Structural Safety Assessment of Bridge By Integration of Wavelet and D-S Evidential Theory

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Summary:
The high nonlinearity and nonstationarity has resulted in the huge challenge in structural safety assessment of bridge in the health monitoring by using the traditional method. In this paper, a novel technique is introduced to solve the problem in the structural safety assessment of bridge by the integration of the wavelet and the D-S evidential theory. In the context, the effects of live-load deflections and strains of the main girder are firstly extracted from the total measurement by wavelet analysis, and then they are used in the primary safety assessment associated with the change of inherent frequency of the bridge by using artificial neural networks. And futher the basic probability assignments are constructed according to results of the primary assessment through the relative artificial neural network. Finally, the integrated assessment of the bridge is completed by the information fusion based on the D-S evidential theory. The result from the application of a real bridge indicate that the proposed method is a promising way in the safety assessment of large scale bridge in the future.

Keywords:
bridge, safety assessment, wavelet, D-S evidential theory.

1. Introduction

Influenced by lots of complex factors in the service, the structural reliability of bridge will be declined and the safety trouble will occur, which will result in gross loss of economy and human life. Consequently, people have paid great attention all the while to the structural safety assessment of bridge in the academe and the engineering. As the traditional local detect and load test can not fit the assessing requirement of the increasingly rapid development of bridge, the technology for structural health monitoring of bridge, which integrates the information technology into the traditional mechanical theory, has received increasing concern and made some development in the recent years. However, the bridge constructed with numbers of components is usually large in size and is heavily impacted by many complicated factors, resulted in high nonlinearity and uncertainty in the structural safety assessment of bridge[1]. And therefore the great difficulty arises by using the classical assessment method with a single parameter and must be removed in the structural safety assessment in the health monitoring of bridge.

In the safety assessment, the effects of live-load deflection and strain of the main girder contain the structural state information of the bridge, by which the safety feature proxy can be constructed and the structural safety assessment of the bridge can be completed. However, because of the influence of the temperature, they are confused in the total measurement of the deflection and strain. Consequently, the temperature effect must be separated firstly from the real measurement in the process of the structural safety assessment .

Considering the muti-scale characteristic of the structural responses and the impact factors, in this paper, a method based on wavelet analysis is presented to extract the effect of live-load deflection and strain of the main girder from the total measurement before the safety assessment.

On the other hand, the information fusion technology based on Dempster-Shafer (D-S) evidential theory[2] that has been applied widely in many fields, such as failure diagnosis, object identification and etc, has the outstanding advantages in reasoning and decision-making of uncertainty, which can overcome the shortcoming of the uncertainty in the safety assessment with only one parameters.

That is said, an integration of the two methods may be a considerable solution for the stuctural safety assessment of bridge.

2. Extracting Effect of live-load deflection and strain

2.1 The Basic wavelet theory

Wavelet transform is a power tool for signal processing and analysis developed rapidly in recent years. Wavelet decomposition of time-varying signals is a kind of localization analysismethod in time and frequency domains, and the time-frequency window can both be changed. This signal processing method has higher frequency and time resolution[3].

For a signal of limited energy $f(t) \in L^2(R)$, its continuous wavelet transfrom is defined as following equation[4]:

$$Wf(a,b) = \frac{1}{\sqrt{a}} \int_R f(t) \overline{\psi\left(\frac{t-b}{a}\right)} dt = \left\langle f(t), \psi_{a,b}(t) \right\rangle \tag{1}$$

Where a is the scale factor and b is the distance factor ,and $\psi(t)$ is the base wavelet. In the engineering practice, the discrete form of the wavelet transform showed below is frequently used.

$$c_{mn} =< f(t), \psi_{mn}(t) > \tag{2}$$

Where <•> expresses the inner accumulation and $\psi_{mn}(t)$ is defined as :

$$\psi_{mn}(t) = 2^{\frac{m}{2}} \psi(2^m t - n), m, n \in z \tag{3}$$

Then the reverse transform of the discrete wavelet transform of an abitrary $f(t) \in L^2(R)$ is expressed as:

$$f(t) = \sum_{m,n} < f(t), \psi_{mn}(t) > \psi_{mn}(t) = \sum_{m,n} c_{mn} \psi_{mn}(t) \tag{4}$$

There is a fast analysis method for the wavelet transform and the reverse transform that is named Mallat arithmetic[5]. By this fast method, an arbitrary signal can be decomposed to be the approximate and detailed components at a free scale, where the approximate part represents the low-frequency component of the signal and the detailed part represents the component of the high-frequency relatively .

2.2 Extraction of the effect of live-load deflection and strain

For achieving the structural safety assessment of bridge, the effect of live-load deflection and strain must be extracted from the total measurement by wavelet analysis. Several things must be done in this process. Firstly, the basis wavelet must be selected according to the characteristics of the wavelet function, including the orthogonality, symmetry, support width, regularity and etc.

Among all of the wavelet characteristics, orthogonality is the most desired property in any signal analysis operation because the orthogonal wavelets can decompose signals into well-behaved orthogonal signal spaces. In practice, finite support and compact wavelets are more popular due to their relations to multiresolution filter banks. These wavelets have finite impulse response (FIR) wavelet filters. For some applications, the symmetry is also one of the key property that must be considered, which will perform an great benefit to the wavelet reconstruction with the most little distortion. In addition, symmetric wavelets make it easier to deal with the boundaries of the image.For example, for image coding in image processing applications, since human vision is more tolerant to symmetric error than asymmetric one, it is very desirable to use symmetric wavelets[6].

However, it is impossible to select a wavelet basis with every characteristic because there is confilct among the properties. For instance, all orthogonal wavelets are asymmetric in general. In view of this status, we should select the wavelet basis in a middle course regarding the wavelet characteristics. For example, we may select a orthogonal wavelet basis with near symetry property.

Secondary, the wavelet order J must be selected to decompose the signal of live-load effect based on the multiresolution analysis of the signal and the real demand.

Finally, decomposition of the signal is processed to the order J, and the reconstruction of the approximation information of the signal is completed and then the effect of live-load deflection or strain can be obtained by the difference between the total measurement and the approximation information.

The extraction is descibed below taking the deflection as example. Let the total deflection at one of the locations at the main girder is y, and the approximation information of the deflection reconstructed by wavelet to order J is y_{Am}, Then the Effect of live-load deflection is :.

$$y_{Dm} = y - y_{Am} \tag{5}$$

For strain: $$S_{Dm} = S - S_{Am} \tag{6}$$

3. D-S Evidential theory

3.1 Basic concepts of D-S theory

As a mathematical theory of evidence, Dempster-Shafer theory is always applied in the decision-making stage of information fusion, in which the uncertainty of system is represented with the upper limit and lower limit of probability. One of the most important features of D-S theory is that the model is designed to cope with varying levels of precision regarding the information and no further assumptions are needed to represent the information.

The basic probability assignment, represented by m, defining a mapping of the power set to the interval between 0 and 1, can be represented with the following three equations:

$$m : u(X) \to [0,1] \tag{7}$$

$$\sum_{A \in u(X)} m(A) = 1 \tag{8}$$

$$m(\Phi) = 0 \tag{9}$$

where $u(X)$ represents the power set of X, Φ is the null set, and A is a set in the power set(A . $u(X)$).

For a given $u(X)$ and basic probability assignment m, the *Belief* function for a set A is defined as the sum of all the basic probability assignments of the proper subsets (B) of the set of interest (A). $(B \subseteq A)$.

$$Bel(A) = \sum_{B \subseteq A} m(B) \tag{10}$$

Which represents the lower bound of the believe interval. And, respectively, the upper bound *Plausibility*, is the sum of all the basic probability assignments of the sets(B) that intersect the set of interest (A) ($B \cap A = \Phi$).

$$Pl(A) = 1 - Bel(\bar{A}) = \sum_{B \cap A \neq \Phi} m(B) \tag{11}$$

Also, for given *Belief* value, the basic probability assignment can be obtain by :

$$m(A) = \sum_{B \subseteq A} (-1)^{|A-B|} Bel(B) \tag{12}$$

where $|A\text{-}B|$ is the difference of the cardinality of the two sets.

3.2 Combination rule of two evidences

In D-S theory of evidence, the combination mass function of two independent basic probability assignments $m1$ and $m2$ can be calculated from the following equation:

$$m = m1 \oplus m2 \tag{13}$$

where the orthogonal sum of $m1$ and $m2$ can be expressed as following:

$$m(C) = \frac{\sum_{A_i \cap B_j = C} m_1(A_i) m_2(B_j)}{1 - K} \tag{14}$$

where $C \subseteq u$ and $C \neq \Phi$, $m(\Phi) = 0$, and K can be expressed by

$$K = \sum_{A_i \cap B_j = \Phi} m_1(A_i) m_2(B_j) \qquad (15)$$

The equation (14) represents the combination of two independent evidences. For combination of more than two evidences, we can complete it one by one using the equation (14).

3.3 The decision-making in D-S theory

For basic probability assignments given, the decision making of fusion can be completed by using the equation (16)-(18). For given $A_1, A_2 \subseteq u$, if:

$$m(A_1) = \max\{\, m(A_i), A_i \subset u \,\} \qquad (16)$$

$$m(A_2) = \max\{m(A_i), A_i \subset u \text{且} A_i \neq A_1 \}(17)$$

$$\begin{aligned} & m(A_1) - m(A_2) > \varepsilon_1 \\ & m(A_1) > m(u) \\ & m(u) < \varepsilon_2 \end{aligned} \qquad (18)$$

then A_1 is the result of decision, where ε_1、ε_2 are the threshold values selected.

4. Safety assessment by data fusion

4.1 Modelling and primary assessment

In order to complete the safety assessment of bridge with the integration of artificial network and D-S evidential theory, we should firstly build a feasible model which includes the object evaluated, the parameters needed, the methods and steps of assessment. Figure 1(Fig.1) shows the diagram of the model applied in this paper. From this diagram, we can see that the state assessment is divided into two basically steps. Firstly, the primary evaluation of the state is completed with the artificial neural networks, which finished the nonlinear mapping between the correspondent subspace P_i of the parameters and the state subspace S_i of the sub-system. And then, with the outputs of the state sub-space taken as the evidences, the information fusion is achieved and the decision-making is finally decided by using the D-S evidential theory[7].

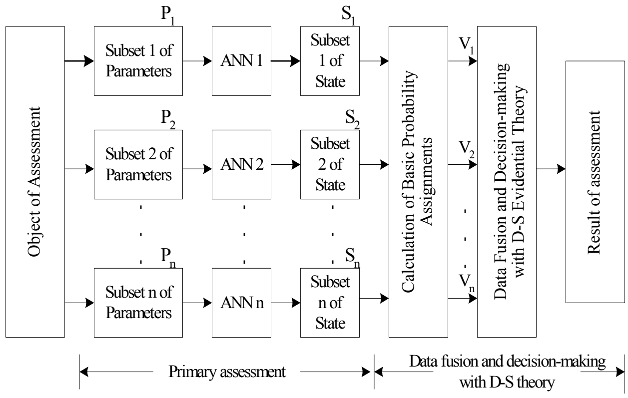

Fig1 Schematic of structural safety assessment of bridge by data fusion

In the primary stage, the correspondent state feature proxies are firstly selected as the inputs of each sub-ANN and the related states of bridge structure are selected as the outputs. Each group of state feature proxies form a parametric subspace P_i while the outputs of the artificial neural network form a state subspace S_i. Then, according to the P_i and S_i, the training samples of each sub-ANN are constructed and applied in the training work and therefore the nonlinear mapping $f: P_i \rightarrow S_i$ is achieved by this artificial neural network. Finally, the test of each sub-ANN should be done and the performance of this ANN is decided for the preparing of information fusion.

4.2 Data fusion and decision-making

For data fusion with D-S evidential theory, the construction of identification frame, evidences and their basic probability assignments must be completed firstly. Here, the identification frame is constructed by combination of the state subspace S_i of the sub-system, denoted by : $U = S_1 \cup S_2 \cup \ldots \cup S_n = \{u_1, u_2, \ldots, u_n\}$. And, the evidence V_i is constructed according to the output vector S_i of each sub-ANN.

The basic probability assignments can be calculated according to the characteristics of assessment and practical demands. In this article, we can calculate the basic probability assignments of evidence V_i to proposition u_j by following equations:

$$m_i(u_j) = \frac{R_i(u_j)}{\sum_j R_i(u_j) + K_i} \tag{19}$$

$$R_i(u_j) = \frac{1/d_{ij}}{\sum_j 1/d_{ij}} \tag{20}$$

where $R_i(u_j)$ is the correlation coefficient between evidence V_i and the proposition u_j in the identification frame, d_{ij} is the *Manhattan* distance between the output vector X_i and the standard expected output vector Y_i of each sub-ANN. And K_i is the total uncertainty of the whole system, expressed by :

$$K_i = 1 - \alpha_i \beta_i \gamma_i \tag{21}$$

where γ_i is the identified precision of the ith sub-ANN, and α_i represents the difference between the two maximum coefficients of evidence V_i to the proposition u_j in the identification frame and β_i is the variance of the coefficients of evidence V_i to all of propositions except the maximum one, which can be expressed by the following equations:

$$\beta_i = [\frac{1}{N-1}\sum_j (R_i(u_j) - \tau_i)^2]^{\frac{1}{2}} \tag{22}$$

$$\tau_i = \frac{1}{N-1}\sum_{\substack{j=0 \\ j\neq m}}^{N} R_i(u_j) \tag{23}$$

where m means that when $j=m$, the correlation coefficient of the evidence i to the proposition m is maximum. After all of the basic probability assignments are calculated, the data fusion with D-S evidential theory can be completed by the equation (14) and (15), and the decision-making can be also achieved with equation (16) to (18).

5. Example

As the key section of Chongqing Masangxi Yangtze River bridge in China, the section in mid-span is selected and the deflection n5 and strain20 in this section are selected for demonstration of the proposed method in this paper.

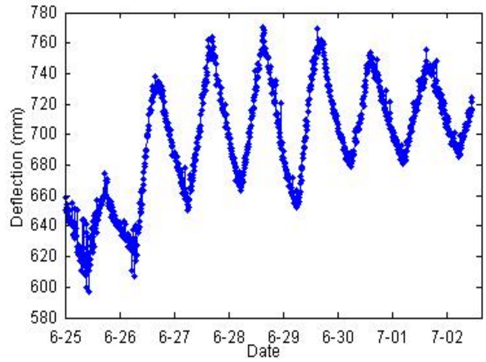

Fig.2 Deflection of n5 at mid-span

According to the analysis of the signal and the wavelet properties, the Sym5 is selected as the basis wavelet and the order is selected as J=5. The effect of live-load deflection and strain after decomposition and reconstruction are showed in figure 2 to figure 5. From the results, we can see the effect of live-load deflection or stain vary with significant random characteristic.

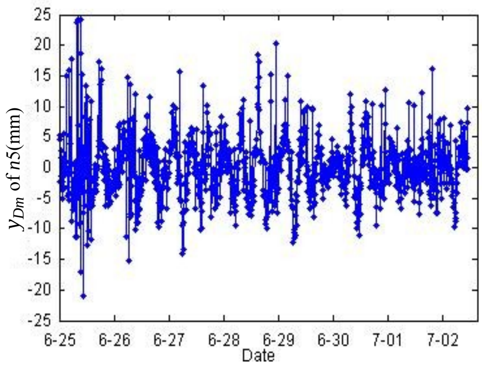

Fig.3 Effect of live-load deflection of n5 at mid-span

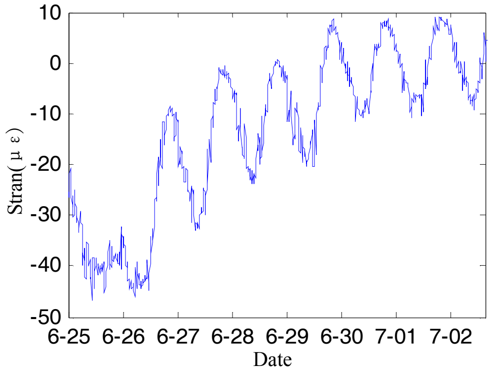

Fig.4 Strain20 at mid-span

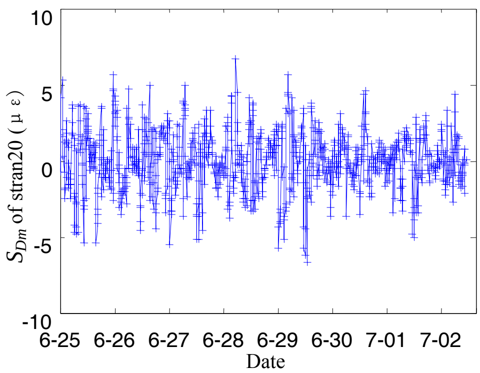

Fig.5 Effect of live-load strain 20 at mid-span

5.1 Primary assessment

Following the steps described in the former context, for a unit of the bridge, three different BP artificial neural networks are selected to finish the nonlinear mapping between P_i and S_i ,and the parameters for state assessment are classified into three correspondent groups. The first group of parameters includes: {temperature, y_{Am}, y_{Dm}}. The second group of parameters is decided by:{temperature, S_{Am}, S_{Dm}}. The third group of parameters includes the first 3 mode shapes of the bridge (vertical bending mode): $\{f_1, f_2, f_3\}$. According to real conditions of the bridge structure, the subspace S_i of state is selected as following:

state Ⅰ: Normal (denoted by u_1, and the standard output of sub-ANN is $(1\ 0\ 0)^T$)

state Ⅱ: unsafe (u_2 ; $(0\ 1\ 0)^T$)

state Ⅲ: uncertain(u_3 ; $(0\ 0\ 1)^T$)

After the subsets of parameters and subspaces of states are decided, the BP ANN can be decided. Here, the structures of the three BP ANN were selected as 3×8×3, with log-sigmoid transfer function in the hidden layer and a linear transfer function in the output layer. Then, each BP ANN is trained with 200 samples constructed by historical parameters and the performance test of the BP ANN trained was completed by 200 test samples. Table 1 showed the test results of the three BP ANN.

Table 1. The test performance of the three BP ANN(%)

ANN	Identified rate	Fault rate	Rejected rate	Identified precision
1	86	5	8	94.50
2	89	8	6	91.75
3	79	10	11	88.76

5.2 Results of Data fusion

In the primary assessment, the identification frame can be decided as following:

$$U = \{u_1, u_2, u_3\}$$

In the three different periods, three different groups of measured structural parameters of the key section of the bridge are input to the correspondent BP ANN trained, and the primary results of safety assessment are obtained. And then, with equation (19) to (23), the correlation coefficient between each evidence V_i and correspondent proposition u_j in the identification frame is calculated and the basic probability assignments (BPAs) of each proposition is therefore decided. The results of the basic probability assignments are shown in Table 2. Finally, the data fusion is achieved by equation (14) and (15), and the decision-making is completed by equation (16) to (18) and the results of data fusion are shown in Table 3 ($\varepsilon_1 = \varepsilon_2 = 0.06$). From Table 2 and Table 3, we can see that the BPAs of the uncertainty is deduced and the BPAs of proposition $u1$ is enhanced significantly after data fusion, and the structural state of the bridge can be easily decided with the judging rule of D-S theory, whereas it can not be decided before data fusion.

Table 2. The results of BPAs of three different evidences

Evidence	$m(u_1)$	$m(u_2)$	$m(u_3)$	Date
V_1	0.73	0.11	0.16	6.25
V_2	0.55	0.33	0.12	
V_3	0.47	0.43	0.10	
V_1	0.69	0.15	0.16	6.29
V_2	0.76	012	0.12	
V_3	0.53	0.12	0.35	
V_1	0.71	0.13	0.16	7.02
V_2	0.47	0.26	0.27	
V_3	0.62	0.18	0.20	

Table 3. The results of BPAs of data fusion

$m(u_1)$	$m(u_2)$	$m(u_3)$	Decision	Date
0.826	0.146	0.028	u1	6.25
0.907	0.069	0.024	u1	6.29
0.846	0.100	0.054	u1	7.02

6. Conclusion

According to the demand of the safety assessment of the bridge, the temperature effect of structural deflection and strain are separated and the effect of live-load deflection and strain are extracted by wavelet analysis. Then the primary safety assessment of the bridge was implemented via three sub-ANNs and the basic probobility assignments are obtained relatively. Finally, the integrated assessment of the bridge is completed by information fusion based on D-S evidential theory. The results indicated that the wavelet analysis is effective to extract the effect of live-load deflection and strain that include the structural safety information. The information fusin can deduce the uncertainty and promote the accuracy of the structural safety assessment of bridge.

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