Referenced Contrast: A New Approach to the Blind Deconvolution of MIMO Channels

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Summary
Contrast-based separation of sources has a number of advantages. Among others, they are optimal in presence of noise. In this work, we propose a new contrast by considering a so-called reference signal that allows not only obtaining the optimal numerical algorithm but also yields better performance in terms of variance of the estimated mixing filter. This contrast needs source kurtosis to have the same sign. Thus, it's appropriate to multichannel blind deconvolution.

Key words:
Reference signals, contrast, convolutive mixture, blind source separation

Introduction

All situations related to the processing of data resulting from the reception of several source signals lead to a difficult analysis. Actually, blind identification, equalization and separation are dedicated to this kind of situations and constitute the most up-to-date methods in signal processing. In recent decade, Blind Source Separation (BSS) became one of the executing new topics in advanced statistics and signal processing and it applies to major fields such as audio, seismology and even within the medical framework. Blind identification methods depend on the nature of the criterion to measure the statistical mutual independence between the output signals. For example, in presence of a known distribution noise, the evaluation of input signals can be performed according to a Maximum Likelihood or a Maximum a Posteriori procedure. If this is not the case, contrast criteria are devoted to this kind of situation. In our project, we would like to apply some contrast functions along with the so called reference systems to improve performances of blind source separation methods. This has been studied in numerous papers, including the first contributions dedicated to the instantaneous source mixtures [1] [2], or more recently concerning convolutive mixtures [3].

Because of their optimization simplicity, these referenced contrasts are extremely appealing. Thus, our main contribution will relate to the generalization of a contrast proposed in [4] from the instantaneous mixture to convolutive one. Then, we use the so called irreducible paraunitary factorization to carry out the equalization. Based on this decomposition, we show that the use of our proposed contrast comes with a decreased computation cost as a weak work of memory in contrary to the standard one [5] [6].

The paper is organized as follows. Section II introduces the assumptions made on the inputs and the mixture model. Section III presents the new proposed contrast, and section IV describes the principles of the numerical algorithm used to solve polynomial systems. In Section V we investigate performances of our proposition with computer experiments. Finally, section VI is dedicated to draw remarks and conclusion

2. System model and assumptions

Let $\mathbf{x}(n)$ be the signals observed at the sensors. It's defined as the output of an unknown N-inputs N-outputs (MIMO) filter with linear transfer function $\mathbf{H}(z)$ driven by a N-variate time series $\mathbf{s}(n) = (s_1(n); s_2(n);\ldots; s_N(n))^T$ 'the sources', the components of which are statistically independent and stationary signals. The goal is to recover the source processes $s_i(n)$, $i \in \{1;,,,;N\}$ from the sole observation of $\mathbf{x}(n)$. The equation that describes the mixing process for convolutive mixing is given by the following model:

$$\mathbf{x}(n) = [\mathbf{H}(z)].\mathbf{s}(n) \qquad (1)$$

The MIMO equalization problem consists of finding a filter $\{\mathbf{S}\} = \{\mathbf{S}(n); n \in \mathrm{Z}\}$ such as the outputs $\mathbf{y}(n)$ of the equalizer are copies of inputs $\mathbf{s}(n)$ under the following assumptions:

- The source signals $s_i(n)$, $i \in \{1;\ldots;N\}$ are mutually independent i.i.d. zero mean process with unit variance.
- Each source signal $s_i(n)$, $i \in \{1;\ldots;N\}$ is stationary complex signal such that $Cum_4\{s_i\} \neq 0$.

Manuscript received March 2, 2006.
Manuscript revised March 25 , 2006.

- The global transfer matrix $\mathbf{G}(z) = \mathbf{S}(z)\mathbf{H}(z)$, satisfies the property $\mathbf{G}(z)\mathbf{G}^H(1/z^*) = I$ where I denotes $N\times N$ identity matrix, (^{H}) the complex conjugation and (H) the conjugate transposition; in other words, $\mathbf{H}(z)$ and $\mathbf{G}(z)$ are paraunitary and hence $\mathbf{S}(z)$.

3. A NEW EQUALIZATION CRITERION

Basically, a contrast function plays the role of an objective function in the sense that its (global) maximization allows us to solve the problem of separation. For definitions of contrast; refer to [7] and references therein. Now, in order to propose the new contrast, we need to define the vector $\mathbf{z}(n)$ as the output of another separating filter defined according to

$$\mathbf{z}(n) = \mathbf{T}[z]\mathbf{s}(n) \qquad (2)$$

where the filter $\mathbf{T}(z) = \mathbf{Hr}(z)\mathbf{H}(z)$, which corresponds to a second global system, is assumed not equal to $\mathbf{G}$. the signal $\mathbf{z}(n)$ is called the reference signal and $\mathbf{Hr}$ the reference matrix.

3.1 A generalized criterion

The main purpose of this section is to propose a new contrast function. To this end, we define first the following 4-th order cumulant,

$$\mathfrak{C}_z\{\mathbf{y}(n)\} = \mathrm{Cum}_4(y_{i_1}(n), y_{i_1}^*(n), z_{i_2}(n), z_{i_2}^*(n)), \qquad (3)$$

and we introduce a technical assumption about the reference system:

Hypothesis 1 $\forall (i,j) \in \{1,...,N\}$ *and* $k \in \mathbb{Z}$:
$0 < |T_{ij}(k)| < |T_{ij}(0)|$

Proposition 1
For a given normalized signal z(n), the function :

$$I_4^z(y(n)) = \varepsilon \sum_{i_1}\sum_{i_2} \mathfrak{C}_z\{y(n)\} \qquad (4)$$

is contrast if all the source kurtosis have the same sign ε.
Proof :
Using the cumulant multilinearity, the source independence and the **Hypothesis 1**, we can write

$$I_4^z(\mathbf{y}(n)) \leq \varepsilon \sum_j \sum_{i_2} T_{i_2 j}(0)T_{i_2 j}^*(0)\mathrm{Cum}_4(s_j) \sum_k \sum_{i_1} G_{i_1 j}(k)G_{i_1 j}^*(k) \qquad (5)$$

and with the paraunitary property of $\mathbf{G}$, it follows

$$I_4^z(\mathbf{y}(n)) \leq \varepsilon \sum_j \sum_{i_2} T_{i_2 j}(0)T_{i_2 j}^*(0)\mathrm{Cum}_4(s_j)$$

The source signals are assumed to be independent sequences of i.i.d. complex random variables, then

$$I_4^z(\mathbf{y}(n)) \leq I_4^z(\mathbf{s}(n)) \qquad (6)$$

The second property of a contrast will be reached if

$$\sum_{i_1}\sum_{i_2}\sum_{k}\sum_{j} G_{i_1 j}(k)G_{i_1 j}^*(k)T_{i_2 j}(k)T_{i_2 j}^*(k)\mathrm{Cum}_4(s_j) - \sum_{i_1}\sum_{i_2} T_{i_2 i_1}(0)T_{i_2 i_1}^*(0)\mathrm{Cum}_4(s_{i_1}) = 0 \qquad (7)$$

Since the terms between summations are positive, (8) is rewritten as

$$G_{i_1 i_1}(0)G_{i_1 i_1}^*(0)T_{i_2 i_1}(0)T_{i_2 i_1}^*(0)cum_4(s_{i_1}) + \sum_{k\neq 0}\sum_{j\neq i_1} G_{i_1 j}(k)G_{i_1 j}^*(k)T_{i_2 j}(k)T_{i_2 j}^*(k)\mathrm{Cum}_4(s_j) = T_{i_2 i_1}(0)T_{i_2 i_1}^*(0)\mathrm{Cum}_4(s_{i_1}) \qquad (8)$$

By identification, we will have:

$$G_{i_1 i_1}(0)G_{i_1 i_1}^*(0) = 1$$

and

$$G_{i_1 j}(k)G_{i_1 j}^*(k) = 0 \ \forall k \neq 0 \text{ and } j \neq i_1.$$

4. NUMERICAL ALGORITHM

The proposed technique, to solve the separation problem, is based on the respect of the paraunitary property of the considered model (1), obtained after whitening observations. In order to realize this, we follow the same factorization procedure as stated in [5]. It consists in factorizing any paraunitary filter of length L as

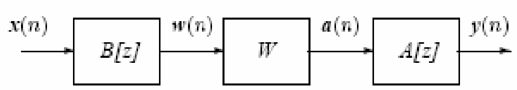

Fig. 1. Factorization of paraunitary equalizer

$$\mathbf{S}[z] = \mathbf{A}[z].\mathbf{W}.\mathbf{B}[z] \qquad (9)$$

where $\mathbf{A}[z]$ and $\mathbf{B}[z]$ are FIR paraunitary filters of length l_a and l_b respectively, with:

$$0 \le l_a \le L$$

and

$$0 \le l_b \le L,\ l_a + l_b = L$$

and **W** is a $N \times N$ unitary matrix.

Now, to address our MIMO equalization problem, we adopt the following notations

$$\mathbf{X}^{kk'}_{jj'q} = \mathsf{cum}(x_j(n-k), x^*_{j'}(n-k'), z_q(n), z^*_q(n)) \quad (10)$$

where *{i; j; q}* take their values in {1;...;*N*}, and *k*, *k'* in Z_+. At present, consider the following input-output relations for the convolutive model

$$y_i(n) = \sum_k \sum_{jj'} A_{ij}(k) W_{jj'} w_{j'}(n-k) \quad (11)$$

Thus, for N=2 :

$$I^z_4(\mathbf{y}(n)) = \mathbf{Y}^{00}_{111} + \mathbf{Y}^{00}_{112} + \mathbf{Y}^{00}_{221} + \mathbf{Y}^{00}_{222} \quad (12)$$

where

$$\begin{aligned} \mathbf{Y}^{00}_{iiq} = &\ \mathsf{cum}(y_i(n), y^*_i(n), z_q(n), z^*_q(n)) \\ = &\ \mathsf{cum}\Bigg(\sum_k \sum_{jj'} A_{ij}(k) W_{jj'} w_{j'}(n-k), \\ & \sum_k \sum_{jj'} A^*_{ij}(k) W^*_{jj'} w^*_{j'}(n-k), z_q, z^*_q\Bigg) \end{aligned} \quad (13)$$

Because of the multilinearity property of cumulants, we have :

$$\begin{aligned} \mathbf{Y}^{00}_{iiq} = & \sum_{kk'} \sum_{jj'j_1j'_1} A_{ij}(k) W_{jj'} A^*_{ij_1}(k') W^*_{j_1j'_1} \\ & \mathsf{cum}(w_{j'}(n-k), w^*_{j'_1}(n-k'), \\ & z_q(n), z^*_q(n)) \end{aligned} \quad (14)$$

According to the notation in (11), we have:

$$\mathbf{Y}^{00}_{iiq} = \sum_{kk'} \sum_{jj'j_1j'_1} A_{ij}(k) W_{jj'} A^*_{ij_1}(k') W^*_{j_1j'_1} \omega^{kk'}_{j'j'_1q} \quad (15)$$

Moreover, **W** is a unitary matrix and it is well known that any $N \times N$ unitary matrix can be written as a product of $M = N(N-1)/2$ Givens rotations, themselves functions of 2 angles (θ,φ). For $N = 2$, unitary matrix $\mathbf{W}_p$ can be generated thanks to one rotation, for all *p*

$$\mathbf{W}_p = \begin{pmatrix} \cos(\theta_p) & \sin(\theta_p)e^{i\phi_p} \\ -\sin(\theta_p)e^{-i\phi_p} & \cos(\theta_p) \end{pmatrix} \quad (16)$$

Thus, the proposed contrast in (4) can be rewritten as

$$\begin{aligned} I^z_4(\mathbf{y}(n)) = &\ \cos(\theta)^2 T_5 - \sin(\theta)^2 T_6 + \cos(\theta) \\ & \sin(\theta) e^{i\phi} T_1 - \cos(\theta)\sin(\theta) \\ & e^{-i\phi} T_2 + \sin(\theta)^2 e^{2i\phi} T_3 \\ & + \sin(\theta)^2 e^{-2i\phi} T_4 \end{aligned} \quad (17)$$

where

$$\begin{aligned} T_1 &= \sum_{iq} \sum_{kk'} (A_{i1}(k) A^*_{i1}(k') \omega^{kk'}_{12q} + A_{i1}(k) A^*_{i1}(k') \omega^{kk'}_{21q} \\ & \quad A_{i1}(k) A^*_{i2}(k') \omega^{kk'}_{22q} + A_{i2}(k) A^*_{i1}(k') \omega^{kk'}_{22q}) \\ T_2 &= \sum_{iq} \sum_{kk'} (A_{i1}(k) A^*_{i2}(k') \omega^{kk'}_{11q} + A_{i2}(k) A^*_{i1}(k') \omega^{kk'}_{11q} \\ & \quad A_{i2}(k) A^*_{i2}(k') \omega^{kk'}_{12q} + A_{i2}(k) A^*_{i2}(k') \omega^{kk'}_{21q}) \\ T_3 &= \sum_{iq} \sum_{kk'} (A_{i1}(k) A^*_{i1}(k') \omega^{kk'}_{22q}) \\ T_4 &= \sum_{iq} \sum_{kk'} (A_{i2}(k) A^*_{i2}(k') \omega^{kk'}_{11q}) \\ T_5 &= \sum_{iq} \sum_{kk'} (A_{i1}(k) A^*_{i1}(k') \omega^{kk'}_{11q} + A_{i1}(k) A^*_{i2}(k') \omega^{kk'}_{12q} \\ & \quad + A_{i2}(k) A^*_{i1}(k') \omega^{kk'}_{21q} + A_{i2}(k) A^*_{i2}(k') \omega^{kk'}_{22q}) \\ T_6 &= \sum_{iq} \sum_{kk'} (A_{i1}(k) A^*_{i2}(k') \omega^{kk'}_{21q} + A_{i2}(k) A^*_{i1}(k') \omega^{kk'}_{12q}) \end{aligned}$$

In this stage, we can notice that the form of the proposed contrast is less complex than the one in [8]. Moreover, one can prove that $\mathbf{Y}^{00}_{iiq}$ is real by applying Cauchy-Schwarz inequality.

$$\begin{aligned} I^z_4(\mathbf{y}(n)) = &\ \cos(\theta)^2 \Re(T_5) - \sin(\theta)^2 \Re(T_6) + \cos(\theta) \\ & \sin(\theta) \Re(e^{i\phi} T_1) - \cos(\theta)\sin(\theta) \Re(e^{-i\phi} T_2) \\ & + \sin(\theta)^2 \Re(e^{2i\phi} T_3) + \sin(\theta)^2 \Re(e^{-2i\phi} T_4) \end{aligned}$$

Then,We have to find the pairs (θ,φ) that maximizes the contrast (18). To reach this goal, we follow th steps described in [8]. Firstly, we make the classical change of variable: θ = arctan(*u*) and φ = 2 ∗ arctan(*t*). After, we differentiate the function acording to variable *u* and *t* indivudially. We obtain 2 polynomials:

$$P1 = \frac{\partial I^z_4(\mathbf{y}(n))}{\partial u} \quad (18)$$

$$P2 = \frac{\partial I^z_4(\mathbf{y}(n))}{\partial t} \quad (19)$$

Polynomial *P*1 is the global degree of 8 (5 in *t* and 3 in *u*) whereas it is of 12 in [8]. While Polynomial *P*2 is the global degree of 10 (8 in *t* and 2 in *u*)whereas it is of 11 in the same reference.

In addition, we need only *NL*×*NL*∗*N* to compute the cumulants matrix of the signals **w** whereas it is the size of *NL*×*NL***NL***NL* in their approach. The roots of the system above can be obtained by using the resultant of a Sylvester matrix of which components are:

$$\begin{pmatrix} q0 & 0 & 0 & d0 & 0 \\ q1 & q0 & 0 & d1 & d0 \\ q2 & q1 & q0 & d2 & d1 \\ 0 & q2 & q1 & d3 & d2 \\ 0 & 0 & q2 & 0 & d3 \end{pmatrix} \qquad (20)$$

where

$$\begin{aligned}
d3 =& \quad 0 \\
d1 =& \quad -2*(1+t^2)^2*\beta+2*(2*(1-t^2)^2 \\
& -(1+t^2)^2)*\gamma+8*t*(1-t^2)*\delta \\
& -2*\alpha*(1+t^2)^2 \\
d2 =& \quad -(1-t^4)*\xi+2*t*(1+t^2)*\zeta \\
d0 =& \quad (1-t^4)*\xi-2*t*(1+t^2)*\zeta \\
q2 =& \quad \gamma*[-(1+t^2)^2*(8*(1-t^2)*t \\
& +4*t*(1+t^2)) \\
& -4*t*(1+t^2)*(t^4-6*t+1)]+ \\
& \delta*[(1+t^2)^2*(4*(1-t^2)-8*t^2) \\
& -16*t^2*(1-t^4)]; \\
q1 =& \quad \xi*[-4*(1+t^2)^2*t^3-4*t*(1+t^2)^2 \\
& *(1-t^2)]+\zeta*[(-2*(1+t^2)-4*t^2) \\
& *(1+t^2)^2+8*t^2*(1+t^2)^2]; \\
q0 =& \quad 0;
\end{aligned}$$

$$\begin{aligned}
\alpha =& \quad \Re(T5) \\
\beta =& \quad \Re(T6) \\
\gamma =& \quad \Re(T3)+\Re(T4) \\
\delta =& \quad \Im(T4)-\Im(T3) \\
\xi =& \quad \Re(T1)-\Re(T2) \\
\zeta =& \quad \Im(T1)+\Im(T2)
\end{aligned}$$

5. SIMULATION

In order to illustrate the performance of the above results, some computer simulations are now presented. The sources were both PSK-4 (figure4) and the mixing filter was a paraunitary channel so that the prewhitening stage is not necessary. Our performance criteria is a simpler version of Distance criterion: ”GAP” and Symbol Error Rate ”SER” used in [5]. Figure 2 shows the mean of ser obtained with reference contrast (CR), with solide line, and the standard one (CS), with dash-dotted line, over 20 Monte Carlo trials with data length N = 1500 for various SNR’s. Figure 3 reports the corresponding GAP. There is a visible advantage of the proposed contrast over the standard one especially at the low SNR. In addition, as shown by 3, the maximization of $I_4^{\tilde{z}}(\mathbf{y}(n))$ yields better results.

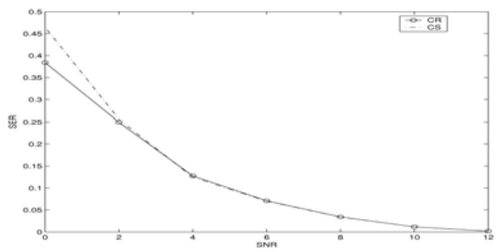

Fig. 2. SNR virsus SER for the two contrasts

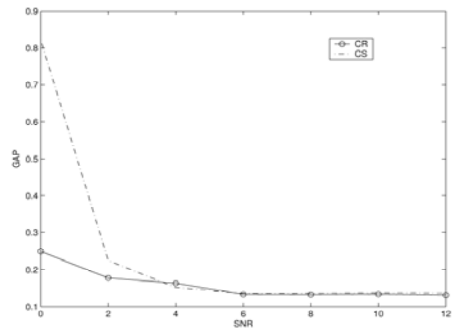

Fig. 3. GAP versus SNR

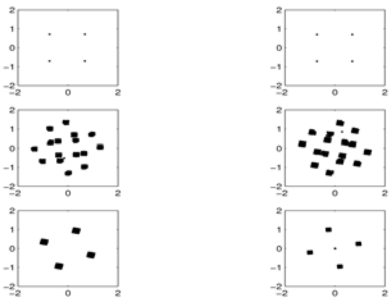

Fig. 4. (top) sources signals (middle) observations signals (below) restored sources

6. CONCLUSION

In this article, we have essentially proposed a generalization of a contrast from the instantaneous to the convolutive domain. As shown by the numerical algorithm, the maximization of $I_4^{\tilde{z}}(\mathbf{y}(n))$ yields better results at a low computational cost.
Finally, the computer simulations have described a simpler variation of the algorithm which is efficient at low SNR.

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