Ultraproduct of first-order lattice-valued logic LF(X) based on finite lattice implication algebra

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Summary
In recent years, model theory has had remarkable success in solving important problems. Its importance lies in the observation that mathematical objects can be cast as models for a language. Ultraproduct is a method of constructing a new model from a family of models, In this paper, we deal with a new form of ultraproduct model for first-order lattice-valued logic LF(X) whose truth-value field is a finite lattice implication algebra. At the same time, Expansion theorem, two forms of fundamental theorem of ultraproducts and consistent theorem are obtained. Finally, another application of ultraproduct to algebra is discussed.
Key words:
Lattice-valued logic; lattice implication algebra; ultrafilter; ultraproduct; consistent theorem

1. Introduction

Lattice-valued logic is an important form of many-valued logic which extends the field of truth-values to lattices. More important, lattice-valued logic can represent the uncertainty, specially the incomparable property of people's thinking, judging and decision. Therefore, lattice-valued logic is studied by many scholars [2,3,4]. However,their work are limited to the interval [0,1] or the finite chain of truth values. In order to establish a logic system with truth-values in a relatively general lattice, lattice implication algebras were defined by Xu Yang in [7] and its many properties were discussed. Based on conclusions on lattice implication algebra, Xu Yang et al established several lattice-valued logic systems [8-12], where they established a first-order lattice-valued logic system LF(X) based on lattice implication algebra. On the basis of a set of grouping sentences or a theory, Wang Shiqiang et al in [2] proved the fundamental theorem of ultraproducts in lattice-valued model whenever the set of truth-values L is finite. Ying Mingsheng gave another form of the fundamental theorem of ultraproducts [13]. We have discussed one form of ultraproduct model for LF(X) [15], which is based on an ultrafilter on L^I, where L is any lattice implication algebra, I is an index set. Moreover, the main difference between the ultroproduct models for classical logic and for LF(X) lies in the ultrafilter, the former is a generalization of the latter. In this paper, by virtue of the idea of [6, 13], we construct another form of ultraproduct model for LF(X) based on finite lattice implication algebra by a classical ultrafilter on the index set I.

2. Preliminaries

Definition 1[7] Let $(L,\vee,\wedge,')$ be a complemented lattice with universal bounds 0, 1. If the mapping $\rightarrow: L\times L\rightarrow L$ satisfies the following conditions: for all $x,y,z\in L$,

(I_1) $x\rightarrow(y\rightarrow z)=y\rightarrow(x\rightarrow z)$,
(I_2) $x\rightarrow x=1$,
(I_3) $x\rightarrow y=y'\rightarrow x'$,
(I_4) If $x\rightarrow y=y\rightarrow x=1$, then x=y,
(I_5) $(x\rightarrow y)\rightarrow y=(y\rightarrow x)\rightarrow x$,
(l_1) $(x\vee y)\rightarrow z=(x\rightarrow z)\wedge(y\rightarrow z)$,
(l_2) $(x\wedge y)\rightarrow z=(x\rightarrow z)\vee(y\rightarrow z)$,

Then $(L,\vee,\wedge,',\rightarrow)$ is said to be a lattice implication algebra (shortly as L).

Definition 2 [1**]** Assume I is a nonempty set and S(I)= {X;X$\subseteq$I}. A filter D over I is defined to be a set D$\subseteq$S(I) such that:

(1) I$\in D$;
(2) If X,Y$\in D$, then $X\cap Y\in D$;
(3) If X$\in D$ and X$\subseteq$Y$\subseteq$I, then Y$\in D$.

D is called an ultrafilter over I, if D is a filter over I and satisfies the following conditions:

(4) For any X$\in$S(I), X$\in D$ if and only if I-X$\notin$D.

In the language of first-order lattice-valued logic LF(X), we will use the following symbols:

(1) An infinite collection V of variables $x_i, i\in N$;
(2) Constant symbols: $c_k, k\in K$, where K is an index set. The set of constant symbols is written as C;
(3) Relation symbols: $P^r_{m_r}, r\in R$, where R is an index set, and $P^r_{m_r}$ is m_r-ary, $m_r\in N_+$;

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(4) Function symbols: F_{m_j}, $j \in J$, where J is an index set and F_{m_j} is m_j-ary, $m_j \in N_+$;
(5) Logical connectives: $\vee, \wedge, \rightarrow, '$;
(6) Quantifiers: $\forall, \exists$;
(7) Technical symbols: (,),′.

We now define terms and formulas.

A term is defined as follows:

(1) Each element of $V \cup C$ is a term, which is said to be a non-superscript term;
(2) For any $j \in J$, if $t_1, \cdots, t_{m_j}$ are non-superscript terms or terms, then $F_{m_j}(t_1, \cdots, t_{m_j})$ is a term;
(3) Any term can be obtained from (1), (2).

A well-formed formula (shortly as formula) is defined as follows.

The set of formulas F is the smallest set F' satisfying the following conditions:

(1) For any $\alpha \in L, \alpha \in F'$;
(2) For any $r \in R$, if $t_1, \cdots, t_{m_r}$ are terms, then $P_{m_r}^{(r)}(t_1, \cdots, t_{m_r}) \in F'$;
(3) If $p, q \in F'$, then $p \wedge q, p \vee q, p \rightarrow q, p' \in F'$;
(4) If $p \in F'$, $x \in V$, then $\forall xp, \exists xp \in F'$.

Definition 3 [11] A model for LF(X) is defined as follows:

μ= $\left\langle A, \{P_{rA}; r \in R\}, \{F_{jA}; j \in J\}, \{c_{kA}; k \in K\} \right\rangle$,

where:

(1) A is a nonempty set which is said to be the universe of the model;
(2) For any $r \in R$, $P_{rA} : A^{m_r} \rightarrow L$ is an m_r-ary relation assigned to each relation symbol $P_{m_r}^{(r)}$;
(3) For any $j \in J$, $F_{jA} : A^{m_j} \rightarrow A$ is a m_j-ary function assigned to each function symbol F_{m_j};
(4) For any $k \in K$, c_{kA} is a constant assigned to each constant symbol c_k.

Definition 4 [11] Let

μ= $\left\langle A, \{P_{rA}; r \in R\}, \{F_{jA}; j \in J\}, \{c_{kA}; k \in K\} \right\rangle$ and

λ = $\left\langle B, \{P_{rB}; r \in R\}, \{F_{jB}; j \in J\}, \{c_{kB}; k \in K\} \right\rangle$ be two models for LF(X).

(1) If A $\subseteq$ B and for any $j \in J$, F_{m_j} is an m_j-ary function symbol, $F_{jA} = F_{jB}\Big| A^{m_j}$, for any $k \in K$, $c_{kA} = c_{kB}$ and for any $r \in R$, $P_{m_r}^{(r)}$ is an m_r-ary relation symbol, $P_{rA} = P_{rB}\Big| A^{m_r}$, then μ is called a submodel of λ or λ is called an extension of μ, denoted as μ $\subset \lambda$.
(2) μ and λ are said to be isomorphic, whenever there is a bijection $g : A \rightarrow B$ such that:

For each m_j-ary function symbol F_{m_j} and any sequence of elements $a_1, a_2, \cdots, a_{m_j} \in A$, we have

$$g(F_{jA}(a_1, a_2, \cdots, a_n)) = F_{jB}(g(a_1), g(a_2), \cdots, g(a_n));$$

For each m_r-ary relation symbol $P_{m_r}^{(r)}$ and any sequence of elements $a_1, a_2, \cdots, a_{m_r} \in A$, we have

$$P_{rA}(a_1, a_2, \cdots, a_n) = P_{rB}(g(a_1), g(a_2), \cdots, g(a_n));$$

For each constant symbol c_k, we have $g(c_{kA}) = c_{kB}$,

We write as $\mu \cong \lambda$ and call g is an isomorphic mapping.

3. Expansion theorem and the fundamental theorem of ultraproducts

Suppose the set of truth-values is a finite lattice implication algebra $(L, \vee, \wedge, ', \rightarrow, 0, 1)$, and L= $\{\alpha_1, \cdots, \alpha_n\}$, i.e., L has n elements, $n \in N$ and I is any nonempty index set, D is any proper filter on I (in classical sense), μ_i (i∈I) is a family of models for LF(X) whose universes are denoted A_i, respectively. Now we give a definition of reduced product.

Definition 5 A reduced product $\prod_D \mu_i$ of μ_i (i∈I) is defined as follows:

(1) Its universe is $\prod_D A_i$;
(2) Suppose $P_{m_r}^{(r)}$ (r∈R, R is an index set) is an m_r-placed relation symbol in LF(X) and its corresponding interpretation in any model μ_i is P_i^r (i∈I), then its interpretation in $\prod_D \mu_i$ is the following relation P:
for any $f_D^1, \cdots, f_D^{m_r} \in \prod_D A_i$, let
$$P(f_D^1, \cdots, f_D^{m_r}) = \alpha_s \text{ if and only if}$$
$$\{i \in I; P_i^r(f^1(i), \cdots, f^{m_r}(i)) = \alpha_s\} \in D, 1 \le s \le n.$$
(3) Suppose F_{m_j} is an m_j-placed functional symbol in LF(X) and its corresponding interpretation in any

model μ_i is F_i^j (i∈I), then its interpretation in $\prod_D \mu_i$ is the following function F: for any $f_D^1,\cdots,f_D^{m_j} \in \prod_D A_i$,

$$F^j(f_D^1,\cdots,f_D^{m_j}) =< F_i^j(f^1(i),\cdots,f^{m_j}(i)); i\in I >_D \quad (1)$$

(4) Suppose c_k (k∈K, K is an index set) is any constant symbol in LF(X), and its interpretation in any model μ_i is c_i^k, then its interpretation in $\prod_D \mu_i$ is the constant c=< c_i^k ; i∈I>$_D$.

In addition, If D is an ultrafilter of I, then the reduced product $\prod_D \mu_i$ of μ_i (i∈I) is called an ultraproduct of μ_i (i∈I).

Lemma 1 Let I be any nonempty index set, D be a proper filter on I, μ_i (i∈I) be a family of models for LF(X).

(1) If $f^1 =_D g^1,\cdots, f^{m_r} =_D g^{m_r}$, then for any s,$1\le s\le n$,

$\{i\in I; P_i^r(f^1(i),\cdots,f^{m_r}(i)) = \alpha_s\}\in D$ if and only if $\{i\in I; P_i^r(g^1(i),\cdots,g^{m_r}(i)) = \alpha_s\}\in D$;

(2) If $f^1 =_D g^1,\cdots, f^{m_j} =_D g^{m_j}$, then

$$< F_i^j(f^1(i),\cdots,f^{m_j}(i)); i\in I >=_D < F_i^j(g^1(i),\cdots,g^{m_j}(i)); i\in I >. \quad (2)$$

According to Lemma 1, the definition of reduced product $\prod_D \mu_i$ is well defined, $P(f_D^1,\cdots,f_D^{m_r})$ and $F_i^j(f_D^1,\cdots,f_D^{m_j})$ depend only on the equivalence classes $f_D^1,\cdots,f_D^{m_r}$ and $f_D^1,\cdots,f_D^{m_j}$, but not on the representatives of these equivalence classes.

Now we give the first conclusion, namely, expansion theorem.

Theorem 1(Expansion Theorem) Let LF(X)* be an expansion of the language of LF(X), I a nonempty index set, D a proper filter on I. If for any i∈I, μ_i is a model for LF(X), λ_i is an expansion of the model μ_i on LF(X)*, then the reduced product $\prod_D \lambda_i$ is an expansion of the model $\prod_D \mu_i$ on LF(X)*.

Proof. It can be proved similarly to that of classical logic.

The first form of the fundamental theorem of ultraproducts is given as follows:

Theorem 2 (the fundamental theorem of ultraproducts I**)** Let L be a finite lattice implication algebra, L=$\{\alpha_1,\cdots,\alpha_n\}$(n is a finite integer), I a nonempty index set, D an ultrafilter on I, μ_i (i∈I) a family of models for LF(X), and π=$\prod_D \mu_i$ an ultraproduct model for LF(X) whose universe is B, then

(1) For any term t($x_1,\ldots,x_m$) and any $f_D^1,\cdots,f_D^m \in B$,

$$\pi(t[f_D^1,\cdots,f_D^m]) =< \mu_i(t[f^1(i),\cdots,f^m(i)]); i\in I >_D; \quad (3)$$

(2) For any formula p($x_1,\ldots,x_m$) and any $f_D^1,\cdots,f_D^m \in B$,

$$\pi(p[f_D^1,\cdots,f_D^m]) = \alpha_s \text{ if and only if } \{i\in I; \mu_i(p[f^1(i),\cdots,f^m(i)]) = \alpha_s\}\in D\ (1\le s\le n);$$

(3) For any sentence p,

$$\pi(p) = \alpha_s \text{ if and only if } \{i\in I; \mu_i(p) = \alpha_s\}\in D\ (1\le s\le n).$$

Proof. It can be proved by induction over the construction of terms, and formulas in similar way to the proof of Theorem 1 [6].

Theorem 3 (the fundamental theorem of ultraproducts II**)** Let L be a finite lattice implication algebra, L=$\{\alpha_1,\cdots,\alpha_n\}$(n is a finite integer), I a nonempty index set, D an ultrafilter on I, μ_i (i∈I) a family of models for LF(X), and π=$\prod_D \mu_i$ an ultraproduct model for LF(X) whose universe is B, then

(1) For any formula p($x_1,\ldots,x_m$), any $f_D^1,\cdots,f_D^m \in B$ and any $\alpha_s \in L$,

$$\pi(p[f_D^1,\cdots,f_D^m]) \ge \alpha_s \text{ if and only if } \{i\in I; \mu_i(p[f^1(i),\cdots,f^m(i)]) \ge \alpha_s\}\in D\ (1\le s\le n);$$

(2) For any sentence p and any $\alpha_s \in L$,

$$\pi(p) \ge \alpha_s \text{ if and only if } \{i\in I; \mu_i(p) \ge \alpha_s\}\in D\ (1\le s\le n).$$

Proof. (1) Suppose there are t elements greater than α_s in L, denoted as $\alpha_{s1},\ldots,\alpha_{st}$. If $\pi(p[f_D^1,\cdots,f_D^m]) \ge \alpha_s$, then there exists $\alpha_{s\theta}$, $\alpha_{s\theta}\ge\alpha_s$, such that $\pi(p[f_D^1,\cdots,f_D^m]) = \alpha_{s\theta}$. By theorem 2,

$$X_\theta = \{i\in I; \mu_i(p[f^1(i),\cdots,f^m(i)]) = \alpha_{s\theta}\}\in D.$$

Since $X_\theta \subseteq \{i\in I; \mu_i(p[f^1(i),\cdots,f^m(i)]) \ge \alpha_s\}$ and D is a filter, we have

$$\{i \in I; \mu_i(p[f^1(i),\cdots,f^m(i)]) \geq \alpha_s\} \in D.$$

Conversely, suppose

$\{i \in I; \mu_i(p[f^1(i),\cdots,f^m(i)]) \geq \alpha_s\} \in D$, then for any $i \in X$, $\mu_i(p[f^1(i),\cdots,f^m(i)]) \geq \alpha_s$.

Let

$X_\theta = \{i \in I; \mu_i(p[f^1(i),\cdots,f^m(i)]) = \alpha_{s\theta}\}, 1 \leq \theta \leq t,$ then $X = X_1 \cup \cdots \cup X_t$. Since D is an ultrafilter, then there exists exactly one among $X_1,\ldots,X_t$ belonging to D, we can suppose $X_{\theta_0} \in D$, i.e.,

$X_{\theta_0} = \{i \in I; \mu_i(p[f^1(i),\cdots,f^m(i)]) = \alpha_{s\theta_0}\} \in D$.

By Theorem 2,

$\pi(p[f_D^1,\cdots,f_D^m]) = \alpha_{s\theta_0} \geq \alpha_s$.

(2) It is inferred from (1) directly.

Definition 6 Let I be an any nonempty index set , D an ultrafilter on I, μ_i (i∈I) a family of models for LF(X), and $\pi = \prod_D \mu_i$ an ultraproduct model for LF(X). If $\mu_i = \mu$ holds for any i∈I , then denote π as $\prod_D \mu$ and call it as an ultrapower of μ.

Corollary 1 Let L be a finite lattice implication algebra and $\prod_D \mu$ an ultrapower of μ. Then

(1) $\prod_D \mu \equiv \mu$;

(2) The natural embedding of μ into the ultrapower $\prod_D \mu$ is an elementary embedding, i.e., $\mu \mathrel{\tilde{<}} \prod_D \mu$.

Proof. (1) It is inferred from Theorem 2 (3) directly.

(2) For any a∈A, let $\hat{a} \in \prod_{i \in I} A$ and $\hat{a} = < a; i \in I >$, i.e., for any i∈I, $\hat{a}(i) = a$. Let d: $A \to \prod_D A$ be a mapping satisfying: for any a∈A, d(a)= $\hat{a}_D$, where $\hat{a}_D = < a; i \in I >_D$. Obviously, d is a one-to-one mapping into $\prod_D A$. For any formula $p(x_1,\ldots,x_m)$ and $a_1,\ldots,a_m \in A$, by Theorem 2, we have

$\prod_D \mu$ (p[d(a_1),…,d(a_m)])= α_s if and only if

$\{i \in I; \mu(p[\hat{a}_1(i),\cdots,\hat{a}_m(i)]) = \alpha_s\} \in D$ if and only if $\mu(p[a_1,\cdots,a_m]) = \alpha_s$. Hence, $\mu \mathrel{\tilde{<}} \prod_D \mu$.

4. Applications of the fundamental theorem of ultraproducts

Lemma 2 [1] Let I be a index set, S(I) be its power set. Then D is an ultrafilter over I if and only if there exists $i \in I$ such that $D = \{X \in S(I) \mid i \in X\}$.

Theorem 3 Let L be a finite lattice implication algebra and I a nonempty index set, D an ultrafilter generated by $i_0 \in I$, and $\{\mu_i;\ i \in I\}$ be any family of models for LF(X). If $\pi = \prod_D \mu_i$ is an ultraproduct of $\{\mu_i;\ i \in I\}$, then $\pi \cong \mu_{i_0}$.

Proof. Suppose the universe of π is B. Define a mapping as follows:

$$\gamma : B \to A_{i_0}$$
$$f_D \mapsto f(i_0).$$

(1) First we prove the mapping γ is a bijection. Let $f_D, g_D \in B$ and $f_D \neq g_D$, then $f =_D g$ does not hold, $i_0 \notin \{i \in I \mid f(i) = g(i)\}$, i.e., $f(i_0) \neq g(i_0)$, hence γ is an injection.

For any $a_{i_0} \in A_{i_0}$, define a mapping f as follows: for any $i \in I$, $f(i) = \begin{cases} a_{i_0}, & i = i_0, \\ a_i, & i \neq i_0, \end{cases}$ where a_i is chosen randomly in $A_i, i \in I \setminus \{i_0\}$. So $\gamma(f_D) = f(i_0) = a_{i_0}$, then γ is a surjection.

(2) Let F_{m_j} is an m_j-placed functional symbol, $j \in J$, for any $f_D^1, f_D^2, \cdots, f_D^{m_j} \in B$, suppose $f = < F_i^j(f^1(i),\cdots,f^{m_j}(i)) \mid i \in I >$. By Definition 5, we have

$\gamma(F_B{}^j(f_D^1, f_D^2,\cdots,f_D^{m_j})) = \gamma(f_D) = f(i_0)$

$= F_{i_0}^j(f^1(i_0),\cdots,f^{m_j}(i_0))$

$= F_{i_0}^j(\gamma(f_D^1),\gamma(f_D^2),\cdots,\gamma(f_D^{m_j}))$.

(3) Let $P_{m_r}^{(r)}$ be an m_r-placed relation symbol , r∈R, for any $f_D^1, f_D^2,\cdots,f_D^{m_r} \in B$, suppose $P(f_D^1,\cdots,f_D^{m_r}) = \alpha_s$, by definition 5 we have $\{i \in I; P_i^r(f^1(i),\cdots,f^{m_r}(i)) = \alpha_s\} \in D$, so $P_{i_0}^r(f^1(i_0), f^2(i_0),\cdots,f^{m_r}(i_0)) = \alpha_s$.

(4) Let $c_k, k \in K$ be a constant symbol, $\gamma(c_{kB}) = c_{k_{i_0}}$ holds obviously.

To sum up, γ is an isomorphism between the models π and μ_{i_0}, so $\pi \cong \mu_{i_0}$.

Definition 7[14] A mapping from the set of all the sentences in LF(X) to lattice implication algebra L is called a (L-valued) theory in LF(X). If T is a theory in LF(X), μ is an (L-valued) model for LF(X), then for every sentence p, μ(p) ≥ T(p), then it is called that μ satisfies T or μ is a model of T.

Definition 8 [14] Let A, B be L-fuzzy subsets of the set of formulas in LF(X). B is called a finite limit of A, if there exists a set of finite formulas $\{p_1,\ldots, p_m\} \subseteq$ SuppA (the support of A), m∈N is finite, such that for any formula p,

$$B(p) = \begin{cases} A(p_i), & if\ p = p_i, 1 \le i \le m \\ O, & otherwise. \end{cases}$$

Now we prove the following consistent theorem mathematically by the fundamental theorem of ultraproducts, which is its very important application.

Theorem 4 (Consistent Theorem) Let L be a finite lattice implication algebra, and T a theory in LF(X), and $I = S_\omega(SuppT)$ a set consisting of all finite subsets of SuppT. If for any i∈I, μ_i is a model of T_i, where T_i is a finite limit of T and it is defined as follows:

$$T_i(p) = \begin{cases} T(p), & if\ p \in i, \\ O, & otherwise, \end{cases}$$

then there exists an ultrafilter on I, such that the ultraproduct $\prod_D \mu_i$ is a model of T.

Proof. For any sentence p∈SuppT, let $\hat{p} = \{i \in I; p \in i\}$, i.e., $\hat{p}$ is a set consisting of finite subsets of SuppT containing p. Suppose E={ $\hat{p}$;p∈SuppT}. Since for any $p_1, \cdots, p_n$ ∈SuppT, $\{p_1, \cdots, p_n\} \in \hat{p}_1 \cap \cdots \cap \hat{p}_n$, E has the finite intersection property. By Ultrafilter Theorem ([20], Proposition 4.1.3), there exists an ultrafilter D on I, such that E⊆D. Construct ultraproduct model $\prod_D \mu_i$ of a family of models μ_i (i∈I), we need to prove $\prod_D \mu_i$ is a model of T. In fact, for any sentence p, let T(p)=α_s and $\|p\| = \{i \in I; \mu_i(p) \ge \alpha_s\}$. Since for any $i \in \hat{p}$, we have $p \in i$, then $\mu_i(p) \ge T_i(p) = T(p) = \alpha_s$, i.e., $i \in \|p\|$, hence $\hat{p} \subseteq \|p\|$. By $\hat{p} \in E$, E⊆D and since D is an ultrafilter, we have $\|p\| \in D$. It is inferred from Theorem 2 that $\prod_D \mu_i$ (p) $\ge \alpha_s$, i.e., $\prod_D \mu_i$ (p)$\ge T(p)$. Therefore, $\prod_D \mu_i$ is a model of T.

Obviously, Theorem 4 can be described as follows: Let L be a finite lattice implication algebra, and T a theory in LF(X). If every finite limit of T has a model, then T has a model.

Now we give another application of the fundamental theorem of ultraproducts. First, we give a notion.

Definition 9 Let μ be a model for LF(X), its universe be A, $\{\mu_i; i\in I\}$ be a family of submodels of μ and their universe be A_i, respectively. If A=$\bigcup_{i\in I} A_i$ and for any i, j∈I, there exists k∈I, such that $A_i \cup A_j \subseteq A_k$, then $\{\mu_i; i\in I\}$ is said to be a local system of μ.

For example, all finitely generated submodels of μ consist of a local system of μ.

Theorem 5 Let μ be a model for LF(X), $\{\mu_i; i\in I\}$ a local system of μ . If there exists a family of models { λ_i ; i∈I}, such that for any i∈I, μ_i is isomorphically embedded in λ_i, i.e., $\mu_i \tilde{\subset} \lambda_i$, then there exists an ultrafilter D on I, such that μ is isomorphically embedded in $\prod_D \lambda_i$, i.e., $\mu \tilde{\subset} \prod_D \lambda_i$.

Proof. Similar to the proof of the corresponding theorem of classical logic and omit it.

5.Conclusions

In this paper, we developed model theory of lattice-valued first-order logic LF(X) and gave an important conclusion, the fundamental theorem of ultraproducts. All of the above work will lay a solid foundation for further study on lattice-valued logic. More important, since a model in many-valued logic is namely a control process, so all the results on model theory that have been obtained here can help us to interpret the results of approximate reasoning on semantic level in some model. One of our future work is to further study model theory and construct approximate reasoning method by it.

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References

[1] C.C. Chang and H.J. Keisler, Model theory, North-Holland, 1973.
[2] J.A.Goguen, "The logic of inexact concepts", Synthese, Vol 19, pp.325--373, 1969.
[3] J.Pavelka, "On fuzzy logic I, II, III", Zeit.Math.Logik u.Grundl.Math., Vol.25, pp. ~45--52, 119--134, 447--

464, 1979.
[4] V.Novak,``First order fuzzy logic'', Studia Logica, Vol.46, No.1, pp. 87--109, 1982.
[5] V.Novak, I.Perfilieva andJ.Mockor, Mathematical Principles of Fuzzy Logic, Kluwer, Boston, 1999.
[6] Wang Shiqiang and Lu Qingbo," the fundamental theorem of ultraproducts in lattice-valued model", Scientia Sinical, 2, 1981, pp. 71-74.
[7] Xu Yang," Lattice implication algebras", Journal of Southwest Jiaotong University Vol.89, No.1, 1993, pp. 20-27 (in Chinese).
[8] Qin Keyun, Study on lattice-valued logic system based on lattice implication algebras and its applications, Ph.D thesis, Southwest Jiaotong University, 1996.
[9] Liu Jun. Study on lattice-valued logic system and lattice-valued resolution principle based on lattice implication algebras. Ph.D thesis, Southwest Jiaotong University, 1998.
[10] Xu Yang, Qin Keyun, Liu Jun and Song Zhenming, "L-valued prepositional logic L_{vpl}", Information Sciences, Vol.114, No.1, 1999, pp.205-235.
[11] Xu.Yang, Liu Jun Song Zhenming and Qin Keyun, "on semantics of L-valued first-order logic L_{vfl}", Int. J. General Sys. , Vol. 29, No.1, 2000, pp. 53-79.
[12] Qin Keyun, Xu Yang, Song Zhenming," Lattice-valued propositional logic LP(X)(I)", Fuzzy Sys. Math., Vol. 11, No.4, 1997, pp. 5-10 (in Chinese).
[13] Ying Mingsheng, "another form of ultraproduct fundamental theorem of lattice-valued model", Science in Bulletin, Vol.37, No.4, 1992, pp.380-380.
[14] Xu Yang, Ruan Da, Qin Keyun, Liu Jun, Lattice-valued logic, Springer-Verlag Berlin Heidelberg, 2003.
[15] Wang Xuefang, Xu Yang, "Ultraproduct Theorem of First-order Lattice-valued Logic FM", Proceeding of SCI 2002/ISAS 2002, Vol.XVI, 2002, pp.125-129.
[16] Wang Xuefang, Qing Ming, Valuation sets in lattice-valued propositional logic LP(X), Proceedings of IEEE International Conference on Fuzzy Systems, pp.114-119,2003.
[17] Wang Xuefang, Liu Peishun, Model theory and closure operators of lattice-valued propositional logic LP(X), 2003 IEEE International Conference on Systems, Man & Cybernetics, pp. 5010-5015, Washington, D.C., USA, October 5-8, 2003.
[18] Wang Xuefang, Meng Dan, Xu Yang, Qin Keyun, Graded consequence relations in lattice-valued propositional logic LP(X), 2003 IEEE International Conference on Systems, Man & Cybernetics, pp. 5004-5009, Washington, D.C., USA, October 5-8, 2003.
[19] Wang Xuefang, Xu Yang and Qin Keyun, Lattice implication algebras and their filters, the Journal of Fuzzy Mathematics, Vol.11, No.4, 893-899, 2003.
[20] Shen Fuxing, Introduction to model theory, Beijing: Beijing Normal University Press, 1995.

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