The Uncertain Two-tuple Ordered Weighted Averaging Operator

***Yao Zhang*†*, Zhi-Ping Fan*††**

School of Business Administration, Northeastern University, Shenyang, 110004 China

Summary
Many multiple attribute group decision making (MAGDM) problems with linguistic information involve uncertainties, which significantly increase the complexity and difficulty in decision analysis. In order to solve such problems, we require appropriate aggregation operators that are capable of dealing with uncertain linguistic information. In this paper, firstly, the uncertain two-tuple linguistic information is described. Then, a new uncertain linguistic aggregation operator called uncertain two-tuple ordered weighted averaging (UTOWA) operator is proposed and some desirable properties of the UTOWA operator are analyzed. Finally, it is pointed out that the proposed operator can be applied to solve MAGDM problems with uncertain linguistic information and also enrich the existing approaches to aggregating uncertain linguistic information.

Key words:
multiple attribute group decision making (MAGDM), uncertain two-tuple ordered weighted averaging (UTOWA) operator, uncertain linguistic information, uncertain two-tuple linguistic information

1. Introduction

In multiple attribute group decision making (MAGDM) analysis with linguistic information, the aggregation operators are required to deal with the aggregation of linguistic information and rankings of alternatives. Since Yager [10,11] developed the ordered weighted averaging (OWA) operator, many operators have been developed to aggregate linguistic information in group decision making such as the linguistic weighted disjunction (LWD) operator [2], linguistic weighted averaging (LWA) operator [2], linguistic OWA (LOWA) operator [1,3], induced LOWA (I-LOWA) operator [7], hybrid linguistic weighted averaging (HLWA) operator [8] and induced uncertain LOWA (IULOWA) operator [9], etc. However, these approaches do not address the problem where the aggregated results are inconsistent with linguistic term in the pre-established discrete linguistic term set, which leads to the loss of information and inaccuracy of the aggregated results. To overcome these drawbacks, a two-tuple OWA (TOWA) operator [4,5] and an expanded TOWA (ETOWA) operator [15] are proposed. However, it is difficult to solve MAGDM problems with uncertain linguistic information using the TOWA and ETOWA operators. In this paper, we shall develop a new uncertain linguistic operator called uncertain two-tuple ordered weighted averaging (UTOWA) operator and study some of their desirable properties of the operator.

2. Preliminaries

For the convenience of analysis, we give several definitions in this section. These definitions are used throughout the paper.

Definition 1. [6] Let $\tilde{x} = [x^l, x^u]$, where $x^l, x^u \in \boldsymbol{R}$, $x^l \le x^u$, and $\boldsymbol{R}$ is a real number set, we then call $\tilde{x}$ an interval number. If $x^l = x^u$, then $\tilde{x}$ is a real number, i.e., $\tilde{x} = x^l = x^u$.

Given any two positive interval numbers $\tilde{a} = [a^l, a^u]$ and $\tilde{b} = [b^l, b^u]$, some main operations of positive interval numbers $\tilde{a}$ and $\tilde{b}$ can be expressed as follows [6]: ① $\tilde{a} + \tilde{b} = [a^l + b^l, \ a^u + b^u]$; ② $\tilde{a} \times \tilde{b} = [a^l b^l, a^u b^u]$; ③ $\tilde{a} \div \tilde{b} = [a^l / b^u, a^u / b^l]$; ④ $k\tilde{a} = [ka^l, ka^u]$, ($k > 0$); ⑤ $1/\tilde{a} = [1/a^u, 1/a^l]$.

Definition 2. [14] Let $\tilde{a} = [a^l, a^u]$ and $\tilde{b} = [b^l, b^u]$ be two positive interval numbers, then, the possible degree of $\tilde{a} > \tilde{b}$, i.e., $P_{\tilde{a}>\tilde{b}}$, is given by

$$P_{\tilde{a}>\tilde{b}} = \begin{cases} 1, & b^u \le a^l \\ \dfrac{a^u - b^u}{a^u - a^l} + \dfrac{b^u - a^l}{a^u - a^l} \cdot \dfrac{a^l - b^l}{b^u - b^l} + 0.5 \cdot \dfrac{b^u - a^l}{a^u - a^l} \cdot \dfrac{b^u - a^l}{b^u - b^l}, & b^l < a^l < b^u \le a^u \\ \dfrac{a^u - b^u}{a^u - a^l} + 0.5 \cdot \dfrac{b^u - b^l}{a^u - a^l}, & a^l \le b^l < b^u \le a^u \end{cases} \quad (1a)$$

and accordingly, the possible degree of $\tilde{b} > \tilde{a}$, i.e., $P_{\tilde{b}>\tilde{a}}$, is given by

Manuscript received September 5, 2006.
Manuscript revised September 25 , 2006.

$$P_{\tilde{b}>\tilde{a}} = \begin{cases} 0, & b^u \le a^l \\ 0.5 \cdot \dfrac{b^u - a^l}{a^u - a^l} \cdot \dfrac{b^u - a^l}{b^u - b^l}, & b^l < a^l < b^u \le a^u \\ \dfrac{b^l - a^l}{a^u - a^l} + 0.5 \cdot \dfrac{b^u - b^l}{a^u - a^l}, & a^l \le b^l < b^u \le a^u \end{cases} \quad (1b)$$

From Definition 2, we can easily obtain the following results:

(1) $0 \le P_{\tilde{a}>\tilde{b}} \le 1$, $0 \le P_{\tilde{b}>\tilde{a}} \le 1$;

(2) $P_{\tilde{a}>\tilde{b}} + P_{\tilde{b}>\tilde{a}} = 1$;

(3) if $P_{\tilde{a}>\tilde{b}} = 1$ (or $P_{\tilde{b}>\tilde{a}} = 0$), then $\tilde{a} > \tilde{b}$;

(4) if $P_{\tilde{a}>\tilde{b}} = 0.5$ (or $P_{\tilde{b}>\tilde{a}} = 0.5$), then $\tilde{a} = \tilde{b}$;

(5) if $0 < P_{\tilde{a}>\tilde{b}} < 0.5$ (or $0.5 < P_{\tilde{a}>\tilde{b}} < 1$), then $\tilde{a} \underset{P_{\tilde{a}>\tilde{b}}}{>} \tilde{b}$;

(6) if $\tilde{a} \underset{P_{\tilde{a}>\tilde{b}}}{>} \tilde{b}$, $\tilde{b} \underset{P_{\tilde{b}>\tilde{c}}}{>} \tilde{c}$, and $P_{\tilde{a}>\tilde{b}} > 0.5$, $P_{\tilde{b}>\tilde{c}} > 0.5$, then $\tilde{a} \underset{P_{\tilde{a}>\tilde{c}}}{>} \tilde{c}$.

Many aspects of different activities in the real world cannot be assessed in a quantitative form, but rather in a qualitative one, i.e., with vague or imprecise knowledge. In that case, a better approach may be to use linguistic assessments instead of numerical values. The linguistic approach represents qualitative aspects as linguistic values by means of linguistic variables [3,12,13].

Suppose that $S = \{s_0, s_1, \cdots, s_T\}$ be the pre-established finite and totally ordered discrete linguistic term set with odd cardings, where s_i denotes the *i*-th linguistic term of *S*. It can be seen that $T+1$ is the cardinality of S. For example, a set of seven terms S, could be $S = \{s_0 = \text{Very Poor (VP)}, s_1 = \text{Poor (P)}, s_2 = \text{Medium Poor (MP)}, s_3 = \text{Fair (F)}, s_4 = \text{Medium Good(MG)}, s_5 = \text{Good (G)}, s_6 = \text{Very Good (VG)}\}$.

Definition 3. [9] Let $\tilde{s}_i = [s_{k_i}, s_{l_i}]$, where $s_{k_i}, s_{l_i} \in S$, $k_i, l_i \in [0, T]$, $k_i \le l_i$. s_{k_i} and s_{l_i} are the lower and the upper limits, respectively, we then call $\tilde{s}_i$ an uncertain linguistic information. Let $\tilde{S}$ be the set of all the uncertain linguistic information.

Let (s_i, α_i) be the linguistic information by means of a two-tuple [4], where s_i ($s_i \in S$) is a linguistic label and α_i is a numerical value that represents the value of symbolic translation. Let β be the result of an aggregation of the indices of a set of labels assessed in a linguistic term set S, i.e., the result of a set of a symbolic aggregation operation, $\beta \in [0, T]$. Let $i = \text{round}(\beta)$ and $\alpha_i = \beta - i$ be two values, such that, $i \in [0, T]$, and $\alpha_i \in [-0.5, 0.5)$, then α_i is called a symbolic translation.

Definition 4. Let $\tilde{s}_{\alpha_i} = [(s_{k_i}, \alpha_{k_i}), (s_{l_i}, \alpha_{l_i})]$, where $s_{k_i}, s_{l_i} \in S$, $k_i, l_i \in [0, T]$, $k_i \le l_i$, and $\alpha_{k_i}, \alpha_{l_i} \in [-0.5, 0.5)$, are the numerical values that represent the values of symbolic translation, respectively. (s_{k_i}, α_{k_i}) and (s_{l_i}, α_{l_i}) are the lower and the upper limits, respectively, we then call $\tilde{s}_{\alpha_i}$ an uncertain two-tuple linguistic information. Let $\tilde{S}_\alpha$ be the set of all the uncertain two-tuple linguistic information.

Definition 5. Let positive interval number $\tilde{\mu} = [\mu^L, \mu^U]$ be the result of an aggregation of the indices of a set of labels assessed in an uncertain linguistic term set $\tilde{S}$, i.e., the result of a set of a symbolic aggregation operation. μ^L, μ^U are the results of an aggregation of the indices of a set of labels assessed in a linguistic term set S, respectively, where $\mu^L, \mu^U \in [0, T]$, being $T+1$ the cardinality of S. Let $k_i = \text{round}(\mu^L)$ and $\alpha_{k_i} = \mu^L - k_i$ be two values, and $l_i = \text{round}(\mu^L)$ and $\alpha_{l_i} = \mu^U - l_i$ also be two values, such that, k_i, $l_i \in [0, T]$, and α_{k_i}, $\alpha_{l_i} \in [-0.5, 0.5)$, then α_{k_i}, α_{l_i} are called symbolic translations, respectively.

Definition 6. Let $S = \{s_0, s_1, \cdots, s_T\}$ be a linguistic term set, $\tilde{s}_i = [s_{k_i}, s_{l_i}]$ an uncertain linguistic information and $\tilde{\mu} = [\mu^L, \mu^U]$ an interval value supporting the result of a symbolic aggregation operation, where $\mu^L, \mu^U \in [0, T]$, then the uncertain two-tuple that expresses the equivalent information to $\tilde{\mu}$ is obtained with the following function:

$$\Delta : [0, T] \to S \times [-0.5, 0.5). \quad (2a)$$

$$\Delta(\mu^L) = \begin{cases} s_{k_i}\ ,\ k_i = \text{round}(\mu^L), \\ \alpha_{k_i} = \mu^L - k_i\ ,\ \alpha_{k_i} \in [-0.5, 0.5)\,. \end{cases} \quad (2b)$$

and

$$\Delta : [0, T] \to S \times [-0.5, 0.5)\,. \quad (3a)$$

$$\Delta(\mu^U) = \begin{cases} s_{l_i}\ ,\ l_i = \text{round}(\mu^U), \\ \alpha_{l_i} = \mu^U - l_i\ ,\ \alpha_{l_i} \in [-0.5, 0.5)\,. \end{cases} \quad (3b)$$

where 'round' is the usual rounding operation, s_{k_i} and s_{l_i} have the closest index labels to "μ^L", "μ^U", respectively, "α_{k_i}" and "α_{l_i}" are values of the symbolic translation, respectively.

Definition 7. Let $S = \{s_0, s_1, \cdots, s_T\}$ be a linguistic term set and $\tilde{s}_{\alpha_i} = [(s_{k_i}\ ,\ \alpha_{k_i})\ ,\ (s_{l_i}\ ,\ \alpha_{l_i})]$ be an uncertain two-tuple linguistic information. There is always a function Δ^{-1}, such that, from an uncertain 2-tuple it returns its equivalent interval value $\tilde{\mu} = [\mu^L\ ,\ \mu^U]$, where $\mu^L, \mu^U \in\ [0\ , T]$

$$\Delta^{-1} : S \times [-0.5, 0.5) \to [0, T]\,. \quad (4a)$$

$$\Delta^{-1}(s_{k_i}, \alpha_{k_i}) = k_i + \alpha_{k_i} = \mu^L. \quad (4b)$$

and

$$\Delta^{-1} : S \times [-0.5, 0.5) \to [0, T]\,. \quad (5a)$$

$$\Delta^{-1}(s_{l_i}, \alpha_{l_i}) = l_i + \alpha_{l_i} = \mu^U. \quad (5b)$$

From the above Definitions, it is obvious that the conversion of a linguistic term into a linguistic two-tuple consists of adding a value 0 as symbolic translation: $s_i \in S \Rightarrow (s_i, 0)$. It is also obvious that the conversion of an uncertain linguistic information into an uncertain two-tuple linguistic information consists of adding a value 0 as symbolic translation: $\tilde{s}_i = [s_{k_i}\ ,\ s_{l_i}] \in \tilde{S} \Rightarrow [(s_{k_i}, 0)\ ,\ (s_{l_i}, 0)]$.

3. The proposed UTOWA operator

Yager provides the following definition of the ordered weighted averaging (OWA) operator [10].

Definition 8. [10] An OWA operator of dimension *n* is a mapping, $\phi : R^n \to R$, that has an associated weighting vector $\boldsymbol{w} = (w_1, w_2, \cdots, w_n)^{\mathrm{T}}$ with the properties $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$, such that

$$\phi(a_1, a_2, \cdots, a_n) = \sum_{i=1}^{n} w_i b_i\,. \quad (6)$$

where b_i is the *i*-th largest of the a_i.

An important feature of the OWA operator is the recording step. During this step, an argument a_i is not associated with a particular weight w_i but rather a weight w_i is associated with a particular ordered position *i* of the arguments.

Herrera and Martinez [4] develop a TOWA operator and a brief description of the TOWA operator is given below.

Definition 9. Let $\{(s_1, \alpha_1), (s_2, \alpha_2), \cdots, (s_n, \alpha_n)\}$ be a collection of two-tuple linguistic information and $s_i \in S$, $\alpha_i \in [-0.5, 0.5)$, $i \in n$, then a TOWA operator ϕ is represented by

$$\phi((s_1, \alpha_1), (s_2, \alpha_2), \cdots, (s_n, \alpha_n)) = \Delta(\sum_{i=1}^{n} h_i c_i)\,. \quad (7)$$

where $\boldsymbol{c} = (c_1, c_2, \cdots, c_n)^{\mathrm{T}}$ is an associated ordered value vector and each element c_i ($c_i \in \boldsymbol{c}$) is the *i*-th largest value in the collection $\{\Delta^{-1}(s_i, \alpha_i) \mid i \in n\}$. $\boldsymbol{h} = (h_1, h_2, \cdots, h_n)^{\mathrm{T}}$ is a weighting vector by means of a fuzzy linguistic quantifier according to Yager's ideas [10]. In the case of a non-decreasing proportional quantifier Q, the weighting vector is calculated using the following expression:

$$h_i = Q(i/n) - Q((i-1)/n)\,, \qquad i \in n\,. \quad (8)$$

where $h_i \in [0,1]$, $\sum_{i=1}^{n} h_i = 1$. The membership function of a non-decreasing relative quantifier Q can be represented as

$$Q(r) = \begin{cases} 0, & \text{if } r < a \\ \dfrac{r-a}{b-a}, & \text{if } a \le r \le b \\ 1, & \text{if } r > b. \end{cases} \quad (9)$$

with $a, b, r \in [0,1]$, several examples of proportional quantifier are "Most", "At least half" and "As many as possible", where the parameters, (a,b) are $(0.3, 0.8)$, $(0, 0.5)$ and $(0.5, 1)$, respectively.

The TOWA operator, however, can only be used in situations where the aggregated arguments are the relatively certain linguistic information. Based on the TOWA operator, in the following we shall propose the UTOWA operator, which can be used in situations where the aggregated arguments are given in the format of uncertain linguistic information.

Definition 10. Let $\{\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n}\}$ be a collection of uncertain two-tuple linguistic information and $\tilde{s}_{\alpha_i} \in \tilde{S}_\alpha$, $i \in n$, then an UTOWA operator ϕ is defined as follows:

$$\phi(\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n}) = \Delta(h_1\tilde{c}_1 \oplus h_2\tilde{c}_2 \oplus \cdots \oplus h_n\tilde{c}_n). \quad (10)$$

where $\tilde{\boldsymbol{c}} = (\tilde{c}_1, \tilde{c}_2, \cdots, \tilde{c}_n)^{\mathrm{T}}$ is an associated ordered interval number vector and each element $\tilde{c}_i$ ($\tilde{c}_i \in \tilde{\boldsymbol{c}}$) is the *i*-th largest interval number using Eq. (1) in the collection $\{[\Delta^{-1}(s_{k_i}, \alpha_{k_i}),\ \Delta^{-1}(s_{l_i}, \alpha_{l_i})] \mid i \in n\}$. The weighting vector $\boldsymbol{h} = (h_1, h_2, \cdots, h_n)^{\mathrm{T}}$ associated with c_i is defined as:

$$h_i = \frac{\Delta^{-1}(w_i)}{\sum_{i=1}^{n} \Delta^{-1}(w_i)}, \qquad i \in n \quad (11)$$

where $h_i \in [0,1]$, $\sum_{i=1}^{n} h_i = 1$. $\boldsymbol{w} = (w_1, w_2, \cdots, w_n)^{\mathrm{T}}$ is a linguistic weighting vector, where each element w_i ($w_i \in \boldsymbol{w}$) is associated with $\tilde{s}_{\alpha_i}$ ($\tilde{s}_{\alpha_i} \in \tilde{S}_\alpha$), and $w_i \in S$.

The UTOWA operator has many desirable properties as follows:

Theorem 1. Let $\{\tilde{s}'_{\alpha_1}, \tilde{s}'_{\alpha_2}, \cdots, \tilde{s}'_{\alpha_n}\}$ is any permutation of $\{\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n}\}$, then

$$\phi(\tilde{s}'_{\alpha_1}, \tilde{s}'_{\alpha_2}, \cdots, \tilde{s}'_{\alpha_n}) = \phi(\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n}). \quad (12)$$

Proof: Let $\phi(\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n}) = \Delta(h_1\tilde{c}_1 \oplus h_2\tilde{c}_2 \oplus \cdots \oplus h_n\tilde{c}_n)$, and $\phi(\tilde{s}'_{\alpha_1}, \tilde{s}'_{\alpha_2}, \cdots, \tilde{s}'_{\alpha_n}) = \Delta(h_1\tilde{c}'_1 \oplus h_2\tilde{c}'_2 \oplus \cdots \oplus h_n\tilde{c}'_n)$. Since $\{\tilde{s}'_{\alpha_1}, \tilde{s}'_{\alpha_2}, \cdots, \tilde{s}'_{\alpha_n}\}$ is a permutation of $\{\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n}\}$, using Eqs. (2)-(5), we have $\tilde{c}_i = \tilde{c}'_i$, $i \in n$, then $\phi(\tilde{s}'_{\alpha_1}, \tilde{s}'_{\alpha_2}, \cdots, \tilde{s}'_{\alpha_n}) = \phi(\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n})$. □

Theorem 2. If $\tilde{s}_{\alpha_i} = \tilde{s}_\alpha$ ($\tilde{s}_{\alpha_i} \in \tilde{S}_\alpha$, $\tilde{s}_\alpha \in \tilde{S}_\alpha$, $i \in n$) for $\forall i$, then

$$\phi(\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n}) = \tilde{s}_\alpha . \quad 13)$$

Proof: Since $\tilde{s}_{\alpha_i} = \tilde{s}_\alpha$ for $\forall i$, using Eqs. (2)-(5) , we have $\phi(\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n}) = \phi(\tilde{s}_\alpha, \tilde{s}_\alpha, \cdots, \tilde{s}_\alpha) = \Delta(h_1\tilde{c} \oplus h_2\tilde{c} \oplus \cdots \oplus h_n\tilde{c}) = \Delta((h_1 + h_2 + \cdots + h_n)\tilde{c}) = \Delta(\tilde{c}) = \tilde{s}_\alpha$. □

Theorem 3.

$$\mathrm{Min}(\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n}) \le \phi(\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n}) \le \mathrm{Max}(\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n}). \quad (14)$$

Proof: Let $\phi(\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n}) = \mathrm{Min}(\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n})$, if $\boldsymbol{h} = \boldsymbol{h}_* = (0, 0, \cdots, 1)^{\mathrm{T}}$, and $\phi(\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n}) = \mathrm{Max}(\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n})$, if $\boldsymbol{h} = \boldsymbol{h}^* = (1, 0, \cdots, 0)^{\mathrm{T}}$. Since each element $\tilde{c}_i$ ($\tilde{c}_i \in \tilde{\boldsymbol{c}}$) is the *i*-th largest interval number using Eq. (1) in the collection $\{[\Delta^{-1}(s_{k_i}, \alpha_{k_i}),\ \Delta^{-1}(s_{l_i}, \alpha_{l_i})] \mid i \in n\}$, using Eqs. (2)-(5), we have

$\phi(\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n}) = \Delta(h_1\tilde{c}_1 \oplus h_2\tilde{c}_2 \oplus \cdots \oplus h_n\tilde{c}_n) \geq \Delta(\tilde{c}_n) = \text{Min}(\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n})$, $\phi(\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n}) = \Delta(h_1\tilde{c}_1 \oplus h_2\tilde{c}_2 \oplus \cdots \oplus h_n\tilde{c}_n) \leq \Delta(\tilde{c}_1) = \text{Max}(\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n})$.

Then,

$\text{Min}(\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n}) \leq \phi(\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n}) \leq \text{Max}(\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n})$. □

Theorem 4. If the weighting vector $\boldsymbol{h} = (\frac{1}{n}, \frac{1}{n}, \cdots, \frac{1}{n})^{\mathrm{T}}$, then the UTOWA operator is reduced to the AA operator, i.e.,

$$\phi(\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n}) = \Delta[\frac{1}{n}(\tilde{c}_1 \oplus \tilde{c}_2 \oplus \cdots \oplus \tilde{c}_n)]. \quad (15)$$

Proof: Since the weighting vector $\boldsymbol{h} = (\frac{1}{n}, \frac{1}{n}, \cdots, \frac{1}{n})^{\mathrm{T}}$, it follows that by Eqs. (2) and (3),

$$\phi(\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n}) = \Delta(h_1\tilde{c}_1 \oplus h_2\tilde{c}_2 \oplus \cdots \oplus h_n\tilde{c}_n) = \Delta[\frac{1}{n}(\tilde{c}_1 \oplus \tilde{c}_2 \oplus \cdots \oplus \tilde{c}_n)].. \quad \square$$

Theorem 5. Let $\{\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n}\}$ and $\{\tilde{s}'_{\alpha_1}, \tilde{s}'_{\alpha_2}, \cdots, \tilde{s}'_{\alpha_n}\}$ be two ordered collections of uncertain two-tuple linguistic information. If $\tilde{s}_{\alpha_i} \leq \tilde{s}'_{\alpha_i}$, for $\forall i$, and the weighting vector $\boldsymbol{h}$ is constant, then

$$\phi(\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n}) \leq \phi(\tilde{s}'_{\alpha_1}, \tilde{s}'_{\alpha_2}, \cdots, \tilde{s}'_{\alpha_n}). \quad (16)$$

Proof: Let $\phi(\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n}) = \Delta(h_1\tilde{c}_1 \oplus h_2\tilde{c}_2 \oplus \cdots \oplus h_n\tilde{c}_n)$, and $\phi(\tilde{s}'_{\alpha_1}, \tilde{s}'_{\alpha_2}, \cdots, \tilde{s}'_{\alpha_n}) = \Delta(h_1\tilde{c}'_1 \oplus h_2\tilde{c}'_2 \oplus \cdots \oplus h_n\tilde{c}'_n)$. Since $\tilde{s}_{\alpha_i} \leq \tilde{s}'_{\alpha_i}$, for $\forall i$, it follows that $\tilde{c}_i \leq \tilde{c}'_i$ using Eq. (1), by Eqs. (2)-(5), then $\Delta(h_1\tilde{c}_1 \oplus h_2\tilde{c}_2 \oplus \cdots \oplus h_n\tilde{c}_n) \leq \Delta(h_1\tilde{c}'_1 \oplus h_2\tilde{c}'_2 \oplus \cdots \oplus h_n\tilde{c}'_n)$, hence $\phi(\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n}) \leq \phi(\tilde{s}'_{\alpha_1}, \tilde{s}'_{\alpha_2}, \cdots, \tilde{s}'_{\alpha_n})$.□

Let $k_i = l_i$, $\alpha_{k_i} = \alpha_{l_i}$, in collection $\{\tilde{s}_{\alpha_1}, \tilde{s}_{\alpha_2}, \cdots, \tilde{s}_{\alpha_n}\}$, $i \in n$, it is obvious that UTOWA operator is reduced to TOWA operator.

4. Conclusion

In this paper, we develop a new UTOWA operator and study some desirable properties of the operator. The proposed operator can be used to aggregate the uncertain linguistic information in MAGDM problems. It also enriches the existing approaches to aggregating uncertain linguistic information. Furthermore, we shall continue working in the extension and application in MAGDM of the UTOWA operator. In this paper, we develop a new UTOWA operator and study some desirable properties of the operator. The proposed operator can be used to aggregate the uncertain linguistic information in MAGDM problems. It also enriches the existing approaches to aggregating uncertain linguistic information. Furthermore, we shall continue working in the extension and application in MAGDM of the UTOWA operator.

Acknowledgment

This work was partly supported by the National Science Fund for Distinguished Young Scholars of China (Project No. 70525002), the National Natural Science Foundation of China (NSFC, Project No. 70371050), the Teaching and Research Award Program for Outstanding Young Teachers (TRAPOYT) in Higher Education Institutions and Research Fund for the Doctoral Program of Higher Education (Project No. 20040145018), Ministry of Education, China.

References

[1] G. Bordogna, M. Fedrizzi, G. Pasi, “A linguistic modeling of consensus in group decision making based on OWA operators”, IEEE Tranactions on Systems, Man and Cybernetics, vol.27, pp.126-132, 1997.
[2] F. Herrera, E. Herrera-Viedma, “Aggregation operators for linguistic weighted information”, IEEE Transactions on Systems, Man and Cybernetics, vol.27, pp.646-656, 1997.
[3] F. Herrera, E. Herrera-Viedma, J. L. Verdegay, “A model of consensus in group decision making under linguistic assessments”, Fuzzy Sets and Systems, vol.78, pp.73-87, 1996.
[4] F. Herrera, L. Martinez, “A 2-tuple fuzzy linguistic representation model for computing with words”, IEEE Transactions on Systems, Man and Cybernetics, vol.8, pp.746-752, 2000.
[5] F. Herrera, L. Martinez, “A model based on linguistic

2-tuple for dealing with multigranularity hierarchical linguistic contexts in multiexpert decision-making", IEEE Transactions on Systems, Man and Cybernetics, vol.31, pp.227-234, 2001.
[6] C. Z. Luo, "Introduction to Fuzzy Set (I)", Beijing, Beijing Normal University, pp.197-199, 1989.
[7] H. B. Mitchell, P. A. Schaefer, "Multiple priorities in an induced ordered weighted averaging operator", International Journal of Intelligent Systems, vol.15, pp. 317-327, 2000.
[8] Z. S. Xu, "The hybrid linguistic weighted averaging operator", Asia Information-Science-Life, in press, 2006.
[9] Z. S. Xu, "Induced uncertain linguistic OWA operators applied to group decision making", Information Fusion, in press, 2006.
[10] R. R. Yager, "On ordered weighted averaging aggregation operators in multi criteria decision making", IEEE Transactions on Systems, Man and Cybernetics, vol.18, pp.183-190, 1988.
[11] R. R. Yager, J. Kacprzyk, "The Ordered Weighted Averaging Operators: Theory and Application", Norwell, Kluwer, 1997.
[12] L. A. Zadeh, "The concept of a linguistic variable and its applications to approximate reasoning", Information Sciences (Part I), vol.8, pp.199-249, 1975.
[13] L. A. Zadeh, "The concept of a linguistic variable and its applications to approximate reasoning", Information Sciences (Part II), vol.8, pp.301-357, 1975.
[14] Q. Zhang, Z. P. Fan, D. H. Pan, "A ranking approach for interval numbers in uncertain multiple attribute decision making problems", Systems Engineering–Theory & Practice, vol.5, pp.129-133, 1999(in Chinese).
[15] Y. Zhang, Z. P. Fan, "An approach to multiple attribute group decision making with linguistic information based on ETOWA operators", International Conference of Computers and Industrial Engineering, Taiwan, China, 2006.

Yao Zhang received the B.E. degree in industrial foreign trade from Northeastern University (NEU) in 1998, the M.S. degrees in Management from NEU in 2000, and the Ph.D degree in Management Science and Engineering from NEU in 2006, respectively. She is currently lecturer in the School of Business Administration, NEU. Her research interests include decision analysis, supply chain and logistics management.

Zhi-Ping FAN received the B.E. degree in industrial automation from Northeastern University (NEU), Shenyang, China, in 1982, the M.S. degree in industrial automation from NEU in 1986, and the Ph.D. degree in control theory and applications from NEU in 1996. He is currently Professor in the Department of Management Science and Engineering, School of Business Administration, NEU. He was Research Fellow at City University of Hong Kong in 2001, 2003, 2004 and 2005, and he has published more than 20 papers in international journals. He is currently interested in decision analysis, operations research, information management and knowledg