Computational simulation of gas transport of human respiratory system using fractal networks

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Summary
The optimal gas transport system in human respiratory network should maximize hydraulic permeability for a given investment in transport tissue. The computer simulation of hydraulic permeability of a branched transport system is proposed in order to investigate how this optimum can be achieved in the human respiratory system. Based on the parallel and series principles and Darcy's law, the equivalent permeability of human respiratory system is developed by using fractal networks. The fractal network model of human respiratory system is presented to model gas transport properties. The influence of fractal parameters on permeability is investigated numerically. The investigation show that the equivalent permeability of respiratory network will increase with the scaling factor β and the number of bifurcating branches M. The finite element model of human respiratory system is presented and the pressure and velocity distribution in the human respiratory system are computed and analyzed.

Key words:
Gas transport, Human respiratory system, Fractal networks, Computational simulation

1. Introduction

The geometry and dimensions of branched structures such as blood vessels or airways are important factors in determining the efficiency of physiological processes. It has been shown that fractal trees can be space filling and can ensure minimal dissipation. The bronchial tree of most mammalian lungs is a good example of an efficient distribution system with an approximate fractal structure. A widely-used model for simulations and predictions of gas transport, particle deposition and dosimetry is that of a regular, dichotomic branching pattern, a representation that has its origin in the so-called trumpet model that is used with geometric adjustments introduced later. This is a simplification of the lung geometry that assumes a spatially symmetric branching, a fixed relation of the cross sections of a parent branch with the daughter branches, and a constant length of all possible spatial trajectories between the trachea and any terminating bronchioli. This model is self-similar regarding spatial scaling properties and as such it reveals fractal properties[1]. Considering a human upper airway model, or equivalently complex internal flow conduits, the transport and deposition of nano-particles in the 1~150 nm diameter range are simulated and analyzed for cyclic and steady flow conditions[2]. The bronchial system, the pulmonary artery and the pulmonary vein of the lung of the domestic pig, *Sus scrofa* were simultaneously cast with silicone rubber[3]. Asymmetrical dichotomous bifurcation preponderated in the tree-like arrangement of the three conducting systems. Lengths and diameters of the various generations were measured. The strong correlations between some of the structural parameters indicated a high level of structural optimization.

Simple branching networks are most easily illustrated by river networks. The merging of small streams gives larger streams, large streams merge to give small rivers, and so forth. But small streams can also merge with larger streams and rivers. The branching statistics of networks can differentiate between alternative models for their formation. The original taxonomy for branching networks was given by Horton(1945). Strahler(1957) introduced a slightly modified system which is now widely used in a variety of applications[4]. In the bronchial tree, airways branch by dichotomy with a systematic reduction of their length and diameter. In the human lung, for example, the conducting airway tree ends at about 2^{17} terminal bronchioles, each of which is followed by roughly six generations of alveolar ducts that constitute the acini and are involved in gas exchange. Considering the lower part of the bronchial tree (generations 6–16) and assuming that air flow in this duct system obeys Poiseuille law (a good approximation below the sixth generation at rest 8), a 'best' structure can be deduced by minimizing the total viscous dissipation in a finite tree volume[5]. Even though the human bronchial tree, at least for the first six generations, exhibits a most symmetrical branching if compared to a wide variety of species, irregular and asymmetric branching pattern cannot be neglected. Therefore, regular symmetric models do not predict gas flow and particle deposition reliably, and more realistic

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models are needed to increase accuracy. Dichotomic branching models also tend to fill space in a spherical fashion and can not be adapted to the form and volume of the human lung. For the local analysis of air flows, more sophisticated models are in use to show that many of the transport processes within the airways depend quite sensitively on the geometry of the bronchial bifurcations and the structure of the boundaries. But in the global modelling of the tracheobronchial tree, recent efforts rely on stochastic models or synthesize the underlying structure from fixed scaling laws based on a small number of anatomic examinations. These models meant to represent a mean adult lung, rather than describing the lung of a specific individual. The aim of the paper is to develop a computing model of bronchial tree of human respiratory system based on fractal networks, to analyze the influence of fractal parameter on the permeability, and simulate the air flow characteristics in the human respiratory system by using computer modeling technology.

2. Gas transport simulation of human respiratory system using fractal networks

In a fractal channel network of human respiratory system, the diameter of the channels decreases as the branching level rises, as shown in Fig.1. This simply causes a significant decreasing effect in the Reynolds number in the channels in the direction of decreasing channel diameters because the total flow rate of the fractal channel network is fixed. Consequently, the thermal and hydrodynamic characteristics of the flow in the channels will show a variation for different branching levels that will also affect the overall thermal and hydrodynamic performance of the heat sink[6].

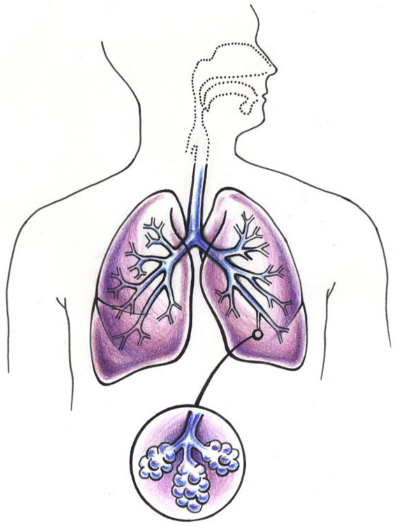

Fig. 1 Self-similar fractal characteristics of human respiratory system

Arterial flow of blood in the human body and air flow in lungs exhibits a tree like flow structure, as shown in Fig. 2. The tree like flow structure is also a characteristic of many other nonliving natural flow systems, such as the flow in a river basin, etc. The tree like flow structure in living and nonliving entities is an invitation to engineers to design their engineering flow systems in a tree like form. The tree like flows in living and nonliving entities are optimized by nature by taking into account “all” the conditions and constraints that affect the flow.

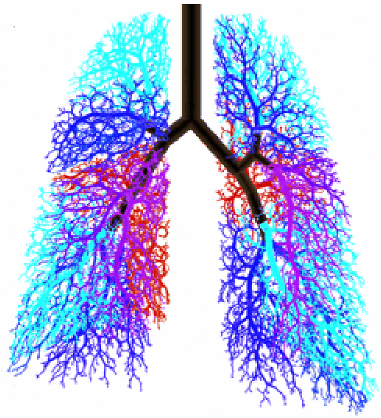

Fig. 2 Respiratory system composed of branching tubes

Scaling laws arise from the interplay between physical and geometric constraints implicit in the following principles. First, in order for the network to supply the entire volume of the organism, a space-filling fractal-like branching pattern is required. Second, the final branch of the network (such as the capillary in the circulatory system) is a size-invariant unit. And third, the energy required to distribute resources is minimized; this final restriction is basically equivalent to minimizing the total hydrodynamic resistance of the system[7]

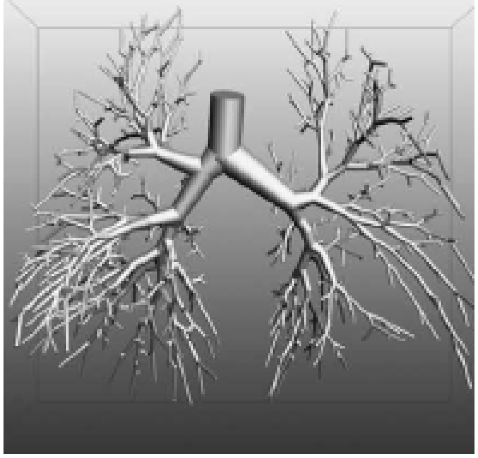

Fig. 3 Topological representation of bronchial tree model

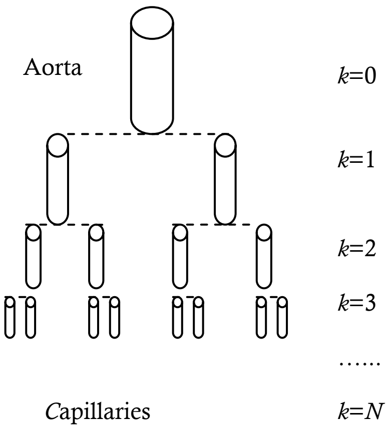

Fig. 4 Binary tree model of human respiratory system

In the general case, the fractal network of human respiratory system is composed of N branchings from the aorta(level 0) to the capillaries(level N, denoted here by a subscript c). To generate a fractal channel network, an initiator channel of length l_0 and diameter d_0 bifurcates at one end as shown in Fig. 4. The new channels of length l_1 and diameter d_1 bifurcate at each end to form the second branching level of the fractal network. The bifurcations at the ends of the newly formed channels may be continued until the required fractal channel network at the specified branching level is obtained. The index k shows the level of branching. A typical branch at some intermediate level k has length l_k, radius r_k, and pressure drop Δp_k, as shown in Fig.5 .

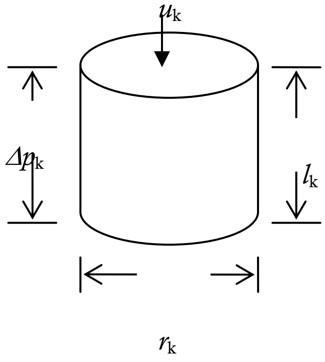

Fig. 5 Basic parameters of fractal model

The volume rate of flow at level k can be expressed as follows

$$q_k = \pi r_k^2 u_k \tag{1}$$

Where q_k is the volume rate of flow of every tube at level k, r_k is the radius of branch at level k; and u_k is the flow speed averaged over the cross section at level k. The total number of braches at level k is determined by follows[8]

$$N_k = M^k \tag{2}$$

Where M represents the number of channels that appear at one end of a channel after branching; N_k is the total number of braches at level k. According to the mass conservation law, the fluid rate of system can be computed as follows

$$q_0 = N_k q_k = N_k \pi r_k^2 u_k \tag{3}$$

Where q_0 is the fluid rate of system. In order to characterize the branching, the two scale factors are introduced

$$\beta_k = \frac{r_{k+1}}{r_k} \tag{4}$$

$$\gamma_k = \frac{l_{k+1}}{l_k} \tag{5}$$

Where β_k and γ_k are the scaling factors associated with the length and diameter of the channels, respectively. In order to minimize the energy dissipated in the system, the fractal network must be a conventional self-similar fractal:

$$\beta_k = \beta \tag{6}$$

$$\gamma_k = \gamma \tag{7}$$

$$n_k = n \tag{8}$$

$$M = \gamma^{-D_l} \tag{9}$$

$$M = \beta^{-Dd} \tag{10}$$

Where D_l and D_d are fractal dimensions associated with the length and diameter of the channels. Ghodoossi(2005) proposed that the values of D_l and D_d are taken as 1.4 and 3, respectively. For a self-similar fractal, the number of branches increases in geometrically decrease form level 0 to level N. Based on the Hagen-poiseulle equation for flow through a straight pipe, the equivalent permeability of the level 0 is expressed as follows [9]

$$K_0 = r_0^2 / 8 \tag{11}$$

Based on the fractal model and scaling factor, the equivalent permeability of any tube in the level k is expressed as follows

$$\begin{aligned}
K_{1j} &= r_0^2 / 8 \times \beta^2 \\
K_{2j} &= r_0^2 / 8 \times \beta^4 \\
&\cdots\cdots \\
K_{kj} &= r_0^2 / 8 \times \beta^{2k} \\
&\cdots\cdots \\
K_{N+1,j} &= r_0^2 / 8 \times \beta^{2(N+1)}
\end{aligned} \tag{12}$$

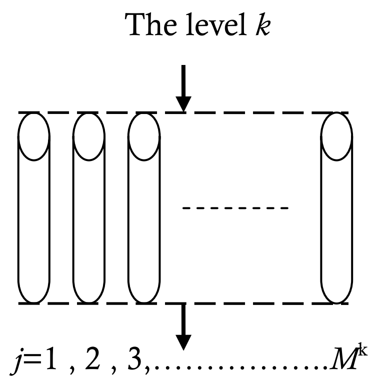

Fig. 6 Parallel network in the level k

There are M^k tubes in the level k, as shown in Fig. 6. Based on the parallel principles, the equivalent permeability of the level k is expressed as follows

$$K_k = r_0^2 / 8 \times \beta^{2k} M^k \tag{13}$$

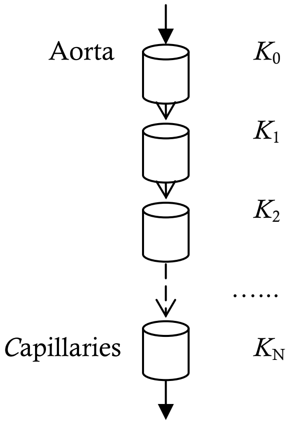

Fig. 7 Series network of respiratory system

Based on the series principles, as shown in Fig. 7, the equivalent permeability of the total network is expressed as follows

$$\frac{1}{K_e} = \frac{1}{K_0} + \frac{1}{K_1} + ... + \frac{1}{K_N} \tag{14}$$

$$\frac{1}{K_e} = \frac{8}{r_0^2}\left(1 + \frac{1}{\beta^2 M} + \frac{1}{\beta^4 M^2} ... + \frac{1}{\beta^{2N} M^N}\right) \tag{15}$$

$$\frac{1}{K_e} = \frac{8}{r_0^2} \frac{1 - \left(\frac{1}{\beta^2 M}\right)^{N+1}}{1 - \frac{1}{\beta^2 M}} \tag{16}$$

$$K_e = \frac{r_0^2}{8}\left[\frac{1 - \frac{1}{M\beta^2}}{1 - \left(\frac{1}{M\beta^2}\right)^{N+1}}\right] \tag{17}$$

Influences of scaling factor β and number of bifurcating branches *M* on permeability is shown in Fig. 8and 9.

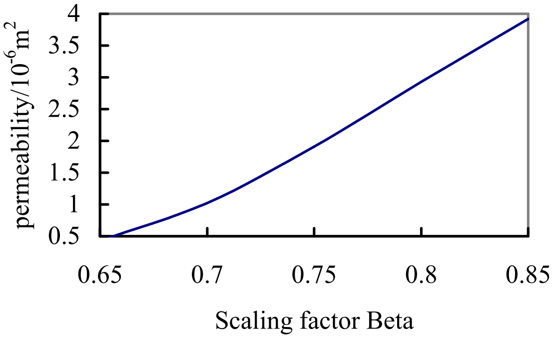

Fig. 8 Influence of scaling factor β on permeability

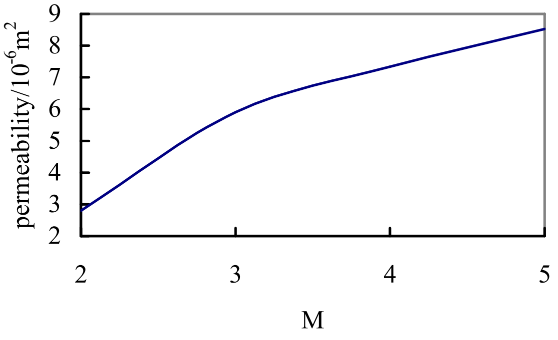

Fig.9 Influence of number of bifurcating branches *M* on permeability

3. Numerical simulation of respiratory system by using finite element method

The understanding of airflow structures in the human airways underlies the basis for analyzing particle transport and deposition. As shown in Fig. 2 and 3, the human respiratory system can be approached as binary tree. A binary tree is made up of a finite set of elements called nodes. In order to capture the air flow structures, the finite element model is meshed as shown in Fig. 10. The airway conduit is assumed to be smooth and rigid. The model parameters are chosen as follows: Scaling factors, γ=0.6059, β=0.7937; the number of channels that appear at one end of a channel after branching, *M*=2; the total number of generation of fractal level, *N*=3; the diameter and length of channel at level 0; r_0=0.01m. l_0=0.1m.

Pressure distribution of respiratory system is shown in Fig. 11. Air speed distribution of respiratory system is shown in Fig. 12.

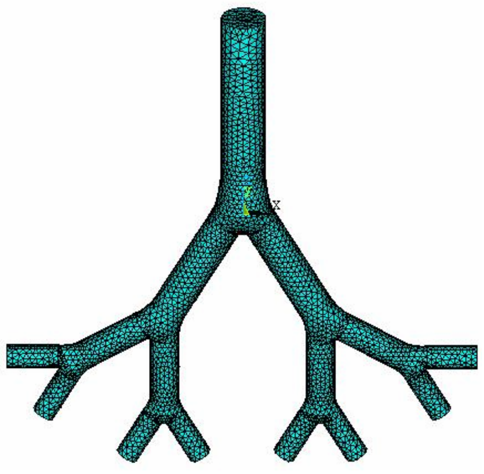

Fig. 10 Meshing of finite element model

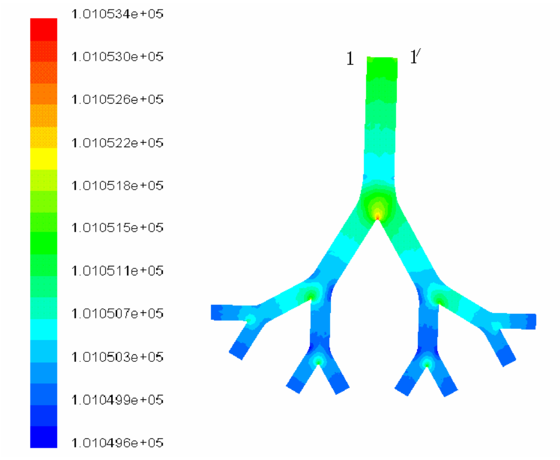

Fig. 11 Pressure distribution of respiratory system(Unit: Pa)

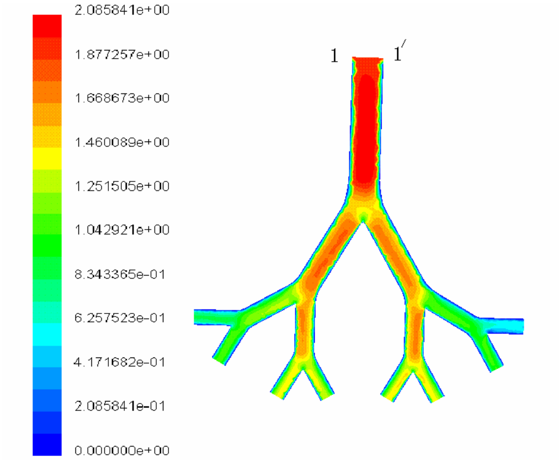

Fig. 12 Air speed distribution of respiratory system(Unit: m/s)

4. Conclusion

Any modeling and functional simulation of human respiratory organs critically relies on the precision and representativeness of the structural models available. The purpose of the investigation is to obtain more precise information of the respiratory network system with characteristic of fractal bronchial tree by means of numerical modeling. A fractal permeability model for human respiratory network is derived by using fractal geometry principles. The air flow characteristics in human respiratory system are modeled and simulated by finite element method. The investigation shows that a modeling procedure based on fractal geometry is quite effective for characterizing human respiratory network. The fractal respiratory network model can be used to estimate the performance of permeability of airflow and heat transport. Both animals and plants require extensive and expensive transport systems, for which an optimal design presumably provides a selective advantage.

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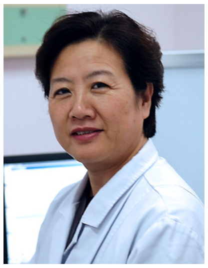

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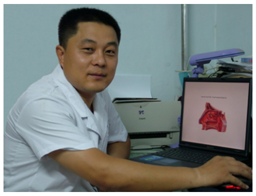

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