# Intercarrier Interference Suppression for OFDM Systems Using Hopfield Neural Network

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**Summary**
In Orthogonal frequency division multiplexing (OFDM) transmission system, channel variations within an OFDM symbol destroy orthogonality between subcarriers, resulting in intercarrier interference (ICI), which increases an error floor in proportional to normalized Doppler frequency. To mitigate the effects of channel variations, in this paper, we propose a novel ICI suppression technique, which realizes near maximum likelihood sequence estimation by using continuous Hopfield neural network (HNN). The obvious advantage of using continuous HNN is speeding up the process of signal detection. The each neuron of continuous HNN herein present has multi-level activation function. The number of levels depends on the modulation format adopted in each subcarrier modulation. The performance of the proposed HNN-based detector is evaluated via computer simulations and compared with both conventional detection and optimal detection. It is shown that the HNN detector has low computational complexity and good performance for most Doppler frequency of practical importance.

***Key words:***
*Intercarrier interference, OFDM, Hopfield network, Neural networks*

## 1. Introduction

The high demand for large volume of multimedia services in wireless communication system requires high transmission rates. However, high transmission rates will result in severer frequency selective fading and intersymbol interference (ISI). To combat these channel impairments, orthogonal frequency division multiplexing (OFDM) has been proposed [1]. In OFDM, the entire signal bandwidth is divided into many narrowbands and transmitted over subcarriers. Each subcarrier has a bandwidth much less than the channel coherent bandwidth, i.e., in time domain the symbol duration of the signal in each subcarrier is increased to be much larger than the maximum multipath delay spread, or equivalently, in the frequency domain each subcarrier band exhibits flat fading. However, its priority comes from the orthogonal division of total system bandwidth.

One of the major problem in OFDM system is the high sensitivity of modulation to frequency-offset error caused by oscillator inaccuracies and the Doppler shift. In such situations, the orthogonality between the subcarriers is no longer maintained, which results in ICI. Depending on the Doppler spread in the channel and the block length chosen for transmission, ICI can potentially cause a severe degradation of QoS in OFDM system. The fading channels generally exhibit both frequency selectivity and time selectivity. OFDM has been proposed to combat the frequency selectivity, but its performance might be affected by the time selectivity. The time selective fading causes a loss of subcarrier orthogonality, thus resulting in ICI[2].

These impairments have already motivated several studies to find solutions. Among the several ICI reduction schemes, ICI self-cancellation [3] or polynomial coded cancellation [4] scheme has received much attention due to its simplicity and its high robustness to frequency offset errors. In this technique, each data symbol is transmitted on two adjacent subcarriers with opposite polarity in order to cancel ICI. The data throughput of this scheme will therefore be half of that of conventional OFDM. Thus this cancellation scheme is also referred to as rate-half repetition coding. This cancellation scheme is further extended to reduce more ICI by mapping data symbols onto a larger group of adjacent subcarriers[3][4]. However, this further reduces the data throughput, despite more ICI reduction. In [5], rate 2/3 and 3/4 coding schemes have been proposed to improve the data throughput with moderate ICI reduction.

In this paper we proposed a HNN based signal detection for OFDM system in time-varying multipath fading channel. OFDM systems that employ the proposed detection scheme have higher frequency utilization efficiency compared to ICI self-cancellation based system because the proposed scheme doesn't need any special assistance from transmitter side, unlike rate-half repetition coding scheme. The simplest (conventional) detection implemented by an FFT and a bank of single carrier decision is not the good choice

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due to the existing ICI. In contrast, the optimal detection based on maximum likelihood sequence estimation has ideal performance in mitigating ICI and in term of BER, but it requires computational complexity, which grows exponentially with increasing the number of subcarriers (more exactly, the number of bits in one OFDM symbol). Since there would be a large number of subcarriers in OFDM systems, it would be impractical to implement the optimal detection in most real systems. Therefore, we investigate the application of continuous HNN to the problem of signal detection in OFDM system. It is shown that the NP-complete problem of minimizing the objective function of the optimal detection can be translated into minimizing an HNN "energy" function, thus allowing to take the advantage of the ability of continuous HNN to perform very fast descent algorithms in analog hardware and to produce in real-time suboptimal solutions to hard combinatorial optimization problems. Such application of HNN in signal detection for DS/CDMA, MC-CDMA systems has been widely investigated [6][7]. And these existing HNN models designed for signal detection are limited in that the activation function of neurons is bistable. As well know, in OFDM system each subcarrier normally adopts high order modulation formats. Multilevel activation function for neurons, therefore, is more suitable for OFDM system. In this paper a systematic method for generating multilevel activation function for neurons is suggested. And the performance of the proposed HNN-based detector is evaluated via computer simulations and compared with that of the optimal signal detection and conventional one.

The rest of this paper is organized as follows. In the next section OFDM transmission system and ICI due to time-varying and multipath fading are briefly described. Then, in section III, the HNN based signal detection scheme is derived for OFDM system where 64-QAM is adopted in sub-carrier modulation. After that, in section IV, simulation results are provided, the performance of HNN based detection is compared to that of both the conventional and the optimal detection schemes. Finally, the paper is concluded in section V together with a consideration about its advantage in both detection speed and power consumption.

## 2. System Model and ICI

In OFDM system the available bandwidth is divided into $N$ sub-channels and the length of the guard interval is $G$ . $D(k)$ represents the transmitted data in the th sub-channel and is related to $d(k)$ as $D(k)=\sum_{i=0}^{N-1} d(i)e^{-j2\pi ki/N}$ . $d_p$ is the added cyclic prefix vector with length $G$ and is related to $d$ as follows:

$$d_p(i) = d(N-G+i) \qquad 0 \le i \le G-1 \tag{1}$$

Let $T$ be the time duration of one OFDM symbol after adding the guard interval. Then, $h_k^{(i)}$ represents the $k$ th channel path attenuation at time $t = i \times T_s$ where $T_s = T/(N+G)$ . In our notation $h_k^{(i)}$ for $-G \le i \le -1$ and $0 \le i \le N-1$ represents the $k$ th channel path attenuation in the guard and data interval respectively. Then the data part of the channel output can be expressed as follows:

$$r(i) = \sum_{k=0}^{N-1} h_k^{(i)} d((i-\tau_k) \bmod N) + w(i) \qquad 0 \le i \le N-1 \tag{2}$$

where $d((i-\tau_k) \bmod N)$ represents cyclic shift in the base of $N$ and $w(i)$ represents a sample of additive white Gaussian noise. Then $R$ , the FFT of sequence $r$ , will be as follows:

$$\begin{aligned} R(k) &= D(k)\cdot X_k(0) \\ &+ \sum_{i=1}^{N-1} D((k-i) \bmod N) X_k(i) + W(k) \\ &0 \le k \le N-1 \end{aligned} \tag{3}$$

where $W$ denotes the FFT of $w$ . Furthermore, the second term on the right hand side of Eq. (3) represents the ICI introduced by fading. It can be easily shown that $X_k(i)$ is as follows for $0 \le k, i \le N-1$:

$$X_k(i) = \frac{1}{N}\sum_{m=0}^{N-1}\sum_{u=0}^{N-1} h_m^{(u)} e^{-j2\pi(i\times(u-m)+m\times k)/N} \tag{4}$$

Let $h_m^{ave} = \frac{1}{N}\sum_{u=0}^{N-1} h_m^{(u)}$ represent the time average of the $m$ th channel path attenuation over the data part of the symbol. Then $X_k(0) = \sum_{m=0}^{N-1} h_m^{ave} e^{-j2\pi km/N}$ is the FFT of $h_m^{ave}$ . Define $f_d$ as the maximum Doppler shift. Then the normalized Doppler shift will be $f_{d,norm} = f_d N T_s$ . As $f_{d,norm}$ increases, the second term on the right hand side of Eq.(3) can not be neglected. Therefore, the simplest signal detection shown in Fig.2 (not including the HNN) is suffered from ICI seriously. The main advantage of this detection is its relatively low complexity. Each branch of the detection operates without knowledge of any other sub-carrier. Except for the signal associated with a particular branch, all other sub-carriers are essentially considered as noise.

The optimal detection performances joint detection for all-subcarrier-carrying information based on maximum likelihood sequence estimation (MLSE). As shown in Fig.1, the front end (FFT block) in the optimal detection is the same as that in the simplest detection. Only maximum likelihood sequence estimation (MLSE) algorithm is instead of the single carrier decision. The computational complexity for MLSE is extremely high, particularly if there are more than ten subcarriers. It is clear that the optimal detection is impractical to implement in a real system. However, it is significant to employ MLSE as a benchmark to evaluate the performance of sub-optimal approaches (such as HNN detection described below).

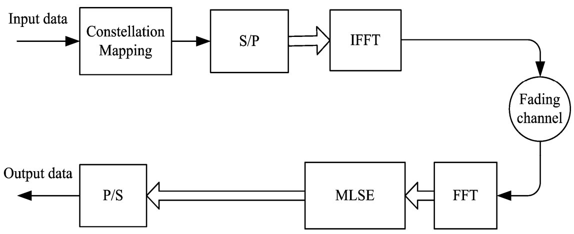


Fig. 1 OFDM signal transmission with optimal detection

## 3. Hopfield Network based Detection

Channel state information is assumed known perfectly at receiver side through channel estimation. Therefore, the optimal or maximum likelihood decision on $\boldsymbol{\alpha}$ is chosen as $\tilde{\mathbf{D}} = [\tilde{D}(0), \tilde{D}(1), ..., \tilde{D}(N-1)]^T$ which maximizes the likelihood function, can be expressed as

$$\tilde{\mathbf{D}} = \arg\{\max_{\mathbf{D}\in\{signal\ set\}} [2\operatorname{Re}(\mathbf{R}^* \times \mathbf{X} \times \mathbf{D}) - \mathbf{D}^* \times \mathbf{X}^* \times \mathbf{X} \times \mathbf{D}]\} \tag{5}$$

where $\operatorname{Re}(\bullet)$ represents the real part of a complex value; $(\bullet)^*$ represents the conjugate transposition of a vector or matrix.

For convenience to describe further, we represent a complex vector (or matrix) as a sum of two real vectors (or matrixes) in the following way:

$$\mathbf{D} = \mathbf{D_R} + j\mathbf{D_I} \tag{6}$$

$$\mathbf{R} = \mathbf{R_R} + j\mathbf{R_I} \tag{7}$$

$$\mathbf{X} = \mathbf{X_R} + j\mathbf{X_I} \tag{8}$$

By combining (5), (6), (7) and (8), we obtain the optimal decision on $\mathbf{D}$ in the form

$$\begin{bmatrix} \tilde{\mathbf{D}}_\mathbf{R} \\ \tilde{\mathbf{D}}_\mathbf{I} \end{bmatrix} = \arg\{\max_{\mathbf{D_R}+j\mathbf{D_I}\in\{signal\ set\}} (2[\mathbf{R}_\mathbf{R}^T \mathbf{R}_\mathbf{I}^T] \times \begin{bmatrix} \mathbf{X_R} & -\mathbf{X_I} \\ \mathbf{X_I} & \mathbf{X_R} \end{bmatrix} \times \begin{bmatrix} \mathbf{D_R} \\ \mathbf{D_I} \end{bmatrix} - [\mathbf{D}_\mathbf{R}^T \quad \mathbf{D}_\mathbf{I}^T] \times \begin{bmatrix} \mathbf{X}_\mathbf{R}^T \times \mathbf{X_R} + \mathbf{X}_\mathbf{I}^T \times \mathbf{X_I} & \mathbf{X}_\mathbf{I}^T \times \mathbf{X_R} - \mathbf{X}_\mathbf{R}^T \times \mathbf{X_I} \\ \mathbf{X}_\mathbf{R}^T \times \mathbf{X_I} - \mathbf{X}_\mathbf{I}^T \times \mathbf{X_R} & \mathbf{X}_\mathbf{R}^T \times \mathbf{X_R} + \mathbf{X}_\mathbf{I}^T \times \mathbf{X_I} \end{bmatrix} \times \begin{bmatrix} \mathbf{D_R} \\ \mathbf{D_I} \end{bmatrix})\} \tag{9}$$

For the sake of simplicity, the Eq. (9) is expressed as follows:

$$\tilde{\mathbf{D}}_\mathbf{RI} = \arg\{\max_{\mathbf{D_{RI}}\in\{signal\ set\}} (2\mathbf{R}_\mathbf{RI}^T \times \mathbf{X_{RI}} \times \mathbf{D_{RI}} - \mathbf{D_{RI}}^T \times \mathbf{W_{RI}} \times \mathbf{D_{RI}})\} \tag{10}$$

where $\mathbf{D_{RI}} = \left[\mathbf{D_R}^T, \mathbf{D_I}^T\right]^T$; $\mathbf{R_{RI}} = \left[\mathbf{R_R}^T, \mathbf{R_I}^T\right]^T$;

$$\mathbf{X_{RI}} = \begin{bmatrix} \mathbf{X_R} & -\mathbf{X_I} \\ \mathbf{X_I} & \mathbf{X_R} \end{bmatrix};$$

$$\mathbf{W_{RI}} = \begin{bmatrix} \mathbf{X}_\mathbf{R}^T \times \mathbf{X_R} + \mathbf{X}_\mathbf{I}^T \times \mathbf{X_I} & \mathbf{X}_\mathbf{I}^T \times \mathbf{X_R} - \mathbf{X}_\mathbf{R}^T \times \mathbf{X_I} \\ \mathbf{X}_\mathbf{R}^T \times \mathbf{X_I} - \mathbf{X}_\mathbf{I}^T \times \mathbf{X_R} & \mathbf{X}_\mathbf{R}^T \times \mathbf{X_R} + \mathbf{X}_\mathbf{I}^T \times \mathbf{X_I} \end{bmatrix}$$

It is obvious that the computational complexity of the optimal detection grows exponentially with the number of bits contained in an OFDM symbol. Therefore, optimal detection scheme is difficult to be used in the existing OFDM system such as WLAN, HiperLAN, DAB, and DVB.

As shown in Fig.2, a Hopfield neural network is set between the FFT and the bank of single carrier demodulators as a component to mitigate the ICI.

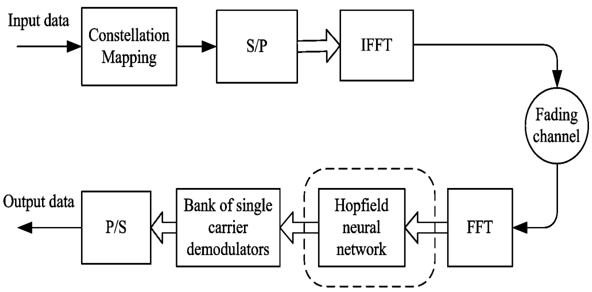


Fig. 2 OFDM signal transmission with HNN detection

The time evolution of the Hopfield network is represented by

$$\frac{du_k(t)}{dt} = -\frac{u_k(t)}{\tau} + \sum_{l=0}^{K-1} W_{k,l} \times v_l(t) + \psi_k \qquad k = 0,1,...,K-1 \tag{11}$$

$$v_k(t) = f[u_k(t)] \qquad k = 0,1,...,K-1 \tag{12}$$

where $u_k(t)$, $v_k(t)$ are the input and output of the $k$th neuron, respectively; $W_{k,l}$ is a connection weight between the output of $k$th neuron and the input of $l$th neuron; $f(\bullet)$ is the neuron activation function, which is a differential monotonic increasing, and bounded function; $\psi_k$ is the bias current of $k$ th neuron; and $\tau = R \times C$ is the time constant of the circuit.

The energy function of the Hopfield network is defined as follows,

$$E = -\sum_{k=0}^{K-1} \psi_k \times v_k(t) - \frac{1}{2}\sum_{k=0}^{K-1}\sum_{l=0}^{K-1} v_k(t) \times W_{k.l} \times v_l(t) \qquad (13)$$

By the appropriate choice of HNN parameters such as bias currents $\psi_k$, $k = 0,1,...K-1$; connection weight $W_{k,l}$, $k = 0,1,...K-1$, $l = 0,1,...K-1$; initial network states $u_k(0)$, $k = 0,1,...K-1$; and activation function $f(x)$, the HNN can help us find out the estimation on $\mathbf{D}$ through network self-evolution.

The Eq. (13) can be expressed in matrix form as:

$$E = -\Psi^T \times \mathbf{V(t)} - \frac{1}{2} \times \mathbf{V(t)}^T \times \mathbf{W} \times \mathbf{V(t)} \qquad (14)$$

where $\Psi = [\psi_0, \psi_1, ..., \psi_{K-1}]^T$; $\mathbf{V(t)} = [v_0(t), v_1(t), ..., v_{K-1}(t)]^T$; $\mathbf{W}$ is a $K \times K$ matrix, the element of $\mathbf{W}$ is $W_{k,l}$, $k = 0,1,...K-1$, $l = 0,1,...K-1$.

After comparing equations (10) and (14) and setting the variables as follows:

$$\Psi = \mathbf{X_{RI}}^T \times \mathbf{R_{RI}} \qquad (15)$$

$$\mathbf{W} = -\mathbf{W_{RI}} \qquad (16)$$

$$\mathbf{V(t)} = \mathbf{D_{RI}} \qquad (17)$$

we have established the mapping between likelihood function and energy function. It is easy to derive from the definition of the connection matrix $\mathbf{W}$ that $\mathbf{W}$ is a symmetrical matrix.

The choice of activation function $f(\bullet)$ depends on the modulation format adopted in sub-carrier modulation. For 64 QAM considered in this paper, the activation function is designed as follows:

$$f(x) = \begin{cases} -6 + \dfrac{1-\exp(-2\times\alpha\times(x+6))}{1+\exp(-2\times\alpha\times(x+6))} & x < -6 \\ k+1-(k+1-x)^{\alpha} & k \le x < k+1, \quad k \in \{-6,-4,-2,0,2,4\} \\ k-1+(x-k+1)^{\alpha} & k-1 \le x < k, \quad k \in \{-4,-2,0,2,4,6\} \\ 6 + \dfrac{1-\exp(-2\times\alpha\times(x-6))}{1+\exp(-2\times\alpha\times(x-6))} & x \ge 6 \end{cases} \qquad (18)$$

where $\alpha$ $(\alpha > 1)$ is a parameter used to control the slope (or gain) of the activation function. As the parameter increases the width of the transition region becomes narrow (i.e. the slope of the curve in the transition region becomes steep). The Fig.3 illustrates neuron activation functions with different gains. The activation function is a differential, monotonic increasing, and bounded function. These properties of the activation function guarantee that the energy of Eq. (13) always decreases with time evolution, and network will be converged [9]. When the width of the transition region of the activation function is narrow, corresponding to large $\alpha$, the stable states of the Hopfield network will converge to a state give by $v_k(t) \in \{\pm1, \pm3, \pm5, \pm7\}$, $k = 0,1,...K-1$. These states coincide with the signal constellation of 64-QAM.

The externally supplied input current for each neuron $\psi_k$, $k = 0,1,...K-1$ and the connection weight between neurons $W_{k,l}$, $k = 0,1,...K-1$, $l = 0,1,...K-1$ are determined by Eq. (15) and (16), respectively. With the network states $u_k(0)$, $k = 0,1,...K-1$ initiated at zero, the input sequence $\psi_k$, $k = 0,1,...K-1$ is applied to the network. After the network is converged the outputs of neurons are fed into the bank of single carrier demodulators, where signal levels are transformed into bit sequences.

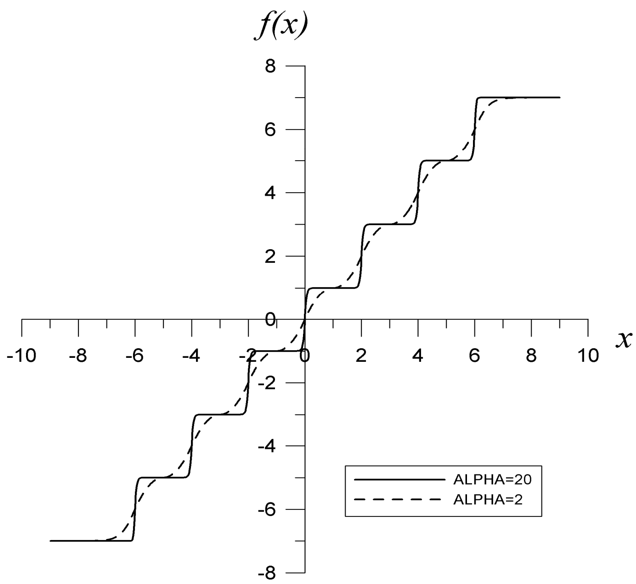


Fig. 3 Activation function of a neuron

## 4. Simulation Results

The HNN based detection is evaluated for time-varying and multipath fading channels by Monte Carlo trials. $T_s$ spaced, tap-delay-line channel model of six paths and exponential power delay profile is employed. Each tap is an independent, zero mean, complex Gaussian variable. An OFDM signal with 48 subcarriers and 64-QAM modulation is used in the simulation to evaluate the performance of the proposed detection.

Figure 4 shows that BER performance for the conventional OFDM system, the proposed HNN based scheme and optimal detection. It is clear to see that due to Doppler shift there exists an error floor in the conventional OFDM system. Optimal detection resolves this problem very well by using Maximum Likelihood Sequence Estimation. However, it is impossible to implement optimal detection in real-time due to high computational complexity. The proposed HNN detection scheme achieves the nearly same BER performance as the optimal detection.

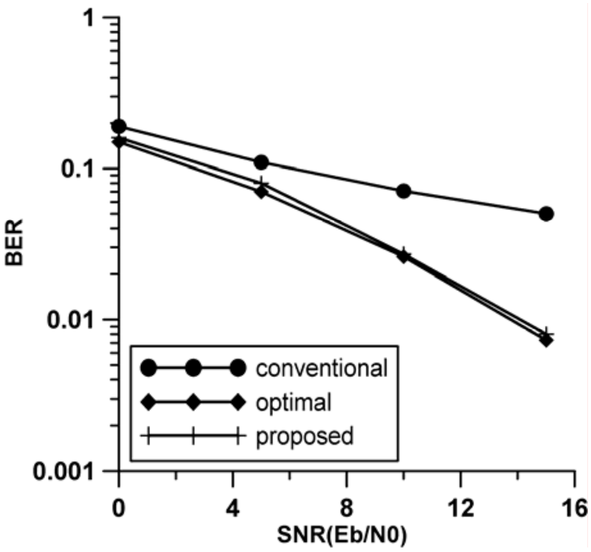


Fig.4. Normalized Doppler frequency Offset = 15%

## 5. Conclusions

We investigate the application of HNN to the problem of signal detection in OFDM system where the orthogonality of sub-carriers is lost due to the time-varying and multipath fading channels. Considering high order modulation format is usually adopted in sub-carrier modulation in OFDM system, the concept of conventional sigmoid function is extended, and a general sigmoid function, shown in Fig.3, is designed as an activation function of neuron. The proposed signal detection scheme employs a HNN to perform likelihood test with possible symbols. Although the designed HNN is only guaranteed to converge to a local minimum of the maximum likelihood objective function, the HNN based detection has been shown to have stronger capability in mitigating the ICI.

On the other hand, the proposed HNN detector has fast detection speed. The main part of the receiver can be implemented by relatively simple analog VLSI hardware with convergence times in the order of a few nanoseconds and less power consumption, which becomes especially important in applications such as handheld and mobile wireless communications.

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