Linguistic Truth-value Lattice-valued Logic System with Important Coefficient and Its Application to Evaluation System

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***Abstract*— In this paper, a linguistic truth-value lattice proposition logic system $LTVLP(X)$ is presented firstly. It is an useful and reasonable tool to deal with the linguistic term set including linear ordered set and non-linear ordered set. Then an operator linguistic truth-value lattice-valued proposition logic $OLTVLP(X)$ is given. The $OLTVLP(X)$ can provide a theoretical model for multi-evaluator single-layer evaluation problem with linguistic term set. Then it follows an evaluation approach in $LTVLP(X)$ and $OLTVLP(X)$ by using logic formula for an application case. Any linear information and incomparable information can be dealt with in the framework under $LTVLP(X)$ and $OLTVLP(X)$.**

Key Words: Artificial intelligence, intelligent evaluation system, linguistic truth-value lattice-valued proposition logic system, operator linguistic truth-value lattice-valued proposition logic system

I. Introduction

In real world, there are many decision situations in which the information cannot be assessed precisely in a quantitative form but may be in a qualitative one, and thus, the use of a linguistic approach is necessary [9]. Similarly, there are many evaluation situation in which the information cannot be evaluated in a quantitative form but may be in a qualitative one, so the use of linguistic approach is necessary end effective. For example, when we try to evaluate "service quality", we tend to use natural language such as {very good, good, medium, poor, very poor} etc. other than precise numerical value. In addition, evaluator's evaluation is often vaguely qualitative and cannot be represented in precise numerical value in a multi-criteria evaluation problem. Therefore, there are some approach which use linguistic assessments instead of numerical values by means of linguistic variables [5], [6], [7], [8], [11], [15], that is, variables whose values are not numbers but words or sentences in a natural or artificial language. So it is necessary to use linguistic variable.

The fuzzy linguistic approach is presented by L.A.Zadeh in 1975[10]. After L.A.Zadeh's work in 1975, a lot of fuzzy linguistic approach has been applied with very good results to different problems, such as, "information retrieval", "clinical diagnosis", "marketing", "risk in software development", "materials selection", "decision-making", "evaluation" etc. [9].

Linguistic variable is necessary and effective for the following reasons: first, some criteria is unquantifiable due to its nature, especially for some criteria related to human perception for feeling (e.g., when we try to evaluate the "sweetness" of a perfume sample, maybe the linguistic term like "fragrant", "gentle", "mild", "soft" would be used in natural language other than numerical value); second, it is unnecessary to get the accurate numerical value(e.g., when we try to evaluate the "height" of a person, maybe intuitive linguistic terms like "very tall", "tall","small","very small" is better than measure value which is measured by scale.) In these situations, a linguistic approach is necessary and helpful. Moreover, some linguistic term seem difficult to distinguish their boundary sometimes, but their meaning of common usage can be understood.

It is natural and reasonable to represent the linguistic term set by using a partially ordered set or lattice[1], [3]. Based on the above feature of linguistic evaluation term set, a proper algebraic and logic model must be chosen to deal with this problem. In this paper, we present the $LTVLIA$, $LTVLP(X)$ and $OLTVLP(X)$ and take them as the representation and operation model of evaluation problem.

In this paper, a linguistic truth-value lattice-valued proposition logic system $LTVLP(X)$ is presented firstly. It is an useful and reasonable tool to deal with the linguistic term set including linear ordered set and non-linear ordered set. Then an operator linguistic truth-value lattice-valued propo-

[1]Manuscript accepted on June 23, 2006.
Manuscript received on June 25, 2006.

sition logic $OLTVLP(X)$ is given. The $OLTVLP(X)$ can provide a theoretical model for multi-evaluator single-layer evaluation problem with linguistic term set. It follows an evaluation approach in $LTVLP(X)$ and $OLTVLP(X)$ by using logic formula for an application case. Any linear information and incomparable information can be dealt with in the framework under $LTVLP(X)$ and $OLTVLP(X)$.

II. Preliminaries

A. Lattice Implication Algebra

Definition 2.1: [17] Let $(L, \vee, \wedge, O, I)$ be a bounded lattice with an order-reversing involution "$'$", I and O the greatest and the smallest element of L respectively, and $\rightarrow: L \times L \rightarrow L$ be a mapping. $(L, \vee, \wedge, \rightarrow, O, I)$ is called a lattice implication algebra if the following conditions hold for any $x, y, z \in L$,

1) $x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z)$;
2) $x \rightarrow x = I$;
3) $x \rightarrow y = y' \rightarrow x'$;
4) $x \rightarrow y = y \rightarrow x$ implies $x = y$;
5) $(x \rightarrow y) \rightarrow y = (y \rightarrow x) \rightarrow x$;
6) (l_1) $(x \vee y) \rightarrow z = (x \rightarrow z) \wedge (y \rightarrow z)$;
7) (l_2) $(x \wedge y) \rightarrow z = (x \rightarrow z) \vee (y \rightarrow z)$;

B. Lattice-valued Propositional Logic $LP(X)$

- Language[18]
  The symbols in $LP(X)$ are
  1) the set of propositional variable:
  $$X = \{p, q, r, \cdots\};$$
  2) the set of constants: L;
  3) logical connectives: $\rightarrow, '$;
  4) auxiliary symbols: $), ($.
  The set F of formulae of $LP(X)$ is the least set Y satisfying the following conditions:
     a) $X \subseteq Y$;
     b) $L \subseteq Y$;
     c) if $p, q \in Y$, then $p', p \rightarrow q \in Y$.
- Semantics
  Lemma 2.2: [18] Let L be a lattice implication algebra, then
  – $\alpha \vee \beta = (\alpha \rightarrow \beta) \rightarrow \beta$;
  – $\alpha \wedge \beta = (\alpha' \vee \beta')'$.

  hold for any $\alpha, \beta \in L$. Hence, L and $LP(X)$ can be looked as algebras with the same type $T = L \cup \{', \rightarrow\}$ and for any $p, q \in F$,
  – $p \vee q \doteq (p \rightarrow q) \rightarrow q$,
  – $p \wedge q \doteq (p' \vee q')'$.

  Definition 2.3: [16] A mapping $v : LP(X) \rightarrow L$ is called a valuation of $LP(X)$, if it is a $T-homomorphism$.
  Lemma 2.4: [16] Let $f : LP(X) \rightarrow L$ be a mapping, then f is a valuation of $LP(X)$ if and only if it satisfies:
  1) $f(\alpha) = \alpha$ for any $\alpha \in L$;
  2) $f(p') = (f(p))'$ for any $p \in F$;
  3) $f(p \rightarrow q) = f(p) \rightarrow f(q)$ for any $p, q \in F$.

III. Linguistic lattice-valued proposition logic system

A. Linguistic Truth-valued Lattice Implication Algebra $LTVLIA$

Definition 3.1: Let
$$(L_1, \wedge_1, \vee_1, \rightarrow_1, I_1, O_1),$$
$(L_2, \wedge_2, \vee_2, \rightarrow_2, I_2, O_2)$ be lattice implication algebra respectively, and its order-reversing involution mapping be $'_1$, $'_2$ respectively, and the product of L_1 and L_2 be as follows:
$$L_1 \times L_2 = \{(a, b) | a \in L_1, b \in L_2\} \quad (1)$$
define the operation $\wedge, \vee, \rightarrow, '$ on $L_1 \times L_2$ as follows: for any (a_1, b_1), $(a_2, b_2) \in L_1 \times L_2$
$$(a_1, b_1) \wedge (a_2, b_2) = (a_1 \wedge_1 a_2, b_1 \wedge_2 b_2); \quad (2)$$
$$(a_1, b_1) \vee (a_2, b_2) = (a_1 \vee_1 a_2, b_1 \vee_2 b_2); \quad (3)$$
$$(a_1, b_1) \rightarrow (a_2, b_2) = (a_1 \rightarrow_1 a_2, b_1 \rightarrow_2 b_2); \quad (4)$$
$$(a_1, b_1)' = ((a_1)'_1, b_1)$$
or
$$(a_1, b_1)' = (a_1, (b_1)'_2)$$
or
$$(a_1, b_1)' = ((a_1)'_1, (b_1)'_2) \quad (5)$$
Lemma 3.2: $(L_1 \times L_2, \wedge, \vee, \rightarrow, (O_1, O_2), (I_1, I_2))$ is a lattice implication algebra.
Proof It can be proved by the definition of lattice implication algebra given in definition 2.1.
Definition 3.3: $(L_1 \times L_2, \wedge, \vee, \rightarrow, (O_1, O_2), (I_1, I_2))$ is called a product lattice implication algebra.
Definition 3.4: Let
$$MT = \{True(\text{Abbr. to T}), False(\text{Abbr. to F})\},$$
the following lattice implication algebra $(MT, \wedge, \vee, \rightarrow, F, T)$ is called meta linguistic truth-value lattice implication algebra, where $\wedge, \vee, \rightarrow$ is the same as that of in classical logic.

Example 3.5: Consider the set $L = \{a_i | i = 1, 2, \ldots, n\}$, define order on L as follows: if $i \leq j$ iff $a_i \leq a_j$, where $i, j \in= \{1, 2, \ldots, n\}$. For any $1 \leq j, k \leq n$,define
1) $a_j \vee a_k = a_{max\{j,k\}}$;
2) $a_j \wedge a_k = a_{min\{j,k\}}$;
3) $a'_j = a_{n-j+1}$;
4) $a_j \rightarrow a_k = a_{min\{n-j+k,n\}}$.

then $(L, \wedge, \vee, \rightarrow, a_1, a_n)$ is a lattice implication algebra, where $\rightarrow$ is Lukasiewicz implication algebra on finite chains, and Hasse graph of L is as in figure 1.

In natural language, we usually take some hedge to modify our natural language, hedge operator and its algebraic structure is discussed in [13], [14]. In this paper, we choose some modifactory word to denote the linguistic term.

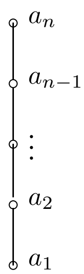

Fig. 1. HASSE Graph of Lattice L

Example 3.6: Let $MW = \{$Slightly (Abbr. to Sl), Somewhat (Abbr. to So), Rather (Abbr. to Ra), Almost (Abbr. to Al), Exactly (Abbr. to Ex), Quite (Abbr. to Qu), Very (Abbr. to Ve), Highly (Abbr. to Hi), Absolutely (Abbr. to Ab.$\}$ be a modifactory word set, then chain $Sl \leq So \leq Ra \leq Al \leq Ex \leq Qu \leq Ve \leq Hi \leq Ab$ is a lattice implication algebra with operation as given in example 3.5, and it is a nine modifactory word set.

Example 3.7: Let $MW = \{$Slightly (Abbr. to Sl), Exactly (Abbr. to Ex), Absolutely (Abbr. to Ab.$\}$ be a modifactory word set, then chain $Sl \leq Ex \leq Ab$ is a lattice implication algebra with operation as given in example 3.5, and it is a three modifactory word set.

Definition 3.8: Let $MW = \{a_1, a_2, \cdots, a_n\}$ be linguistic hedge lattice implication algebra, MT be meta linguistic lattice implication algebra, then the product $MW \times MT$ of MW and MT is a linguistic truth-value lattice implication algebra (abbr. to LTVLIA), its Hasse graph is as figure 2:

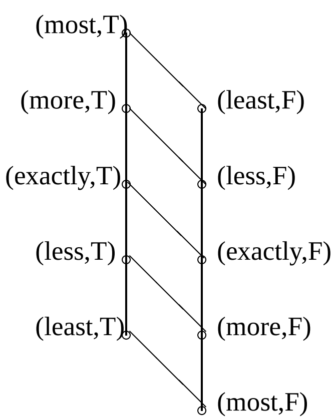

Fig. 2. linguistic truth-value Lattice implication algebra $MW \times MT$

B. Linguistic Truth-value Lattice-valued Proposition Logic System $LTVLP(X)$

- Language

  The symbols in $LTVLP(X)$ are

  1) the set of propositional variable:

  $$X = \{p, q, r, \cdots\};$$

  2) the set of constants: $LTVLIA$;
  3) logical connectives: $\wedge, \vee, \rightarrow, '$;
  4) auxiliary symbols: $), ($.
- Semantics

  Definition 3.9: The set F of formulae of $LTVLP(X)$ is the least set Y satisfying the following conditions:

  1) $X \subseteq Y$;
  2) $LTVLIA \subseteq Y$;
  3) if $p, q \in Y$, then $p', p \wedge q, p \vee q, p \rightarrow q \in Y$.

  Definition 3.10: A mapping $v : LP(X) \rightarrow LTVLIA$ is called a linguistic lattice-valued valuation of $LP(X)$, if it is a $T-homomorphism$.

  Lemma 3.11: Let $f : LP(X) \rightarrow LTVLIA$ be a mapping, then f is a linguistic lattice-valued valuation of $LP(X)$ if and only if it satisfies:

  1) $f(\alpha) = \alpha$ for any $\alpha \in LTVLIA$;
  2) $f(p') = (f(p))'$ for any $p \in F$;
  3) $f(p \rightarrow q) = f(p) \rightarrow f(q)$ for any $p, q \in F$.

C. Operator Linguistic Truth-value Lattice-valued Logic System $OLTVLP(X)$

Definition 3.12: Let $(LTVLIA, \wedge, \vee, \rightarrow, O, I)$ be a linguistic truth-value lattice implication algebra, $\lambda \in [0, 1]$, the symbols in operator linguistic truth-value lattice-valued proposition logic system $\lambda LTVLP(X)$ are

1) the set of lattice-valued atom propositional variable :$\lambda X = \{\lambda p, \lambda q, \lambda r, \cdots\}$;
2) the set of lattice-valued atom constants: $\lambda LTVLIA$;
3) logical connectives: $\vee, \wedge, \rightarrow, '$;
4) auxiliary symbols: $), ($.

The set F of formulae of $\lambda LP(X)$ is the least set λY satisfying the following conditions:

1) $\lambda X \subseteq \lambda Y$;
2) $\lambda L \subseteq \lambda Y$;
3) if $\lambda_1 p, \lambda_2 q \in \lambda Y$, then $(\lambda p)', \lambda_1 p \rightarrow \lambda_2 q \in \lambda Y$, $\lambda_1 p \vee \lambda_2 q \in \lambda Y$, $\lambda_1 p \wedge \lambda_2 q \in \lambda Y$.

IV. Single-layer Evaluation System by Using Linguistic Truth-valued Proposition Logic System

A. Problem Formulation of Single-layer Evaluation System

Let the evaluation problem involve a set of n alternatives (objects) $A_i (i = 1, 2, \cdots, n)$ to be evaluated and r evaluators $r_k, k = (1, 2, \cdots, r)$. Let these alternatives to be evaluated be based on a set of m criterion(attribute) $C_j (j = 1, 2, \cdots, m)$ and every criteria(attribute) can't be divided further, then it is a single-layer evaluation problem, it can be expressed as in figure 3 .

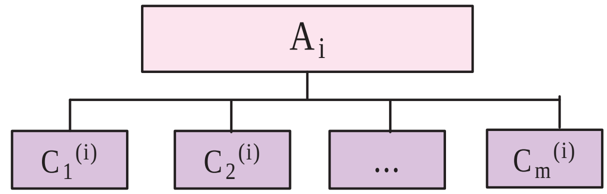

Fig. 3. demonstration graph of single-layer evaluation problem

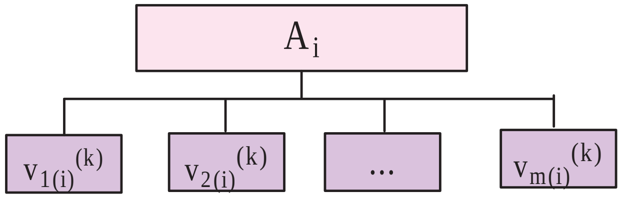

Fig. 4. demonstration graph of single-layer evaluation problem

For every alternative $A_i(i = 1, 2, \cdots, n)$, the $k^{th}(k = 1, 2, \cdots, r)$ evaluator will give the evaluation value for m evaluated attributes (or criteria) and n alternatives. For the i^{th} alternative, the k^{th} evaluator will give evaluation value for each criteria as shown in figure 4 , where $v_{j(i)}^{(k)}, j = 1, 2, \cdots, m, i = 1, 2, \cdots, n$ represents the linguistic evaluation value of the performance rating of alternative $A_i, (i = 1, 2, \cdots, n)$ with respect to criterion $C_j(j = 1, 2, \cdots, m)$.

Different evaluator may have different opinion on importance of different criterion, i.e. they may have different preference on various criterion because of their academic background knowledge or experience difference. For example, when evaluating a performance of a public transportation company, government may think "social duty" is the most important, but customer may think "comfort" and "safety" is the most important. So the importance coefficient must be determined when we try to evaluate alternative. Here, for the $k^{th}(k = 1, 2, \cdots, r)$ evaluator, we denote the importance coefficient of each criteria by the weighting vector

$$\omega^{(k)} = (\omega_1^{(k)}, \omega_2^{(k)}, \cdots, \omega_m^{(k)}). \quad (6)$$

In an evaluation problem, different evaluator may be not the same important. So the importance coefficient of each evaluator can be given in the following weighting vector:

$$e = (e_1, e_2, \cdots, e_r). \quad (7)$$

Note 4.1: For different evaluation problem, there may have multi-layer criteria, and each criterion in each layer can be divided to some different sub-criterion, it is a multi-layer problem. This case will be given in another paper.

B. Transformation of Satisfaction-based with Linguistic Truth-value Lattice-valued

As mentioned in [15], in any evaluation problem based on linguistic variable and its semantics, it is very important thing to choose an appropriate linguistic term set. The term set should satisfy the following conditions [4], [15]:

- The term set should be rich enough to distinguish different attribute value, i.e. it should cover or represent all instances;
- The term set should not be so detailed that it will lead to unnecessary precision.

In addition, for different evaluated criteria, the evaluation term set may be different. The difference comes from the following two ways, one is from the different granular or evaluation measurement, the other is from the structure which is a linear ordered set or non-linear ordered set.

Example 4.2: For example, when we try to evaluate a supply chain, we have several evaluation criteria such as "tangibles", "reliability", "responsiveness", "competence", "courtesy", "credibility", "security", "communication", "understanding the customer"[2]. Accordingly, we have different evaluation linguistic term set such as {very high, high, medium, low, very low} for reliability, and the linguistic term set is an linear order set and can be expressed as in figure 5.

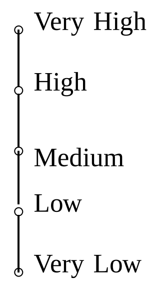

Fig. 5. Linguistic terms in a linear order

However, there are some "vague overlap districts" among some words which cannot be strictly linearly ordered, a more general structure of linguistic terms is given as in figure 6. And the linguistic term set {most high, more high,

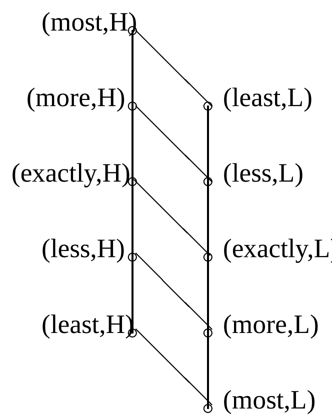

Fig. 6. Linguistic terms in a non-linear order

where H denotes High, L denotes low.

exactly high, less high, least high, most low, more low, exactly low, less low, least low} for competence is a non-linear set.

In this section, we take the $LTVLIA$, $LTVLP(X)$ and $OLTVLP(X)$ as the representation and operation model.

Note 4.3: The linguistic term set is different in most evaluation cases. In the proposed evaluation system, every linguistic evaluation term will be transformed to the satisfaction-based linguistic truth-value proposition logic system $SLTS$, then the $SLTS$ can be transformed to linguistic truth-value term set $LTVTS$.

Example 4.4: Here we take a seven term set as as example, let set of degree of satisfaction term be $SDST$ = {most satisfactory, more satisfactory, satisfactory, average, dissatisfactory, more dissatisfactory, most dissatisfactory}, the linguistic truth-value set be $LTVS$ ={most T., more T., T., α, F., more F., most F.}, where α is between "satisfactory" and "unsatisfactory". When we evaluate the "service quality", we can use the term set {best, better, good, medium, bad, worse, worst}, if an evaluator think the service is "worse", then its corresponding value is v_6, its corresponding semantic is that the degree of satisfaction of "service quality" is "more dissatisfactory". Accordingly, the truth-value of proposition "the service is good" is "more F".

Example 4.5: Take the evaluation of competence in supply chain evaluation in example 4.2 as an example. Its linguistic term set $LTS = \{$most high, more high, exactly high, less high, least high, most low, more low, exactly low, less low, least low$\}$ for competence is a non-linear set, its satisfaction-based linguistic term set $SLTS$ can be transformed to linguistic truth-value set as $SLTS$ ={most satisfactory, more satisfactory, exactly satisfactory, less satisfactory, least satisfactory, most dissatisfactory, more dissatisfactory, exactly dissatisfactory, less dissatisfactory, least dissatisfactory}, then the satisfaction-based linguistic term set $SLTS$ can be transformed to linguistic truth-value term set $LTVTS = \{$most T, more T, exactly T, less T, least T, most F, more F, exactly F, less F, least F$\}$ accordingly.

C. Evaluation System Based on Linguistic Truth-valued Lattice-valued Logic System

After every linguistic evaluation value has been transformed to the linguistic truth-value, the adjacent key step of evaluation process is the aggregation of every linguistic truth-value for each criterion of every alternative.

In this section, we present a new approach by using linguistic truth-value lattice proposition logic formula from the point of linguistic truth-value lattice-valued proposition logic system. For the i^{th} alternative, the k^{th} evaluator will give the linguistic truth-value lattice evaluation vector, as expressed in (8):

$$(v_{1(i)}^{(k)}, v_{2(i)}^{(k)}, \cdots, v_{m(i)}^{(k)}) \quad (8)$$

Definition 4.6: For the i^{th} alternative A_i, we can get r vector:

$$V_{j(i)}^{(k)} = (v_{1(i)}^{(k)}, v_{2(i)}^{(k)}, \cdots, v_{m(i)}^{(k)}), \quad (9)$$

where $k = 1, 2, \cdots, r$, $v_{j(i)}^{(k)} \in LTVLP(X)$.every vector express as in (9) is called linguistic truth-value lattice-valued evaluation vector.

Definition 4.7: The importance coefficient of each criteria given by the $k^{th}(k = 1, 2, \cdots, r)$ evaluator is $\omega^{(k)} = (\omega_1^{(k)}, \omega_2^{(k)}, \cdots, \omega_m^{(k)})$, linguistic truth-value lattice evaluation vector is $(v_{1(i)}^{(k)}, v_{2(i)}^{(k)}, \cdots, v_{m(i)}^{(k)})$, then

$$(\omega_1^{(k)} v_{1(i)}^{(k)}, \omega_2^{(k)} v_{2(i)}^{(k)}, \cdots, \omega_m^{(k)} v_{m(i)}^{(k)}) \quad (10)$$

is called linguistic truth-value lattice-valued evaluation vector with criterion important coefficient, where $\omega_1^{(k)}(v_{1(i)}^{(k)} \in OLTVLP(X)$.

TABLE I

LINGUISTIC TRUTH-VALUE EVALUATION DISTRIBUTION

linguistic term	v_1	v_2	$\cdots$	v_s
distribution value	$g(1)$	$g(2)$	$\cdots$	$g(s)$

Definition 4.8: The importance coefficient of each criteria given by the $k^{th}(k = 1, 2, \cdots, r)$ evaluator is $\omega^{(k)} = (\omega_1^{(k)}, \omega_2^{(k)}, \cdots, \omega_m^{(k)})$, linguistic truth-value lattice-valued evaluation vector is $(v_{1(i)}^{(k)}, v_{2(i)}^{(k)}, \cdots, v_{m(i)}^{(k)})$, the important coefficient of k^{th} evaluator is $e^{(k)}$, then

$$(e^{(k)}\omega_1^{(k)} v_{1(i)}^{(k)}, e^{(k)}\omega_2^{(k)} v_{2(i)}^{(k)}, \cdots, e^{(k)}\omega_m^{(k)} v_{m(i)}^{(k)}) \quad (11)$$

is called linguistic truth-value lattice-valued evaluation vector with criterion and evaluator important coefficient, where $\omega_1^{(k)} v_{1(i)}^{(k)} \in OLTVLP(X)$.

Definition 4.9: The evaluation result is determined by the aggregation of linguistic truth-value lattice evaluation vector with criterion important coefficient, it can be expressed as :

$$\wedge_{j=1}^{m} e^{(k)} \omega_j^{(k)} v_{j(i)}^{(k)}, \quad (12)$$

where $k = 1, 2, \cdots, r$.

Definition 4.10: 1) Let

$$LTVTLIA = \{v_1, v_2, \cdots, v_s\} = \{v_o | o = 1, 2, \cdots, s\}$$

be a linguistic truth-value lattice implication algebra, define a series of mapping $g_o, (o = 1, 2, \cdots, s)$ from $([0, 1] \diamond LTVLIA)^m$ to R as follows:

$$g_o : ([0, 1] \diamond LTVS)^m \to R :$$

$$(c_1 v_{i1}, c_2 v_{i2}, \cdots, c_n v_{im}) \longmapsto \sum c_l, \text{if } v_{il} = v_o. \quad (13)$$

2) Let $(e^{(k)}\omega_1^{(k)} v_{1j}^{(k)}, e^{(k)}\omega_2^{(k)} v_{2j}^{(k)}, \cdots, e^{(k)}\omega_m^{(k)} v_{mj}^{(k)})$ where $j \in \{1, 2, \cdots, n\}$, $k \in \{1, 2, \cdots, r\}$ be an any linguistic truth-value lattice-valued evaluation vector with double important coefficient, define linguistic evaluation value distribution on $LTVS$ as in table I, and vector

$$(g(1), g(2), \cdots, g(s)) \quad (14)$$

is called linguistic truth-value lattice-valued evaluation distribution vector.

Definition 4.11: Let $\{(a_{i1}, a_{i2}, \cdots, a_{is}) | i = 1, 2, \cdots, n\}$ be n linguistic truth-value lattice-valued evaluation distribution vector, define $\oplus$ from $(R^s)^n$ to R^s as follows:

$$\oplus : (R^s)^n \to R^s :$$

$$(a_{11}, a_{12}, \cdots, a_{1s}) \oplus (a_{21}, a_{22}, \cdots, a_{2s}) \oplus \cdots$$
$$\oplus (a_{n1}, a_{n2}, \cdots, a_{ns}) \longmapsto$$

$$(1/n(a_{11} + a_{21} + \cdots + a_{n1}), (1/n(a_{12} + a_{22} + \cdots + a_{n2}), \cdots,$$
$$(1/n(a_{n1} + a_{n2} + \cdots + a_{ns}), \quad (15)$$

Definition 4.12: Let the importance coefficient of each criterion given by the $k^{th}(k = 1, 2, \cdots, r)$ evaluator be $\omega^{(k)} = (\omega_1^{(k)}, \omega_2^{(k)}, \cdots, \omega_m^{(k)})$, linguistic truth-value lattice-valued evaluation vector be $(v_{1(i)}^{(k)}, v_{2(i)}^{(k)}, \cdots, v_{m(i)}^{(k)})$, the important coefficient of k^{th} evaluator is $e^{(k)}$, then the final evaluation result is as follows:

$$\oplus g((\wedge_{j=1}^{m} e^{(k)} \omega_j^{(k)} v_{j(i)}^{(k)}))), \quad (16)$$

V. Conclusion and future works

In this paper, a linguistic truth-value lattice-valued proposition logic system $LTVLP(X)$ is presented firstly. It is an useful and reasonable tool to deal with the linguistic term set including linear ordered set and non-linear ordered set. Then an operator linguistic truth-value lattice-valued proposition logic $OLTVLP(X)$ is given. The $OLTVLP(X)$ can provide a theoretical model for multi-evaluator single-layer evaluation problem with linguistic term set. It follows an evaluation approach in $LTVLP(X)$ and $OLTVLP(X)$ by using logic formula for an application case. Any linear information and incomparable information can be dealt with in the framework under $LTVLP(X)$ and $OLTVLP(X)$.

However, there is still some work need to be finished. The double-layer, multi-layer and mixed-layer evaluation problem with linguistic value based on satisfaction and linguistic truth-value will be given in another paper by the author and her colleagues. In addition, the model can be used to the group decision making with linguistic term set. These related work will be given by the author and her colleagues in the following published paper in the near future.

VI. Acknowledgements

This work was supported by the Scientific Research Fund of Southwestern University of Finance and Economics and the National Natural Science Fund of P.R.China (Grant No. 60474022).

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