Investigation of Chromatic Dispersion and Pulse Broadening Factor of Two New Multi-clad Optical Fibers

Ali Rostami and Morteza Savadi Oskouei

Photonics and Nanocrystals Research Lab. (PNRL), Faculty of Electrical and Computer Engineering, University of Tabriz, Tabriz 51664, Iran
Tel/Fax: +98 411 3393724

Summary
In this paper two new and interesting multi-clad optical fibers are introduced and their properties from many aspects are investigated and compared together. For the proposed optical fibers two main characteristics such as dispersion and pulse broadening are calculated and demonstrated. Sensitivity of the proposed fibers to optical and geometrical parameters are calculated and discussed. We show that introduced type-II optical fiber is so sensitive to optical and geometrical parameters than type-I. So, type-II is better than type-I from control of the characteristic of the fiber point of view and is easy for tuning. Also, in small wavelengths effects of optical and geometrical parameters on dispersion and pulse broadening is considerable than large wavelengths. Our simulations show that in type-I zero-dispersion wavelength moves to large wavelengths.
Key words
Multi-clad optical Fibers, Dispersion, Pulse Broadening

1. Introduction

Recently high-speed and broadband optical communication is basic demand in industry. High-speed data communication is one of serious request for real time processing. Optical fiber based communication is one of best alternatives for these purposes. Optical fiber based high-speed communications need low dispersion as well as dispersion slope and large bandwidth supported by optical physical medium. Nowadays, applications such as time division multiplexing (OTDM) and dense wavelength division multiplexing (DWDM) are usual tasks in industry. Thus, with considering these applications providing a large bandwidth and high-speed communication possibility using optical fibers is highly interesting. For communication using single wavelength, dispersion shifted fibers is enough. But for applications such as DWDM since there are many wavelengths as carriers this method can't provide high-speed possibility. In these applications physical mediums ideally should be provide flat, minimum and uniform dispersion as well as dispersion slope. Also, with developing flat dispersion characteristic nonlinear effects such as Four Wave Mixing (FWM) because of phase mismatching doesn't limit the bandwidth of the fiber and the number of channels in DWDM applications.

For this purpose there are some published papers that discussed new fiber structures to obtain these properties. As a first and interesting work, we can point out to paper presented by R. K. Varshney et al [1]. In this paper an optical flat fiber was presented to minimize dispersion and dispersion slope. In this design, core radius, effective area and carrier wavelength are $1\mu m, 56.1\mu m^2$ and $1.55\mu m$ respectively, which are used and according to their calculation, dispersion duration within $1530-1610\,nm$ and dispersion slope at $1.55\,\mu m$ are $2.7-3.4\,ps/km.nm$ and $0.01\,ps/km.nm^2$ respectively. The presented paper introduces $80\,nm$ bandwidth that is small for today DWDM applications. Also, the reported dispersion is enough high for high-speed data transmission. Finally, the presented work includes only C and L bands for data transmission. A second work reported by X. Tian et al [2] that discuss about increasing of the effective area for RI and RII triple-clad fibers. This paper reported $4.5\,ps/km.nm$ for dispersion within $1540-1620\,nm$ wavelength duration. Also, for this design, dispersion slope reported about $0.006\,ps/km.nm^2$ within $1540-1620\,nm$ wavelength duration. The proposed design has small bandwidth for DWDM applications and also high dispersion in this duration. The calculated dispersion slope in this paper is not so small for high-speed data transmission. There are other papers presented to minimize dispersion and shift to requested values [3-6]. In these works dispersion calculation, minimization and shifting were discussed. The obtained results don't satisfactory. Also, in [5, 7] there are very interesting methods presented for dispersion compensation and management.

For this purpose, in this paper, two new fiber structures are considered and investigated from dispersion and pulse broadening factor point of views. For this purpose, fields distribution inside the proposed fibers based on the Maxwell's equations are calculated. Then using boundary conditions guided modes and wave vectors are obtained. Using waveguide and material dispersion relations the proposed structures are evaluated and simulated results are presented in results section. Also, for pulse

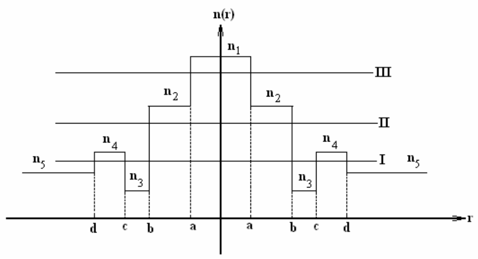
broadening similar treatment is done. Our results show the capability of introduced type-II fiber for tuning of zero-dispersion wavelength and uniformity of dispersion. Thus, the proposed is capable for large range of tuning.
Organization of the paper is as follows.
Mathematical modeling for dispersion analysis and pulse broadening is presented in section II. In section III, simulation results and discussion are illustrated. Finally the paper ends with a conclusion.

2. Mathematical Modeling

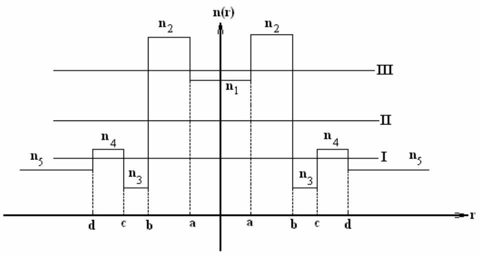
In this section the following special fiber structures (Fig. 1) for dispersion curve investigation are considered. The refractive index distribution function is defined as follows.

$$n(r)=\begin{cases} n_1,\ 0<r<a, \\ n_2,\ a<r<b, \\ n_{3,}\ b<r<c, \\ n_4,\ c<r<d, \\ n_5, d<r, \end{cases} \quad (1)$$

Where, r is the radius position of optical fiber.



Type- I



Type-II

Fig. 1 The index of refraction profiles of two fibers with defined parameters a) Type-I and b) Type-II

For these structures the effective index of refraction is given by $n_{eff}=\beta/k_0$, where β is the propagation wave vector of guided modes and k_0 is the wave number in vacuum. Based on the effective refractive index the proposed structures (Type-I and Type-II) can be divided to three regions of operation respectively, which is defined as follows.

$$Type-I\ \ I:\ n_5<n_{eff}<n_4\ ,\ II:n_4<n_{eff}<n_2\ , \quad (2)$$
$$III:n_2<n_{eff}<n_1$$

$$Type-II\ \ I:\ n_5<n_{eff}<n_4\ ,\ II:n_4<n_{eff}<n_1\ , \quad (3)$$
$$III:n_1<n_{eff}<n_2$$

Now, according to the Maxwell's equations the following table shows field distribution for these structures (type-I and type-II respectively).
Also, for these structures the following normalized frequencies and index of refractions are used.
Type-I:

$$V=k_0a\sqrt{n_1^2-n_5^2}\ ,B=\frac{(\frac{\beta}{k_0})^2-n_5^2}{n_1^2-n_5^2}, \quad (5\text{-a})$$

Type-II:

$$V=k_0a\sqrt{n_2^2-n_5^2}\ ,\ B=\frac{(\frac{\beta}{k_0})^2-n_5^2}{n_2^2-n_5^2}. \quad (5\text{-b})$$

Now, boundary conditions (electric and magnetic fields continuities) in a,b,c and d can be applied to the presented field distributions in Table-I, below determinants (6, 7, 8, 9) are obtained. Finding zeros of the mentioned above determinants will introduce guiding wave vectors at any given wavelengths. Please refer to the appendix.
In these tables, transversal wave vectors are given as follows.

$$U_1=\sqrt{k_0^2n_1^2-\beta^2}a,$$
$$W_1=\sqrt{\beta^2-k_0^2n_1^2}a,\ U_2=\sqrt{k_0^2n_2^2-\beta^2}a,$$
$$W_2=\sqrt{\beta^2-k_0^2n_2^2}a,$$
$$U_4=\sqrt{k_0^2n_4^2-\beta^2}c,\ W_3=\sqrt{\beta^2-k_0^2n_3^2}b,$$
$$W_4=\sqrt{\beta^2-k_0^2n_4^2}c,\quad W_5=\sqrt{\beta^2-k_0^2n_5^2}d.$$
$$\overline{W_2}=W_2\left(\frac{P}{Q}\right),\ \overline{U_2}=U_2\left(\frac{P}{Q}\right),\ \overline{W_3}=W_3\left(\frac{1}{P}\right),$$
$$\overline{W_4}=W_4\left(\frac{1}{L}\right),\ \overline{U_4}=U_4\left(\frac{1}{L}\right), \quad (10)$$

Based on obtained guided wave vectors and basic relations for waveguide and material dispersions the following relation gives total dispersion for the introduced single mode multi-clad optical fibers.

$$D=-\frac{\lambda}{c}\frac{d^2n_5}{d\lambda^2}[1+\Delta\frac{d(VB)}{dV}]-\frac{N_5}{c}\frac{\Delta}{\lambda}V\frac{d^2(VB)}{dV^2}, \quad (11)$$

where $N_5=n_5-\lambda\frac{dn_5}{d\lambda}$ is the group index of the outer cladding layer. Also, the Sellmeier formula can be used for calculation of material dispersion ($\frac{dn_5}{d\lambda}$ and $\frac{d^2n_5}{d\lambda^2}$). For pulse broadening factor the following relation is used.

$$\frac{\sigma}{\sigma_0}=[(1+\frac{C\beta_2(\lambda)Z}{t_i^2})^2+(\frac{\beta_2(\lambda)Z}{t_i^2})^2 +(1+C^2)^2(\frac{\beta_3(\lambda)Z}{2t_i^3})^2]^{\frac{1}{2}}, \quad (12)$$

where $\frac{\sigma}{\sigma_0}, C, t_i, \lambda, Z, \beta_2$ and β_3 are pulse broadening factor, chirp parameter, initial full width at half maximum of input pulse, wavelength, distance, second derivative of the guided wave vector and third derivative of the guided wave vector respectively. For incorporating the optical and geometrical parameters in simulation and easy conclusions, we define the following parameters and in simulation section the effects of these parameters are investigated.

Type-I: Optical Parameters:

$$R_1=\frac{n_1-n_2}{n_1-n_3}, R_2=\frac{n_1-n_4}{n_1-n_5}, R_3=\frac{n_2-n_3}{n_2-n_4}, \Delta=\frac{n_1-n_5}{n_5} \quad (13)$$

Geometrical Parameters: $P=\frac{b}{c}$, $Q=\frac{a}{c}$, $L=\frac{c}{d}$

Type-II: Optical Parameters:

$$R_1=\frac{n_2-n_1}{n_2-n_3}, R_2=\frac{n_2-n_4}{n_2-n_5}, R_3=\frac{n_1-n_3}{n_1-n_4}, \Delta=\frac{n_2-n_5}{n_5} \quad (14)$$

Geometrical Parameters: $P=\frac{b}{c}$, $Q=\frac{a}{c}$, $L=\frac{c}{d}$

3. Simulation Results and discussion

In this section simulated results are presented and discussed. First we consider dispersion behavior of the proposed two structures. These simulations have been done according to $a=1\mu m$, $Q=0.3$, $P=0.7$, $L=0.6$, $R_1=0.2$, $R_2=0.6$, $R_3=7$ geometrical and optical parameters. Each parameter is varied one at a time to investigate the effect of this parameter on dispersion curve and broadening factor at $1.55\mu m$ after 30 km transmission. Dispersion of the proposed structures versus core radius is illustrated in Fig. 2. As it is seen, for these three core radius in type-II the dispersion try to be uniform for $a=0.75\mu m$ and zero-dispersion wavelength is shifted to higher wavelengths with increase of core radius. So, these structures can be optimized to obtain optimum flat and smooth dispersion characteristic. Also, wavelength duration of tuning for type-II is wide compared type-I. The effect of Δ on dispersion characteristic in the fibers is illustrated in Fig. 3. Zero-dispersion wavelength is increased with increase of Δ. Also, tuning range for type-II is wide compared type-I.

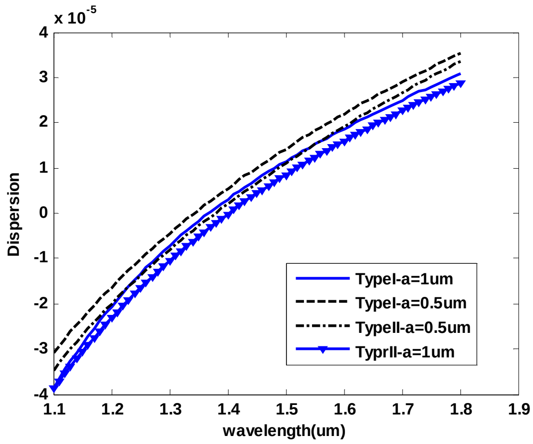


Fig. 2 Dispersion Vs. Wavelength for Introduced

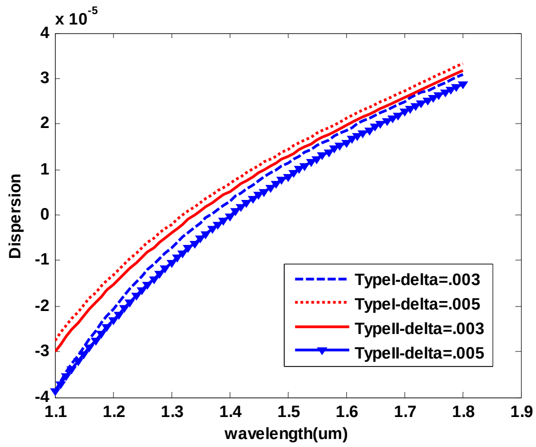


Fig. 3 Dispersion Vs. Wavelength for Introduced

Effect of L on dispersion curve also is illustrated in Fig.4. Effect of P and Q on dispersion curves are illustrated in Figs. (5, 6). Increase of P cause an increase in zero-dispersion wavelength. But increase of Q decrease the zero-dispersion wavelength.

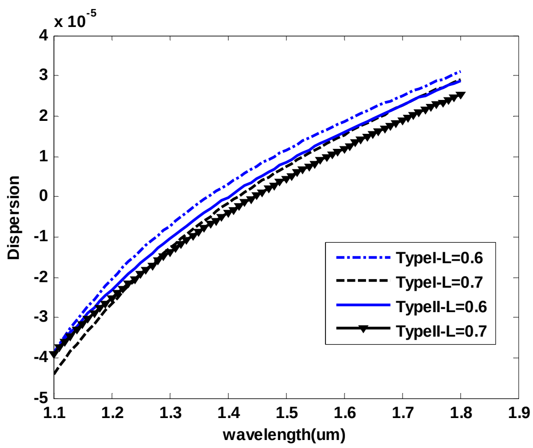


Fig. 4 Dispersion Vs. Wavelength for Introduced

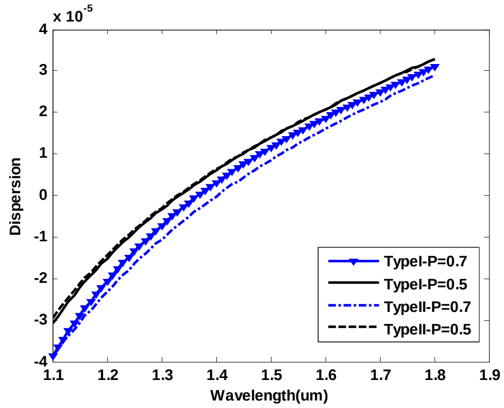


Fig. 5 Dispersion Vs. Wavelength

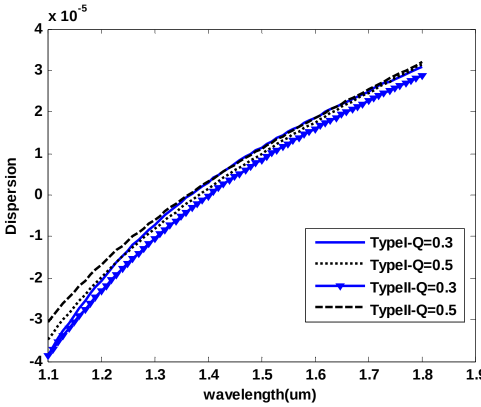


Fig. 6 Dispersion Vs. Wavelength

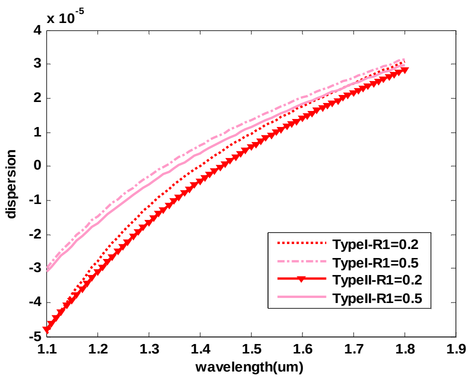


Fig. 7 Dispersion Vs. Wavelength

Effects of R_1, R_2 and R_3 on dispersion curve are illustrated in the following. From these parameters R_2 has strong effect on zero-dispersion shift in type-II optical fiber. Also, R_3 hasn't any considerable effect on type-I.

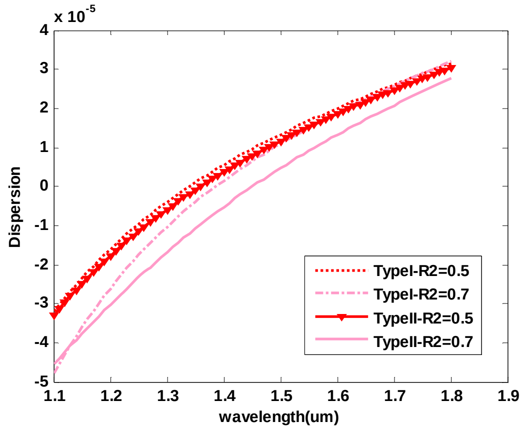


Fig. 8 Dispersion Vs. Wavelength

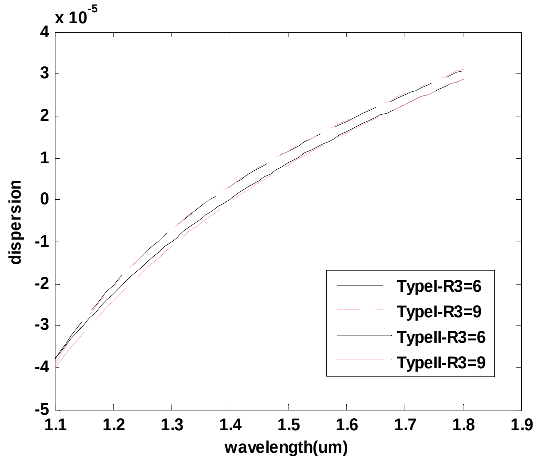


Fig. 9Dispersion Vs. Wavelength

Pulse broadening for these fibers ($C = 0, t_i = 5ps$) is considered with change of system parameters. Clearly type-II has small broadening factor, but it is sensitive to core radius compared type-I, that illustrated in Fig. 10. Effect of Δ on pulse broadening for two fibers is investigated and illustrated in Fig. 11. In this figure also, similar to the previous figure, the situation for type-II is better than type-I but type-II is sensitive

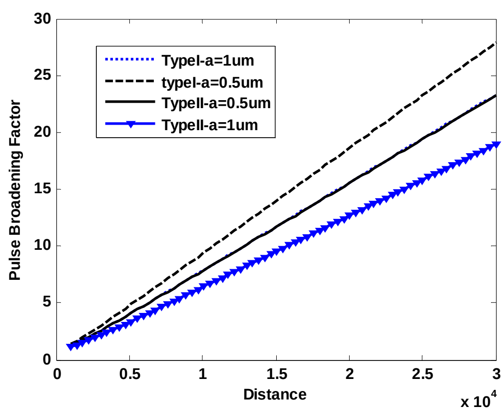


Fig. 10 Pulse Broadening Factor Vs. Distance

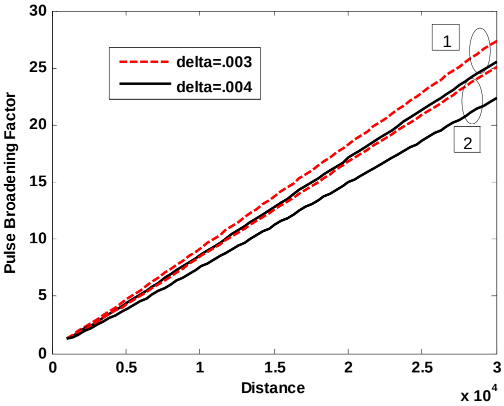


Fig. 11 Pulse Broadening Factor Vs. Distance

Effect of L on pulse broadening is illustrated in Fig. 12. In this case also, the situation for type-II is better than type-I. Pulse broadening is decreased with increase of L. Effects of P and Q on pulse broadening factor are illustrated in Figs. (13, 14). These parameters haven't considerable effects on pulse broadening. Anyway the situation for type-II is better than type-I.

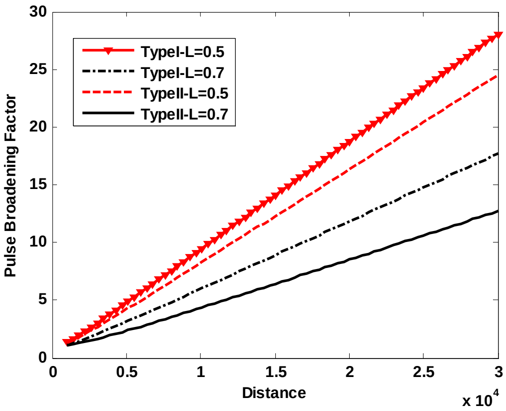


Fig. 12 Pulse Broadening Factor Vs. Distance

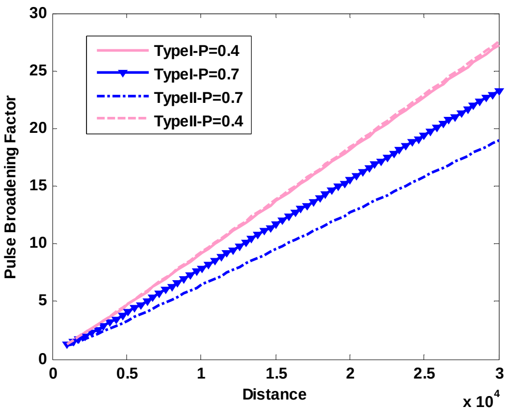


Fig. 13 Pulse Broadening Factor Vs. Distance

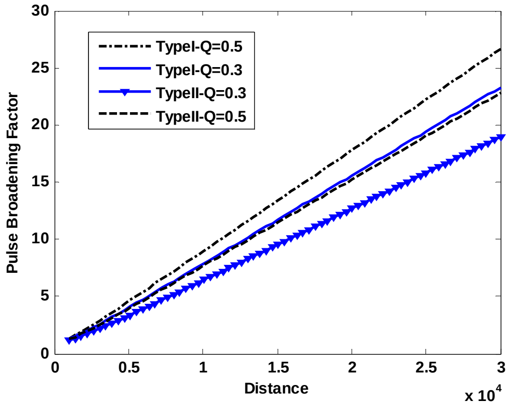


Fig. 14 Pulse Broadening Factor Vs. Distance

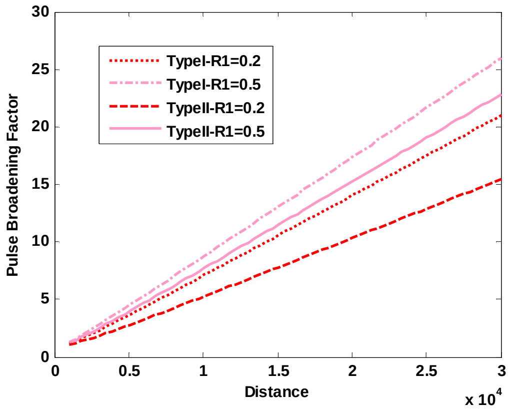


Fig. 15 Pulse Broadening Factor Vs. Distance

Effects of R_1, R_2 and R_3 on pulse broadening are illustrated in Figs. (15, 16, 17). Small R_1 and big $R_{2,3}$ are better from pulse broadening point of view. Also, situation for type-II is better compared type-I.

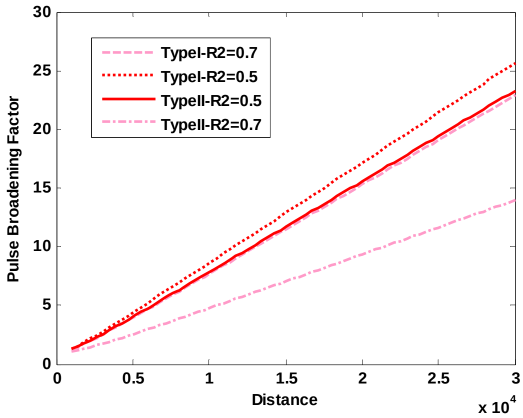


Fig. 16 Pulse Broadening Factor Vs. Distance

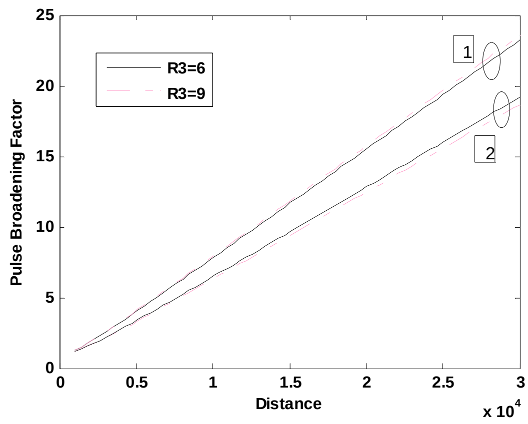


Fig. 17 Pulse Broadening Factor Vs. Distance

According to the simulated results the following conclusions can be obtained.

- Zero-dispersion wavelength is shifted to higher wavelengths with decrease of R_1 in both type of fibers and the effect in low small wavelengths is stronger. Also, with increase of R_1 the slope of dispersion is decreased. Finally, effect of R_1 on type-II is serious than on type-II.
- Zero-dispersion wavelength is shifted to higher wavelengths with decrease of R_2 in both type of fibers and the effect on type-II is serious.
- Change of R_3 haven't considerable effect on zero-dispersion wavelength in both fibers.
- Increase of Δ, P, L, a have considerable effect on type-II fiber and increase the zero-dispersion wavelength to higher wavelengths.

4. Analysis and Simulation of Dispersion length, Higher-order dispersion length and Nonlinear-effect length

According to Eqs. (15-18), the effective-area A_{eff}, dispersion length L_D, the higher order dispersion length L'_D and nonlinear effect length L_{Nl} as function of wavelength λ are showed respectively in figs. (4-1)-(4-4) for Type-I and Type-II optical fibers.

$$A_{eff} = 2\pi \frac{\left[\int_0^\infty |\Psi(r)|^2 r dr\right]^2}{\int_0^\infty |\Psi(r)|^4 r dr}, \qquad (15)$$

$$L_D = \frac{t_i^2}{|\beta_2|}, \qquad (16)$$

$$L'_D = \frac{t_i^3}{|\beta_3|}, \qquad (17)$$

$$L_{NL} = \frac{1}{P_0 \gamma} \qquad (18)$$

Where, $\gamma = \frac{k_0 n_2}{A_{eff}}$, P_0, n_2 are nonlinear coupling coefficient, input pulse power and nonlinear refractive index coefficient. In our simulation, $P_0 = 1mw$ and $n_2 = 1.3e-22(m^2/V^2)$. These simulations have been done according to $a = 1\mu m$, $Q = 0.3$, $P = 0.7$, $L = 0.6$, $R_1 = 0.2$, $R_2 = 0.6$, $R_3 = 7$ geometrical and optical parameters. Introduced Type-I and II. Fig. 18 shows that the effective area of Type-I is larger than effective area of Type-II. It is useful to maintain that, the higher effective area, the longer nonlinear–effect length. It is obvious in Fig (4-4). With considering of Figs. (18-21), it can be seen that $L_D << L'_D$ and $L_D << L_{NL}$. Consequently, the main limitation factor in long haul communication systems is dispersion value and for common transmission lengths, higher order dispersion and nonlinear effect on the pulse broadening can be ignored with defined parameters in this paper.

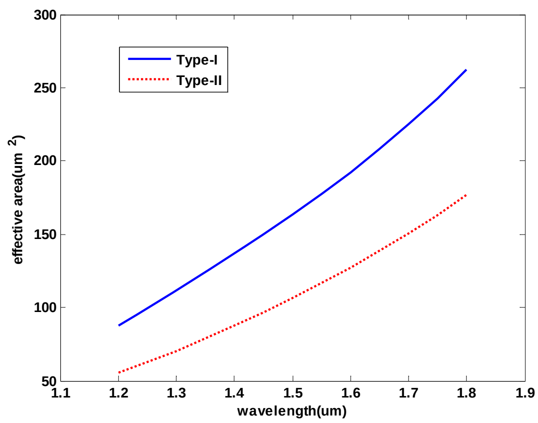


Fig 18 Effective area Vs. wavelength

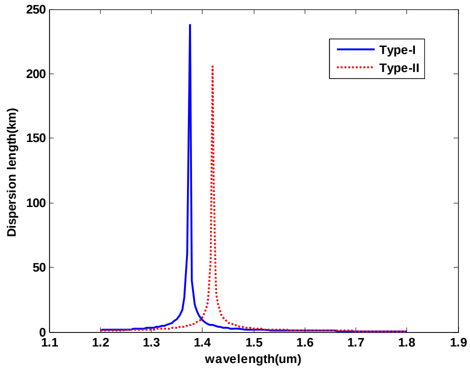


Fig 19 Dispersion length Vs. wavelength

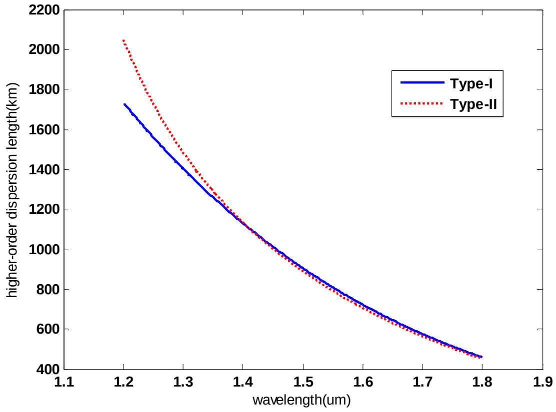


Fig 20 higher- order Dispersion length Vs. wavelength

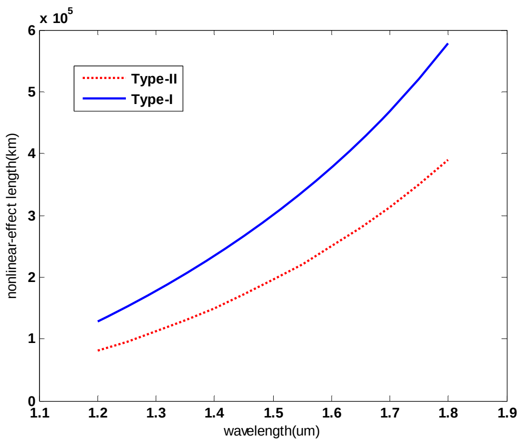


Fig 21 nonlinear- effect length Vs. wavelength

Conclusion

In this paper, two new multi-clad optical fibers have been considered and their dispersion and pulse broadening were studied. Our simulated results have shown that introduced type-II optical fiber has high sensitivity to optical and geometrical parameters. Also, in wide range it characteristics can be tuned. This is interesting point for tuning purpose. Also, the other fiber has small sensitivity to those parameters. From tolerance of parameters point of view this fiber is stable. Also, in small wavelengths the effects of optical and geometrical parameters are strong compared to higher wavelengths that are saturation in dispersion behavior for both structures. Finally, type-II has small dispersion slope compared to type-I and large zero-dispersion wavelength.

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$$\begin{vmatrix} J_m(U_1) & J_m(U_2) & Y_m(U_2) & 0 & 0 & 0 & 0 & 0 \\ 0 & J_m(\overline{U}_2) & Y_m(\overline{U}_2) & I_m(W_3) & K_m(W_3) & 0 & 0 & 0 \\ 0 & 0 & 0 & I_m(\overline{W}_3) & K_m(\overline{W}_3) & J_m(U_4) & Y_m(U_4) & 0 \\ 0 & 0 & 0 & 0 & 0 & J_m(\overline{U}_4) & Y_m(\overline{U}_4) & K_m(W_5) \\ U_1 J'_m(U_1) & U_2 J'_m(U_2) & U_2 Y'_m(U_2) & 0 & 0 & 0 & 0 & 0 \\ 0 & \overline{U}_2 J'_m(\overline{U}_2) & \overline{U}_2 Y'_m(\overline{U}_2) & W_3 I'_m(W_3) & W_3 K'_m(W_3) & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{W}_3 I'_m(\overline{W}_3) & \overline{W}_3 K'_m(\overline{W}_3) & U_4 J'_m(U_4) & U_4 Y'_m(U_4) & 0 \\ 0 & 0 & 0 & 0 & 0 & \overline{U}_4 J'_m(\overline{U}_4) & \overline{U}_4 Y'_m(\overline{U}_4) & W_5 K'_m(W_5) \end{vmatrix} = 0 \qquad (6)$$

$n_4 < n_{eff} < n_5 : Type - I, II$

$$\begin{vmatrix} J_m(U_1) & J_m(U_2) & Y_m(U_2) & 0 & 0 & 0 & 0 & 0 \\ 0 & J_m(\overline{U}_2) & Y_m(\overline{U}_2) & I_m(W_3) & K_m(W_3) & 0 & 0 & 0 \\ 0 & 0 & 0 & I_m(\overline{W}_3) & K_m(\overline{W}_3) & I_m(W_4) & K_m(W_4) & 0 \\ 0 & 0 & 0 & 0 & 0 & I_m(\overline{W}_4) & K_m(\overline{W}_4) & K_m(W_5) \\ U_1 J'_m(U_1) & U_2 J'_m(U_2) & U_2 Y'_m(U_2) & 0 & 0 & 0 & 0 & 0 \\ 0 & \overline{U}_2 J'_m(\overline{U}_2) & \overline{U}_2 Y'_m(\overline{U}_2) & W_3 I'_m(W_3) & W_3 K'_m(W_3) & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{W}_3 I'_m(\overline{W}_3) & \overline{W}_3 K'_m(\overline{W}_3) & W_4 I'_m(W_4) & W_4 K'_m(W_4) & 0 \\ 0 & 0 & 0 & 0 & 0 & \overline{W}_4 I'_m(\overline{W}_4) & \overline{W}_4 K'_m(\overline{W}_4) & W_5 K'_m(W_5) \end{vmatrix} = 0 \qquad (7)$$

$n_4 < n_{eff} < n_2 : Type - I, II$

$$\begin{vmatrix} J_m(U_1) & I_m(W_2) & K_m(W_2) & 0 & 0 & 0 & 0 & 0 \\ 0 & I_m(\overline{W}_2) & K_m(\overline{W}_2) & I_m(W_3) & K_m(W_3) & 0 & 0 & 0 \\ 0 & 0 & 0 & I_m(\overline{W}_3) & K_m(\overline{W}_3) & I_m(W_4) & K_m(W_4) & 0 \\ 0 & 0 & 0 & 0 & 0 & I_m(\overline{W}_4) & K_m(\overline{W}_4) & K_m(W_5) \\ U_1 J'_m(U_1) & W_2 I'_m(W_2) & W_2 K'_m(W_2) & 0 & 0 & 0 & 0 & 0 \\ 0 & \overline{W}_2 I'_m(\overline{W}_2) & \overline{W}_2 K'_m(\overline{W}_2) & W_3 I'_m(W_3) & W_3 K'_m(W_3) & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{W}_3 I'_m(\overline{W}_3) & \overline{W}_3 K'_m(\overline{W}_3) & W_4 I'_m(W_4) & W_4 K'_m(W_4) & 0 \\ 0 & 0 & 0 & 0 & 0 & \overline{W}_4 I'_m(\overline{W}_4) & \overline{W}_4 K'_m(\overline{W}_4) & W_5 K'_m(W_5) \end{vmatrix} = 0 \qquad (8)$$

$n_2 < n_{eff} < n_1 : Type - I$

$$\begin{vmatrix} I_m(W_1) & J_m(U_2) & Y_m(U_2) & 0 & 0 & 0 & 0 & 0 \\ 0 & J_m(\overline{U}_2) & Y_m(\overline{U}_2) & I_m(W_3) & K_m(W_3) & 0 & 0 & 0 \\ 0 & 0 & 0 & I_m(\overline{W}_3) & K_m(\overline{W}_3) & I_m(W_4) & K_m(W_4) & 0 \\ 0 & 0 & 0 & 0 & 0 & I_m(\overline{W}_4) & K_m(\overline{W}_4) & K_m(W_5) \\ W_1 I'_m(W_1) & U_2 J'_m(U_2) & U_2 Y'_m(U_2) & 0 & 0 & 0 & 0 & 0 \\ 0 & \overline{U}_2 J'_m(\overline{U}_2) & \overline{U}_2 Y'_m(\overline{U}_2) & W_3 I'_m(W_3) & W_3 K'_m(W_3) & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{W}_3 I'_m(\overline{W}_3) & \overline{W}_3 K'_m(\overline{W}_3) & W_4 I'_m(W_4) & W_4 K'_m(W_4) & 0 \\ 0 & 0 & 0 & 0 & 0 & \overline{W}_4 I'_m(\overline{W}_4) & \overline{W}_4 K'_m(\overline{W}_4) & W_5 K'_m(W_5) \end{vmatrix} = 0 \qquad (9)$$

$n_1 < n_{eff} < n_2 : Type - II$

Table-1-a Transversal Field Distribution for different regions and effective-refractive index ranges

Region	$n_4 < n_{eff} < n_5$ (Type-I)	$n_4 < n_{eff} < n_2$ (Type-I)	$n_2 < n_{eff} < n_1$ (Type-I)
I	$AJ_m(\frac{U_1 r}{a})$	$AJ_m(\frac{U_1 r}{a})$	$AJ_m(\frac{U_1 r}{a})$
II	$BJ_m(\frac{U_2 r}{a}) + CY_m(\frac{U_2 r}{a})$	$BJ_m(\frac{U_2 r}{a}) + CY_m(\frac{U_2 r}{a})$	$BI_m(\frac{W_2 r}{a}) + CK_m(\frac{W_2 r}{a})$
III	$DI_m(\frac{W_3 r}{b}) + EK_m(\frac{W_3 r}{b})$	$DI_m(\frac{W_3 r}{b}) + EK_m(\frac{W_3 r}{b})$	$DI_m(\frac{W_3 r}{b}) + EK_m(\frac{W_3 r}{b})$
IV	$FJ_m(\frac{U_4 r}{c}) + GY_m(\frac{U_4 r}{c})$	$FI_m(\frac{W_4 r}{c}) + GK_m(\frac{W_4 r}{c})$	$FI_m(\frac{W_4 r}{c}) + GK_m(\frac{W_4 r}{c})$
V	$HK_m(\frac{W_5 r}{d})$	$HK_m(\frac{W_5 r}{d})$	$HK_m(\frac{W_5 r}{d})$

Table-1-b Transversal Field Distribution for different regions and effective-refractive index ranges

Region	$n_4 < n_{eff} < n_5$ (Type-II)	$n_4 < n_{eff} < n_2$ (Type-II)	$n_1 < n_{eff} < n_2$ (Type-II)
I	$AJ_m(\frac{U_1 r}{a})$	$AJ_m(\frac{U_1 r}{a})$	$AI_m(\frac{W_1 r}{a})$
II	$BJ_m(\frac{U_2 r}{a}) + CY_m(\frac{U_2 r}{a})$	$BJ_m(\frac{U_2 r}{a}) + CY_m(\frac{U_2 r}{a})$	$BJ_m(\frac{U_2 r}{a}) + CY_m(\frac{U_2 r}{a})$
III	$DI_m(\frac{W_3 r}{b}) + EK_m(\frac{W_3 r}{b})$	$DI_m(\frac{W_3 r}{b}) + EK_m(\frac{W_3 r}{b})$	$DI_m(\frac{W_3 r}{b}) + EK_m(\frac{W_3 r}{b})$
IV	$FJ_m(\frac{U_4 r}{c}) + GY_m(\frac{U_4 r}{c})$	$FI_m(\frac{W_4 r}{c}) + GK_m(\frac{W_4 r}{c})$	$FI_m(\frac{W_4 r}{c}) + GK_m(\frac{W_4 r}{c})$
V	$HK_m(\frac{W_5 r}{d})$	$HK_m(\frac{W_5 r}{d})$	$HK_m(\frac{W_5 r}{d})$