus, in the second action, there are $x_1^2 p$ discriminators would cooperate and $x_1 - x_1^2 p$ discriminators would defect among the population.

Let P'_{1t} and P'_{2t} be the payoffs of discriminators and defectors in the second round at time t respectively. We find that

$$P'_{1t} = pbx_1^2 - cp(1-x_1) \tag{5}$$

$$P'_{2t} = pbx_1^2 \tag{6}$$

Assume there are two rounds in a generation,

$\sum_i P_i = P_{it} + P'_{it} (i = 1, 2)$. From replication dynamics

equation, we can find the Nash equilibrium frequencies x_1^* satisfy

$$x_1^* = \frac{c(1-p)}{2pb-3pc-b+c} \tag{7}$$

It is similar to the result of one round. The ultimate difference to the one round is incremental threshold. That is to say, increasing interaction rounds not always facilitates cooperation establishment under the condition of no reputation system.

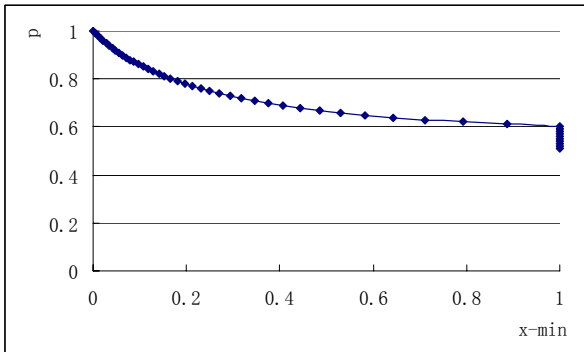


Fig. 1 The relation between *x-min* and *p* (*b/c=3*)

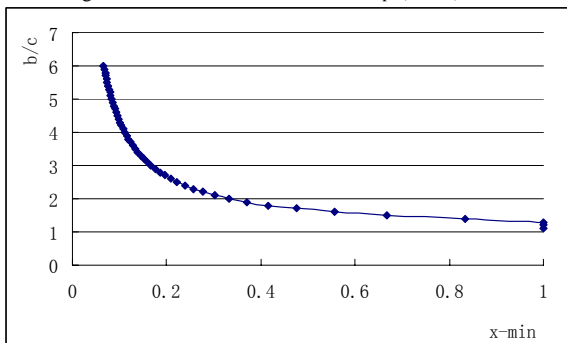


Fig. 2 The relation between *x-min* and *b/c* (*p=0.8*)

4. Conclusion

In a group, each kind of individuals behaves some traits which can be used to be classified into two types by another individual. We name those traits signal quality. Some individuals come into being a subjective probability to distinguish his potential co-player according to the signal quality.

During the course of cooperation evolution, discrimination mechanism is prerequisite, especially for exact discrimination. According to replication dynamics equation, we find that exact discrimination can decrease the threshold of cooperation. In other words, when the signal quality is greater, cooperation mechanism is easier to establish. At the same time, cooperation mechanism highly relates to the value of $\frac{b}{c}$.

If the interaction is more than one round in a generation, the threshold of cooperation would increase with the best

response learning. Namely, cooperation mechanism becomes more difficult to establish.

To sum up, maybe it is an effective means to enhance the signal quality for the cooperation establishment.

Note

1. "Cooperation" has a broad and a narrow definition. The broad definition includes all forms of mutually beneficial joint action by two or more individuals. The narrow definition is restricted to situations in which joint action poses a dilemma for at least one individual such that, at least in the short run, that individual would be better off not cooperating. We employ the narrow definition in this paper. The "cooperate" vs. "defect" strategies in the Prisoner's Dilemma and Commons games anchor our concept of cooperation, making it more or less equivalent to the term "altruism" in evolutionary biology. Thus, we distinguish "coordination" (joint interactions that are "self-policing" because payoffs are highest if everyone does the same thing) and division of labor (joint action in which payoffs are highest if individuals do different things) from cooperation.

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