Multi-resolution Compression of Meshes based on Reverse Interpolatory $\sqrt{3}$ Subdivision Scheme*

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Summary
A new arbitrary triangle mesh compression algorithm based on reverse interpolation $\sqrt{3}$ subdivision scheme is presented in this study. This algorithm is a lossless compression algorithm. The study iterates reverse interpolation $\sqrt{3}$ subdivision scheme to simplify the detailed model into a sequence of approximate multi-resolution. A progressive method is constructed to store the meshes and transmit them via wired and wireless network environment using the sequence and the errors. The result shows the proposed method for complex model simplification is efficient and can be used in many relative fields of 3D graphics applications. The increment and decrement of the face number used this algorithm is slow when compared to other methods. It generates more levels of details which are more suitable for multi-resolution representation.

Key words:
Mesh compression, Multi-resolution Presentation, Progressive Transmission

Introduction

To accommodate the need for detailed mesh model and to maintain a convincing level of realism in the computer graphics application, the models are often created at a very high resolution. However, the cost of storing and transmitting the full detailed models, which are often represented as complex triangle meshes that are very high [14]. It is a challenge to store, transmit and render the graphics models through network, especially in wireless network [16, 20]. For 3D graphics visualization applications on mobile devices [24], a simpler version of complex models aids to reduce storing space and save transmission bandwidth. It's more important that the original complex models will be reconstructed from the coarse ones with few data. The significance of mobile graphics applications are mesh simplification, progressive representation for transmission and view-dependant rendering [17].

Substantial results had been reported in the last few years on surface simplification [1, 2, 7]. Hoppe [3] first introduced the concept of Progressive Mesh (PM) as a multi-resolution framework to represent triangular meshes based on edge collapse and vertex split transformations. Bartels [4] and Samavati [5] had introduced reverse subdivision rules to produce multi-resolution presentation. Lifting wavelet representations were the most efficient multi-resolution method [6, 19, 21, 23]. Some researchers had described methods to simplify surfaces and to represent or optimize the meshes [13, 14, 18].

Subdivision was derived from B-spline [12]. It had utilized refinement rules to address the challenge of building smooth surfaces of arbitrary topology. The smooth surfaces were constructed through a limiting process of repeated refinement using an arbitrary connectivity control mesh. Subdivision method was also popular in geometric modeling [8].

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Subdivision generates smooth surfaces from a given control mesh M_0. By this coarse mesh a sequence of refined meshes $M_1, \cdots, M_n$ is computed. Under the limit this sequence of meshes converges to a continuous smooth surface. Each refinement step is divided into two different operations. First, a topological operation is performed. Therefore, new vertices are added to the mesh and the triangles are split. Then the geometry of the mesh is changed by a smoothing operation. In order to converge to a smooth limit surface M_∞ the subdivision scheme has to satisfy certain necessary conditions.

The present study introduces a new reverse interpolatory $\sqrt{3}$ subdivision method to simplify triangular meshes, and constructs a progressive mesh representation to store and to transmit triangular meshes.

The rest of this paper is organized as follows. Section 2, describes related works. Section 3 outlines the details of the proposed method for meshes decomposition and reconstruction based on interpolatory $\sqrt{3}$ subdivision method. The progressive representation and transmission are also described. Section 4, introduces the progressive applications. Sketch out the detailed framework of the progressive source and channel coding of wireless network. Section 5, shows the implementation and simulation results of this study. Section 6, summarizes main concepts of this study and future works.

2. Related Works

2.1 Interpolatory $\sqrt{3}$ subdivision

Subdivision methods are popular for generating curves and surfaces in computer graphics. This process is achieved by linear operations; each subdivision is interpreted as a matrix transformation which applied to the vertices. For example, if V^i is the vertex at the ith level, and V^{i+1} is obtained by a subdivision of V^i, then

$$V^{i+1} = P^i V^i \tag{1}$$

where V^i is an r-vector, V^{i+1} is an s-vector (where s>r), and P^i, $s \times r$, is the subdivision matrix. P^i depends on i. In subdivision schemes each vertex of V^{i+1} depends only on a few vertexes of V^i that is suitable in geometric proximity, and this implies P^i is sparse. Since the points of the vectors are ordered according to geometric proximity, P^i becomes a banded matrix. For the simple subdivision of curves, each column in the matrix P^i is a shifted version of the previous column, except for a few initial and final columns, hence, it is easy to generate.

Interpolatory $\sqrt{3}$ subdivision [9] is an interpolating subdivision scheme, only the positions of new vertices have to be computed. The stencil is shown in Figure 1. The new q^{k+1} is calculated by following equation [9].

$$\begin{aligned} q^{k+1} = {} & \tfrac{32}{81}(p_1^k + p_2^k + p_3^k) \\ & - \tfrac{1}{81}(p_4^k + p_5^k + p_6^k) \\ & - \tfrac{2}{81}(p_7^k + p_8^k + p_9^k + p_{10}^k + p_{11}^k + p_{12}^k). \end{aligned} \tag{2}$$

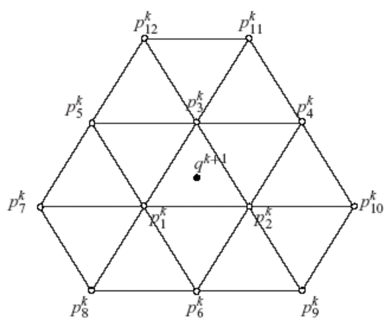

Fig.1. Stencil for the computation of a new vertex.

This subdivision scheme does not use the normal 1-to4 splitting operator where the mesh is refined by inserting one new vertex at each edge. In this scheme, the middle of every triangle t_j of a mesh M_k, a new vertex q_j is computed. This vertex is connected to the old vertices of the triangle. To achieve a regular mesh structure, all old edges of the mesh are now flipped. This means an edge between two triangles t_{j_1} and t_{j_2} is removed and an edge between the new vertex p_{j_1} and p_{j_2} is inserted. This process is illustrated in Figure 2.

The interpolatory $\sqrt{3}$ subdivision stencil is used in the present study to simplify and figure out the errors during the process.

2.2 Progressive Mesh Construction and Mesh Simplification

Hoppe [3] presented a progressive mesh to represent triangular meshes. An initial denser mesh $M = M_n$ was simplified into a coarse mesh M_0 by iterating simplification operations.

$$M_n \rightarrow M_{n-1} \rightarrow \cdots \rightarrow M_0 \quad (3)$$

Hoppe used a method based on edge collapse and vertex split transformations. While most earlier simplification methods were based on vertex decimation process and Jian-Hua Wu et al. [2] introduced a face construction process (FCP).

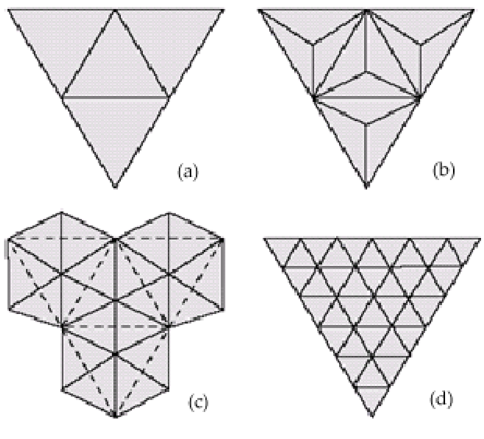

Fig.2. Splitting of the mesh. (a) Original triangle;(b) One new vertex per triangle is computed; (c) Connect the new vertexes with the previous vertexes and the previous edges are flipped; (d) After two refinement steps every triangle is devided into 9 new triangles.

The proposed method is not the same as FCP, the following section gives the details. The proposed mesh method represented is similar to PM of Hoppe, except for the error representation and it is described in the proceeding section.

2.3 Reverse Subdivision

Samavati [5] constructed multi-resolution surfaces of arbitrary topologies by locally reversing the Doo subdivision scheme, and converting a fine surface into a coarse one with an error. The conversion has the property that the subdivision of the resulting coarse surface is closest to the original fine surface, in the least squares sense, for two important face geometries. In the process, first find the faces of the fine surface which are produced by the contraction of a coarse face in a Doo subdivision scheme. Then, expand these faces. Since the expanded faces are not joined properly, several candidates are usually retained for a single vertex of the coarse surface. Finally, vertices of the coarse surface are obtained by taking the average of their corresponding candidates.

Zheng et al. [10] constructed a progressive meshes transmission using reverse butterfly subdivision scheme, and simplified a denser mesh to a coarser one plus a set of errors. Butterfly subdivision is a 1-to-4 subdivision [11], every edge generates a new vertex according to the butterfly-like stencil. Thus the face number is quadrupled in each split process. So, it is not suitable to create more multi-resolution levels within bearable computing time.

The proposed method is based on interpolatory $\sqrt{3}$ subdivision scheme, the study does not utilize the expanding method to determine the coarse mesh since there are many difference between Doo and interpolatory $\sqrt{3}$ subdivision. Figure 2, shows the face numbers have been tripled after each interpolatory $\sqrt{3}$ subdivision. So the increasing speed of the faces is reduced to generate more multi-resolution levels.

3. Mesh Simplification and Transmission

3.1 Reverse Interpolatory $\sqrt{3}$ subdivision

Using equation (2), subdivide a coarser mesh to a denser mesh. Mesh Simplification is a reverse process, and it converts a given denser mesh into a coarser mesh. Figure 3 shows a mesh generated by interpolatory $\sqrt{3}$ subdivision scheme where the circle vertex is generated by the quadrate vertexes. It is found that the new generated vertexes are distributed regularly. Thereby, the reverse interpolatory $\sqrt{3}$ subdivision decimates those circle vertexes efficiently.

An error metrics defined with topological operation is a significant feature in the mesh simplification. The quality of the resulting meshes mainly depends on the underlying error metrics. In this study, the prediction and update processes are used for the error metrics. Update the vertexes of the coarse mesh locally to reduce the error, and the error is recorded and used for reconstruction.

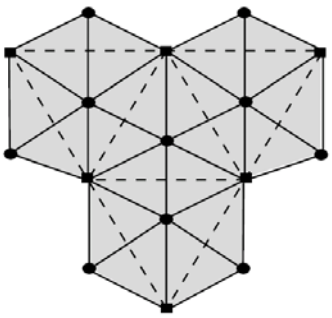

Fig.3. Semi-regular mesh with vertex valence 6.

Consider a mesh M_j which has to be simplified into a coarse mesh M_{j-1} plus an error e_{j-1}. The proposed approach consists of three steps: vertexes division, subdivide vertex prediction and vertex modification.

3.1.1 Vertexes Division

The step to find vertexes to eliminate is called as vertexes division, which disjoins all vertexes of the mesh into two groups. One group consists of vertexes to be eliminated, called as odd group, even they are not the odd index vertexes. The other group consists of vertexes of coarse mesh is called even group. Then the split process is denoted by the following equation.

$$(Even_{j-1}, Odd_{j-1}) := Split(M_j) \tag{4}$$

For regular mesh whose valence of each vertex is 6, easily divides the whole point set into even and odd groups. The circle vertexes in Figure 3 belong to odd group.

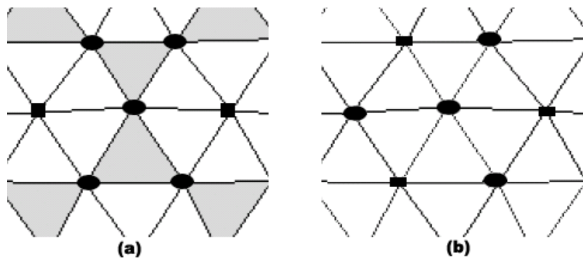

Fig.4. (a) Odd point has 4 neighbor points around it which are eliminated in reverse butterfly subdivision scheme (b) Odd point has 3 neighbor points around it which are eliminated in the proposed method

When compare with the previous work [10], this study defines a new method to divide odd points from the whole point sets, they are as follows. The neighbor points of the even group will be in the odd group. The each point in the odd group has neighbor points which are divided into two parts as odd group and even group. For regular mesh with valence of 6 [10], 4 vertexes of 6 are divided into odd group that are eliminated in the next compress. In this study, only 3 vertexes of 6 are eliminated in the following compress. Thus the points to be eliminated are reduced in the proposed method when compare to the previous methods.

A semi-regular mesh with some extraordinary vertexes whose valence is not 6 is placed in the extraordinary vertexes of the even group. The other vertexes are easily split into even and odd groups if the semi-regular mesh satisfies the following condition:

$$Distance(P^i, P^j) = 3^k, k = 1,2,\cdots \quad (5)$$

where $Distance(P,Q)$ is the distance between vertex P and Q, P^i and P^j are two vertexes in the extraordinary vertex set. The vertexes, which does not satisfy the above condition (5), are classified as irregular mesh.

Additional works are done to make the split procedure run in the irregular mesh. Next convert them to regular or semi-regular meshes. Following the above use re-sample method to convert dense meshes to subdivision connectivity meshes.

3.1.2 Subdivided Vertex and Error Prediction

This process is used to calculate the error before decimating the odd vertexes. It is known that the odd vertexes are generated by even vertexes in interpolation $\sqrt{3}$ subdivision scheme. So, each odd vertex is subdivided by the corresponding 12 even vertexes, i.e. using equation (2), and predicts a new value which is approximate to it. If the denser mesh is exactly subdivided from the coarser one, the predict value will be equal to the odd vertex. However, they are often different after editing. Then the difference e_{j-1} is figured out using following equation, where P calculates the predict value from equation (2).

$$e_{j-1} = Odd_{j-1} - P(Even_{j-1}) \quad (6)$$

3.1.3 Vertexes Modification

To reduce the geometric error, update the even vertexes locally before eliminating the odd ones. Thus, defined an operator U to update M_{j-1} as follows:

$$M_{j-1} = Even_{j-1} - U(e_{j-1}) \quad (7)$$

where e_i is the error generated in the predict process. The next section determines value of U by analysis.

From equation (2), derived the following

$$\begin{aligned}\Delta q^{k+1} &= \tfrac{32}{81}(\Delta p_1^k + \Delta p_2^k + \Delta p_3^k) \\ &- \tfrac{1}{81}(\Delta p_4^k + \Delta p_5^k + \Delta p_6^k) \\ &- \tfrac{2}{81}(\Delta p_7^k + \Delta p_8^k + \Delta p_9^k + \Delta p_{10}^k + \Delta p_{11}^k + \Delta p_{12}^k)\end{aligned} \quad (8)$$

where Δq^{k+1} is the error of vertex q^{k+1}, The vertexes with different distance to q^{k+1} contributes different error to q^{k+1} and the ones with distance larger than 3 contributes nothing. That is, P_1^k, P_2^k and P_3^k with distance of 1 contributes $\frac{32}{81}\Delta$ to Δq^{k+1}, P_4^k, P_5^k and P_6^k with distance of 2 contributes $-\frac{1}{81}\Delta$ to Δq^{k+1}, while P_7^k, P_8^k, P_9^k, P_{10}^k, P_{11}^k and P_{12}^k with distance of 3 contributes $-\frac{2}{81}\Delta$ to Δq^{k+1}.

By observing the odd vertexes around the middle even vertex, derived the following

$$U(e) = \tfrac{32}{81}\sum e^1 - \tfrac{1}{81}\sum e^2 - \tfrac{2}{81}\sum e^3 \quad (9)$$

where $e_i, i = 1,2,3$ denotes the predict error of the odd vertexes with distance i to the even vertex of update.

3.2 Mesh Reconstruction

The reconstruction scheme is immediately built from the above description:

(i) Undo Modify Process:

$$Even_{j-1} = M_{j-1} - U(e_{j-1})$$

(ii) Undo Predict Process:

$$Odd_{j-1} = e_{j-1} + P(even_{j-1})$$

(iii) Merge Process:

$$M_j := Merge(Even_{j-1}, Odd_{j-1})$$

The equations above reconstruct the original dense mesh exactly.

4. Progressive Applications

By iterating the reverse interpolatory $\sqrt{3}$ subdivision process, a more coarse mesh and a sequence of errors are obtained, which are the details of the original mesh. i.e. the progressive mesh representation of the original mesh is as follows.

$$\begin{aligned}(M_n) &\to (M_{n-1}, e_{n-1}) \to \cdots \\ &\to (M_0, e_0, e_1, \cdots, e_{n-1})\end{aligned} \tag{10}$$

When turning to wire cum wireless network environment, the transmission bandwidth is not always stable to send the massive data of the complex meshes. So, the progressive transmission method is brought forward.

When a transmission session is started, the coarsest base mesh is sent first. The client end displays the base mesh when the transmission is finished. Then, if a more detailed surface is required, a command string is sent from client to server by requesting the next level of details. When the detail is obtained, the client reconstructs a denser mesh which is the same as the original one.

The steps of the proposed algorithm are given below.

(a) Find a vertex that has a valence not equal to 6 and mark it as preserved vertex.

(b) The vertexes that are adjacent to the preserved vertex in step 1 must have a valence of 6. Choose one vertex from the adjacent vertexes arbitrarily and mark it as an eliminated vertex. Next, proposed algorithm begins with the chosen vertex.

(c) The chosen vertex must be a vertex with valence 6 and will be eliminated. Then select 3 preserved vertexes and mark the left 3 vertexes as eliminate vertexes from its adjacent vertexes as mentioned before.

(d) Process the 3 eliminated vertexes that selected in step 3 and thereby getting 9 new eliminated vertexes. Recursively process the eliminated vertexes until all of vertexes have been processed. Then divide all the vertexes into 2 groups as preserved ones and eliminated ones.

(f) Remove the eliminated vertexes and construct new mesh faces.

A new simplified model with less vertexes and faces is achieved.

5. Implementation and Simulation

To demonstrate the progressive mesh presentation and transmission over wired to wireless network, implement the scheme for reverse interpolatory $\sqrt{3}$ subdivision scheme and the reconstruction algorithm.

Table 1. Comparison between reverse butterfly subdivision method and Interpolatory $\sqrt{3}$ subdivision scheme

		Interpola-tory $\sqrt{3}$	Butterfly Subdivision
Original Mesh	Number of vertexes	3475	6182
	Number of faces	6942	12352
After reverse subdivi-sion	Number of vertexes	1161	1548
	Rate of decreased vertexes	66.59%	74.96%
	Number of faces	2314	3088
	Rate of decreased faces	66.67%	75%
After twice reverse subdivi-sion	Number of vertexes	389	389
	Rate of decreased vertexes	66.49%	74.87%
	Number of faces	772	772
	Rate of decreased faces	66.64%	75%

Obviously, refine the coarsest mesh without error information cannot reconstruct the original mesh. It is an important feature that the original mesh can be rebuilt by the proposed reconstruction process with the progressive presentation of mesh.

To demonstrate the progressive mesh presentation and to prove the proposed method, this study has implemented the scheme for simplification and reconstruction algorithm in VC++ 6.0, using OpenGL. The results are shown in Figure. 5. When compared with the reverse butterfly subdivision, the proposed algorithm preserves more vertexes, faces and edges, as shown in Table. 1.

6. Conclusion and Future works

This study describes a fast and easy operating mesh simplification method of reverse interpolatory $\sqrt{3}$ subdivision, and reconstruction scheme. An error metrics based on subdivision scheme is defined. A multi-resolution representation method for progressive transmission applications is proposed in the study. From the results of implementation, the proposed algorithm is efficient and fast method for multi-resolution application, especially for wireless network applications. The proposed method reduces vertexes slower than the previously methods.

The possible future works are to find an appropriate method to compress errors which are nearly zero. View-dependent rendering is an important method for mobile applications. To transmit a small size portion for the limitation display or to convert the mesh into a long strip before transmission will be considered in the near future.

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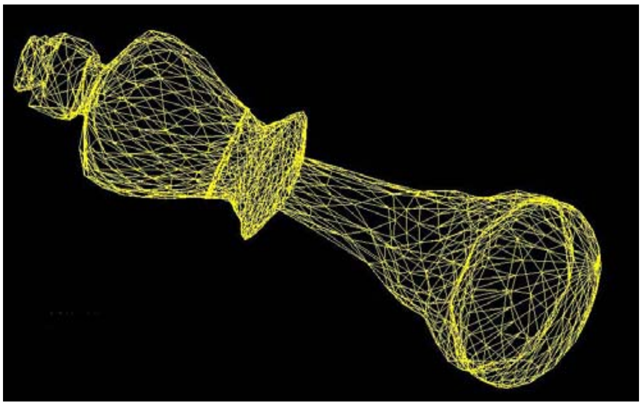
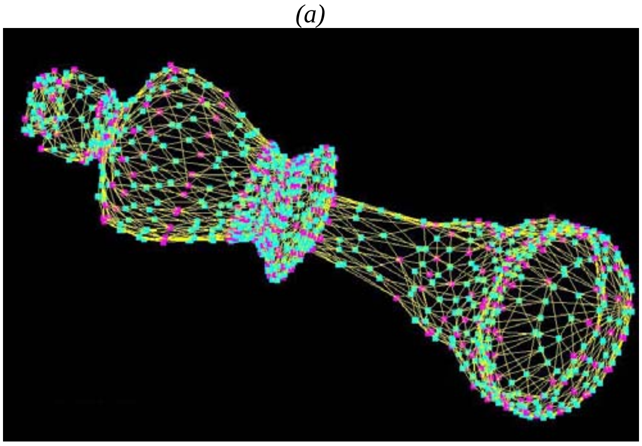
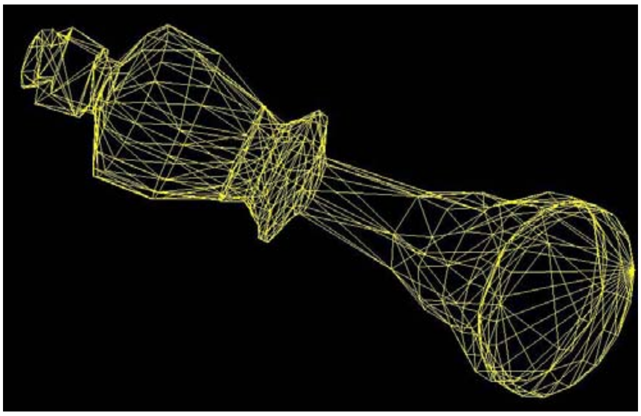


(a)



(b)



(c)

Fig.5.Illustration of the reverse interpolatory $\sqrt{3}$ subdivision: (a) Initial mesh (b) Vertex division (c) Coarse mesh after one step of simplification

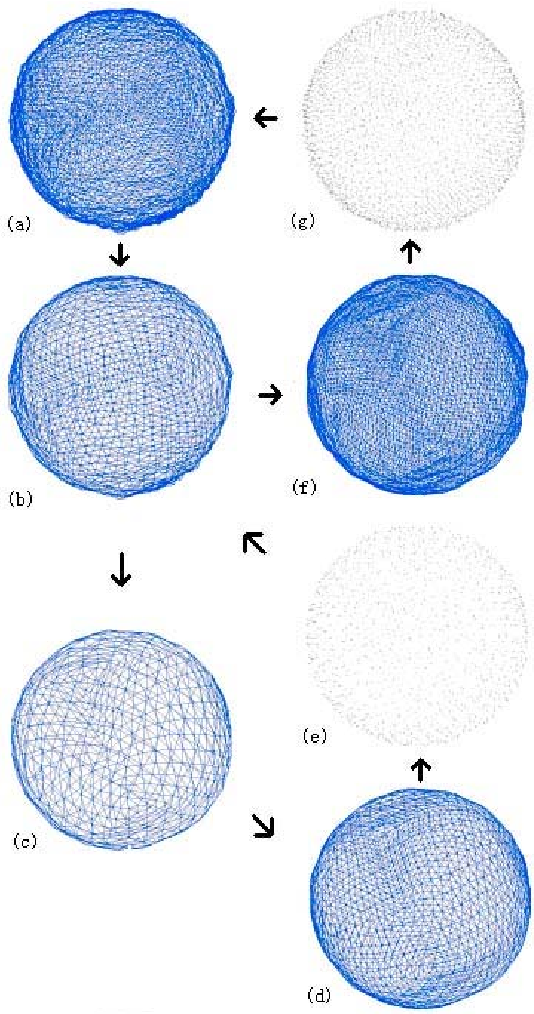


Fig. 6. Progressive Transmission of a modified planet model...(a) Original mesh;(b) Simplified mesh from (a);(c) Ssimplified mesh from (b); (d) Subdivision result of (c);(e) Error between (d) and (b); (f) Subdivision of (b); (g) Error between (f) and (a)

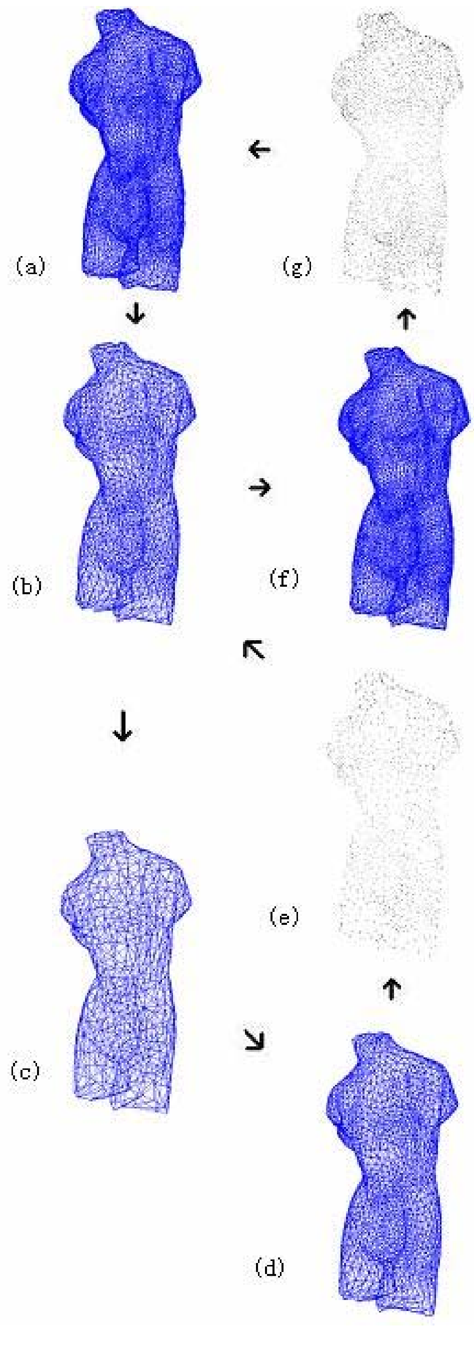


Fig. 7. Progressive Transmission of venus model…(a) Original mesh; (b) Simplified mesh from (a);(c) Simplified mesh from (b); (d) Subdivision result of (c);(e) Error between (d) and (b); (f) Subdivision of (b); (g) Error between (f) and (a)