Double-layer Modificatory Linguistic Truth-value Lattice-valued Evaluation Method

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Summary
In this paper, a double-layer modificatory linguistic truth-value lattice-valued evaluation method is presented. Its logic basic is based on modificatory linguistic truth-value lattice-valued proposition logic system *MLTVLP(X)* and operator modificatory linguistic truth-value lattice-valued proposition logic *OMLTVLP(X)*. *MLTVLP(X)* and *OMLTVLP(X)* are similar to *LTVLP(X)* and *MLTVLP(X)*, which have been given in our previous research paper. But they have different valuation field, i.e. different type of Lattice Implication Algebra. It is an attempt to solve the double-layer evaluation problem with incomparable linguistic qualitative information. In this evaluation method, every valuator can specify a modificatory linguistic truth-value for every sub-criterion. Likewise, different valuator can be endowed with different importance according to their different background, knowledge or experience. Moreover, different valuator can specify different weight for every sub-criterion according to their experience or preference. All different importance is aggregated to the final evaluation result. It is shown that incomparable linguistic information can be dealt with in this evaluation method.

Key words:
Artificial Intelligence, Lattice Implication Algebra, Lattice-valued Logic system, Double-layer Evaluation Method with Linguistic Truth-value

1. Introduction

There are many evaluation situations, in which the information cannot be evaluated in a quantitative form but may be in a qualitative one, so it is necessary to use qualitative value in an evaluation system. For example, when we try to evaluate “comfort of a car seat”, we tend to use natural language such as {most comfortable, more comfortable, uncomfortable} etc. other than precise numerical value. Consider another example, when we evaluate one’s ability, we tend to use {competent, able, incapable, unable} etc. other than numerical value.

One of the qualitative information processing methods is linguistic variable and approaches based on linguistic variable. The fuzzy linguistic variable and related approach is presented by L. A. Zadeh in 1975[16, 17]. After L. A. Zadeh's work in 1975, a lot of fuzzy linguistic approaches have been proposed and applied with very good results to different problems, such as, “education[1]”, “software usability-evaluation[4, 20]”, “information retrieval[11]”, “risk evaluation in software development[13]”, “evaluation of user interface[24]” and “decision-making system[2, 5, 6, 7, 8, 9, 12, 25, 26, 28]” etc.. Whether in evaluation system or in decision-making system, information fusion plays an important role. Some solutions to this problem are given in [5, 10, 27]. In addition, there are some research work which try to use hedge algebra to describe fuzzy linguistic variable in fuzzy logic in [21, 22, 23]. However, all these methods are based on linear ordered set. In other words, incomparable information can’t be dealt with in this linguistic variable framework.

In fact, incomparable information does exist in natural language. For example, it is difficult to distinguish between “almost True” and “more or less True”. In order to describe and deal with incomparable information, Xu proposed Lattice Implication Algebra in 1993[30]. Xu etc. have established lattice-valued proposition logic system *LP(X)* [29] and lattice-valued first-order logic system *LF(X)* [29] based on Lattice Implication Algebra since 1993. It provides a necessary foundation for the processing of incomparable information. In addition, there are some research work on incomparable information processing. An evaluation method with incomparable information is presented in [3]. Lattice-valued linguistic-based decision-making method is discussed in [14]. A model for handling linguistic terms in the framework of lattice-valued logic LF(X) is presented in [15]. A framework of linguistic truth-valued propositional logic based on Lattice Implication Algebra is given in [18]. In addition, incomparable hedge is proposed based on lattice Implication Algebra in [19]. Similarly, incomparable information does exist in evaluation system. In this paper,

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we proposed modificatory linguistic truth-value Lattice Implication Algebra, abbr. to *MLTVLIA*. Based on *MLTVLIA*, we establish modificatory linguistic truth-value lattice-valued proposition logic system *MLTVLP(X)* and operator modificatory linguistic truth-value lattice-valued proposition logic *OMLTVLP(X)* to represent linguistic truth-value of a proposition and take the truth-value as the evaluation value of each sub-criterion. Based on this representation, we try to construct an evaluation method to solve double-layer evaluation problem with incomparable information by using modificatory linguistic truth-value lattice-valued logic System and operator linguistic truth-value lattice-valued logic system.

The paper is organized as follows. We introduce some necessary definitions first. Then we give modificatory linguistic truth-value lattice-valued proposition logic system *MLTVLP(X)* and operator modificatory linguistic truth-value lattice-valued proposition logic *OMLTVLP(X)*. After that, we propose the double-layer modificatory linguistic truth-value lattice-valued evaluation method. It is an attempt to solve incomparable information in a double-layer evaluation system.

2. Preliminaries

In this section, we will give some necessary definitions for the readability and intelligibility of this paper.

Definition 2.1[30]. Let $(L,\wedge,\vee,\rightarrow,O,I)$ be a bounded lattice with an order-reversing involution " ′ ", I and O be the greatest and the least element of L respectively, and $\rightarrow: L\times L\rightarrow L$ be a mapping. If the following conditions hold for any $x,y,z\in L$:

(1) $x\rightarrow(y\rightarrow z)=y\rightarrow(x\rightarrow z)$;

(2) $x\rightarrow x=I$;

(3) $x\rightarrow y=y'\rightarrow x'$;

(4) $x\rightarrow y=y\rightarrow x=I$ implies $x=y$;

(5) $(x\rightarrow y)\rightarrow y=(y\rightarrow x)\rightarrow x$;

(6) $(x\vee y)\rightarrow z=(x\rightarrow z)\wedge(y\rightarrow z)$;

(7) $(x\wedge y)\rightarrow z=(x\rightarrow z)\vee(y\rightarrow z)$;

then $(L,\wedge,\vee,',\rightarrow,O,I)$ is called a lattice implication algebra.

Definition 2.2[3]. Let MT= {True (abbr. to T), False (abbr. to F)}, the following lattice implication algebra $(MT,\wedge,\vee,',\rightarrow,F,T)$ is called a meta linguistic truth-value lattice implication algebra, where $\wedge,\vee,',\rightarrow$ is defined the same as that of in classical logic.

In Classical Logic, the truth-value of a proposition is "True" or "False". However, we often use some modificatory word to decrease or increase the meaning of a word in daily life and work. Naturally, a model must be built to describe this case. In this paper, we try to establish this model named modificatory linguistic truth-value lattice-value logic system. We introduce some modificatory words (abbr. To MW) and combine the modificatory word (MW) and truth-value {True, False} to a modificatory linguistic truth-value in modificatory linguistic truth-value Lattice-value logic system.

Example 2.3[29] Consider the set $L=\{a_i\mid i=1,2,\cdots,n\}$, define order on L as follows: if $i\le j$ iff $a_i\le a_j$, where $i,j\in\{1,2,\cdots,n\}$. For any $1\le i,j\le n$, define:

(1) $a_i\vee a_j=a_{\max\{i,j\}}$;

(2) $a_i\wedge a_j=a_{\min\{i,j\}}$;

(3) $a_i'=a_{n-i+1}$;

(4) $a_i\rightarrow a_j=a_{\min\{n-i+j,n\}}$,

then $(L,\wedge,\vee,',\rightarrow,O,I)$ is a Lattice Implication Algebra. We will use this Lattice Implication Algebra to describe the hedge set.

Example 2.4

(1) Let MW = {Slightly (Abbr. to Sl), Somewhat (Abbr. to So), Rather (Abbr. to Ra), Almost (Abbr. to Al), Exactly (Abbr. to Ex), Quite (Abbr. to Qu), Very (Abbr. to Ve), Highly (Abbr. to Hi), Absolutely (Abbr. to Ab.} be a modifactory word set, then chain Sl ≤ So ≤ Ra ≤ Al ≤ Ex ≤ Qu ≤ Ve ≤ Hi ≤ Ab is a lattice implication algebra with operation as given in example 2.3, and it is a nine modifactory word set;

(2) Let MW = {least, less, more or less, more, most} be a modificatory word set, then chain least ≤ less ≤ more or less ≤ more ≤ most is a lattice implication algebra with operation as given in example 2.3, and it is a five modificatory word set.

Definition 2.5. Let L_1=$\{a_1,a_2,\cdots,a_s\}$ be a modificatory word set, $(\{a_1,a_2,\cdots,a_s\},\wedge_1,\vee_1,'_1,\rightarrow_1,O_1,I_1)$ be a lattice implication algebra as defined in Example 2.3, L_2={True, False} be a truth-value set, $(\{True,False\},\wedge_2,\vee_2,'_2,\rightarrow_2,O_2,I_2)$ be a meta linguistic truth-value lattice implication algebra, define the product of L_1 and L_2 be as follows:

$L_1\times L_2=\{(a,b)\mid a\in L_1,b\in L_2\}$, define the operation $\wedge,\vee,',\rightarrow$ on $L_1\times L_2$ as follows:

for any $(a_1,b_1),(a_2,b_2)\in L_1\times L_2$,

$(a_1,b_1)\wedge(a_2,b_2)=(a_1\wedge_1 a_2,b_1\wedge_2 b_2)$,

$(a_1,b_1)\vee(a_2,b_2)=(a_1\vee_1 a_2,b_1\vee_2 b_2)$,

$(a_1,b_1)'=(a_1,(b_1)'_2)$,

$(a_i, T) \to (a_j, F) = (a_{\max\{0, i+j-s\}}, F)$
$(a_i, F) \to (a_j, T) = (a_{\min\{s, i+j\}}, T)$
$(a_i, T) \to (a_j, T) = (a_{\min\{s, s-i+j\}}, T)$
$(a_i, F) \to (a_j, F) = (a_{\min\{s, s-j+i\}}, T)$

, then $(L_1 \times L_2, \wedge, \vee, ', \to, (a_n, F), (a_n, T))$ is a lattice implication algebra[18], $(L_1 \times L_2, \wedge, \vee, ', \to, (a_n, F), (a_n, T))$ is called a modifactory linguistic truth-value lattice implication algebra, abbr. to *MLTVLIA*, its Hasse Graph is as in fig. 1.

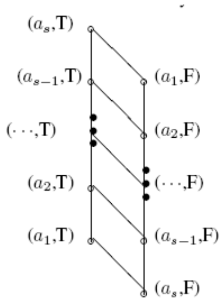

Fig. 1 Hasse Graph of Modificatory Linguistic Truth-value Lattice Implication Algebra.

Note 2.6 For Descartes product of two lattice implication algebras, we can define different negation, disjunction, conjunction, implication operations on it, more than one kind of lattice implication algebra can be defined as a result of different way of definition, more details will be discussed in our other paper.

3. *MLTVLP(X)* and *OMLTVLP(X)*

Based on the modificatory linguistic truth-value lattice implication algebra *MLTVLIA*, modificatory linguistic truth-value lattice-value proposition logic *MLTVLIA* can be defined as the following.

Definition 3.1 Let $(L_1 \times L_2, \wedge, \vee, ', \to, (a_s, F), (a_s, T))$ be a modifactory linguistic Truth-value Lattice Implication algebra, the definition of modificatory linguistic Truth-value lattice-valued proposition logic system *MLTVLP(X)* is defined as follows:

Syntax:

The symbols in *MLTVLP(X)* are:

(1) the set of propositional variable: $X = \{p, q, r, \ldots\}$;
(2) the set of constants: *MLTVLIA*;
(3) logical connectives: $\wedge, \vee, \to, '$;
(4) auxiliary symbols:), (.

The set *F* of formulae of *MLTVLP(X)* is the least set Y satisfying the following conditions:

(1) $X \subseteq Y$;
(2) *MLTVLIA* $\subseteq$ *Y*;
(3) if $p, q \in Y$, then $p', p \wedge q, p \vee q, p \to q \in Y$.

A mapping *v*: *MLTVLP(X)* → *MLTVLIA* is called a modificatory linguistic lattice-valued valuation of *LP(X)*, if it is a *T-homomorphism*.

Based on *MLTVLP(X)*, we can describe some modificatory linguistic truth-value, but we can't use *MLTVLP(X)* to describe the modificatory linguistic truth-value with importance coefficient. In the following, we will introduce operator modificatory linguistic truth-value lattice-valued proposition logic system *λMLTVLP(X)*. For any $\lambda \in [0,1]$, we define $\lambda MLTVLIA = \{\lambda\alpha \mid \lambda \in [0,1], \alpha \in MLTVLIA\}$, $\lambda MLTVLP(X) = \{\lambda F \mid F \in MLTVLP(X), \lambda \in [0,1]\}$.

Definition 3.2 Let $(L_1 \times L_2, \wedge, \vee, ', \to, (a_s, F), (a_s, T))$ be a modificatory linguistic truth-value Lattice Implication Algebra, $\lambda \in [0,1]$, the definition of *OMLTVLP(X)* is defined as follows:

Syntax:

The symbols in *OMLTVLP(X)* are:

(1) the set of propositional variable: $X = \{\lambda p, \lambda q, \lambda r, \ldots\}$;
(2) the set of constants: *λMLTVLIA*;
(3) logical connectives: $\wedge, \vee, \to, '$;

auxiliary symbols:), (.

The set *F* of formulae of *OMLTVLP(X)* is the least set *Y* satisfying the following conditions:

(1) (1)$\lambda X \subseteq \lambda Y$;
(2) *λOMLTVLIA* $\subseteq \lambda Y$;
(3) if $\lambda_1 p, \lambda_2 q \in \lambda Y$; , then $(\lambda p)', \lambda_1 p \wedge \lambda_2 q, \lambda_1 p \vee \lambda_2 q, \lambda_1 p \to \lambda_2 q \in \lambda Y$.

A mapping *v*: *OMLTVLP(X)* → *λMLTVLIA* is called a linguistic lattice-valued valuation of *LP(X)*, if it is a *T-homomorphism*.

4. Double-layer Modificatory Linguistic Truth-value Lattice-valued Evaluation Method

4.1 Formulation of Double-Layer Evaluation Method

Let the evaluation problem involve *n* alternatives (objects) $\{A_i \mid i = 1, 2, \cdots, n\}$ to be evaluated and *r* valuators (experts) $\{e_k \mid k = 1, 2, \cdots, r\}$. Let each alternative will be evaluated based on a set of *m* criteria (attributes) $\{C_j \mid j = 1, 2, \cdots, m\}$ and each criterion C_j can be divided to $p_j (j = 1, 2, \cdots, m)$ sub-criterion $\{C_{j1}, C_{j2}, \cdots, C_{jp_j}\}$ and each sub-criterion can't be divided further, it is a double-layer evaluation problem. The double-layer evaluation problem can be shown in figure 2. In this paper, each criterion C_j ($j = 1,2,\cdots,m$) is also called upper-layer criterion. Each sub-criterion C_{jt} ($j = 1,2,\cdots,m, t = 1,2,\cdots,p_j$) is also called lower-layer sub-criterion.

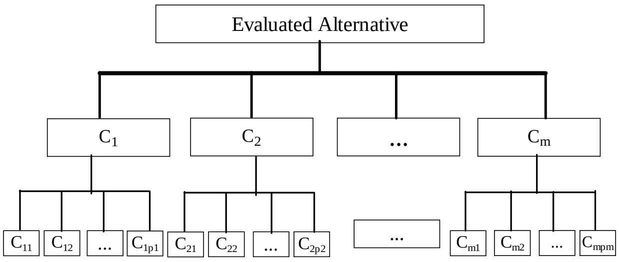

Fig. 2 Demonstration graph of double-layer evaluation problem.

4.2 Selection of Evaluation Term Set and Designation of Evaluation Value

In our approach, we focus on processing the information with linguistic value especially information with incomparable linguistic value other than numerical information. Actually, the evaluated term set is usually different for different evaluated criteria in an evaluation problem in most cases.

We adopt the following method to denote and designate the evaluation value in our evaluation method.

(1) Selection of Evaluation Term set.

We use the following modificatory lattice implication algebra H × {True, False} = {(a_s, False), (a_{s-1}, False), …, (a_1, False), (a_1, True), (a_2, True), …, (a_{s-1}, True), (a_s, True)} as defined in definition 2.5 to describe the degree of truth-value of each sub-criterion. And select H × {True, False} = {(a_s, False), (a_{s-1}, False), …, (a_1, False), (a_1, True), (a_2, True), …, (a_{s-1}, True), (a_s, True)} as the evaluation term set in this evaluation problem.

(2) Designation the Evaluation Value

All valuators (experts) come to an agreement on the exclusive standard evaluation adjective without any modificatory adverb for every sub-criterion in positive view. For example, "satisfactory" will be used as the exclusive adjective when we try to evaluate service quality other than "dissatisfactory". Based on the exclusive adjective, we will take the truth-value of each proposition as the evaluation value. We denote the exclusive adjective by "*A*" without loss of generality. For the alternative *x*, then the standard proposition is as "*x* is *A*". The evaluation value will be given based on the truth-value of proposition "*x* is *A*".

For example, a standard proposition is given as in "*x* is *A*". One valuator (expert) may think proposition p: "*x* is *very A*", then the truth-value of this proposition *p* given by this valuator is "very True"; Otherwise, if he thinks proposition p "*x* is slightly *A*", then the truth-value of this proposition p is "slightly True". Similarly, if a standard proposition is given as in "*y* is *B*". If valuator (expert) thinks proposition q "*y* is exactly *B*", then the truth-value of proposition q is "exactly True"; Otherwise, if he thinks proposition q "*y* is slightly not *B*", then the truth-value of proposition q is "slightly False". By using this method, the evaluation value can be given by each valuator (expert) according to their agreement regardless of the standard adjective is *A* or *B* and each evaluation value belongs to the evaluation term set, which is a modificatory linguistic lattice implication algebra.

4.3 Double-Layer Evaluation Process Based on Modificatory Linguistic Truth-value Lattice-valued Logic System

We give the following double-layer evaluation process on modificatory linguistic truth-value lattice-valued logic system.

(1) Designate the evaluation value of each alternative for each lower-layer sub-criterion and get sub-evaluation value matrix.

For double-layer evaluation problem, each valuator (expert) e_k (k=1, 2, …, r) should give evaluation value based on the description in 4.2 for each sub-criterion C_{jt}, where j=1, 2, …, m, t = 1, 2, …, p_j) .

The following sub-criterion evaluation value vector $\begin{pmatrix} v_{j1}^{(i)(k)} \\ v_{j2}^{(i)(k)} \\ \vdots \\ v_{jp_j}^{(i)(k)} \end{pmatrix}$ denotes the evaluation value of the i^{th} alternative for the sub-criterion $C_{j1}, C_{j2}, \cdots, C_{jp_j}$ of the j^{th} criterion C_j given by the k^{th} valuator, where $j = 1,2,\cdots,m,\ i = 1,2,\cdots,n,\ k = 1,2,\cdots,r$.

For $i = 1,2,\cdots,n,$ take each sub-criterion evaluation value vector as a column vector, we can get the sub-evaluation value matrix for $p_j\ (j = 1,2,\cdots,m)$ sub-criterion on n alternatives given by the k^{th} valuator (expert) as in (1):

$$\begin{pmatrix} v_{j1}^{(1)(k)} & v_{j1}^{(2)(k)} & \cdots & v_{j1}^{(n)(k)} \\ v_{j2}^{(1)(k)} & v_{j2}^{(2)(k)} & \cdots & v_{j2}^{(n)(k)} \\ \vdots & \vdots & \vdots & \vdots \\ v_{jp_j}^{(1)(k)} & v_{jp_j}^{(2)(k)} & \cdots & v_{jp_j}^{(n)(k)} \end{pmatrix} \quad (1)$$

Then, for each of n evaluated alternatives $\{A_1, A_2, \cdots, A_n\}$, r valuators (experts) will give $m \times r$ sub-criterion evaluation value matrices as in (1), where $j = 1,2,\cdots,m,\ k = 1,2,\cdots,r$.

(2) Specify Importance Coefficient

In double-layer evaluation problem, the following three kinds of important coefficient should be considered and included in the final aggregation process.

The first one is importance coefficient of each lower-layer sub-criterion for each upper-layer criterion; the second one is importance coefficient of each expert for each upper-layer criterion, i.e. for each group of lower-layer sub-criterion of each upper-layer criterion; the last one is the importance coefficient of each criterion.

A) For the j^{th} criterion C_j ($j = 1,2,\cdots,m$), the valuator (expert) may give different importance coefficient for each sub-criterion $\{C_{j1}, C_{j2}, \cdots, C_{jp_j}\}$ of the j^{th} criterion C_j according to their different background, preference or experience, where $j = 1,2,\cdots,m$. We use the following sub-criterion weight vector $(\omega_{j1}^{(k)},\ \omega_{j2}^{(k)},\ \cdots\ \omega_{jp_j}^{(k)})$ to denote the importance for each group of sub-criteria $\{C_{j1}, C_{j2}, \cdots, C_{jp_j}\}$ of the j^{th} criterion C_j given by the k^{th} valuator (expert), where $k = 1,2,\cdots,r,\ j = 1,2,\cdots,k$. There are $m \times r$ sub-criterion weight vectors given by r valuators (experts) individually for each group sub-criteria of m criteria as follows: $(\omega_{j1}^{(k)},\ \omega_{j2}^{(k)},\ \cdots\ \omega_{jp_j}^{(k)})$, where $j = 1,2,\cdots,m,\ k = 1,2,\cdots,r$.

B) In addition, different expert have different importance for different criteria C_j according to their own knowledge or background, so the importance coefficient of the k^{th} valuator (expert) for j^{th} criterion C_j is $e_{(j)}^{(k)}$, where $j = 1,2,\cdots,m$, $k = 1,2,\cdots,r$. The importance coefficient of valuator can be given by a third-party or other weight getting method.

C) The importance coefficient of each upper-layer criterion is described as $\omega_j,\ j = 1,2,\cdots,m$. The weight can be given by a consensus of all valuators (experts) or using other existing weight computation methods.

(3) Aggregate and get final evaluation result.

I. Aggregate the importance coefficient of the k^{th} valuator (expert) for the j^{th} criterion C_j and the sub-criterion weight vector $(\omega_{j1}^{(k)},\ \omega_{j2}^{(k)},\ \cdots\ \omega_{jp_j}^{(k)})$.

The importance coefficient of the k^{th} valuator (expert) for j^{th} criterion C_j is $e_{(j)}^{(k)}$, where $j = 1,2,\cdots,m$, $k = 1,2,\cdots,r$. The importance of each sub-criterion of the j^{th} criterion C_j given by the k^{th} valuator (expert) , i.e. sub-criterion weight vector is $(\omega_{j1}^{(k)},\ \omega_{j2}^{(k)},\ \cdots\ \omega_{jp_j}^{(k)})$, then the double weight vector is as in (2), where × is multiplication on R.

$$\begin{aligned} & e_{(j)}^{(k)} \bullet (\omega_{j1}^{(k)},\ \omega_{j2}^{(k)},\ \cdots\ \omega_{jp_j}^{(k)}) \\ & = (e_{(j)}^{(k)} \times \omega_{j1}^{(k)},\ e_{(j)}^{(k)} \times \omega_{j2}^{(k)},\ \cdots,\ e_{(j)}^{(k)} \times \omega_{jp_j}^{(k)} \\ & \trianglerighteq (e_{(j)}^{(k)} \omega_{j1}^{(k)},\ e_{(j)}^{(k)} \omega_{j2}^{(k)},\ \cdots,\ e_{(j)}^{(k)} \omega_{jp_j}^{(k)} \end{aligned} \quad (2)$$

The aggregation includes two main steps: lower-layer aggregation and upper-layer based on lower-layer aggregation results. We will describe them respectively as in II and III.

II. Aggregate of sub-criterion evaluation value matrices and double weight vector.

For the convenience of the following depiction, we have to index the modificatory linguistic truth-value set. There are a lot of methods to index this truth-value, without loss of generality, we index the truth-value set described in fig. 1 as follows:

$$\begin{aligned} & V_1 = (a_s, T) \\ & V_2 = (a_{s-1}, T) \quad V_3 = (a_1, F) \\ & V_4 = (a_2, F) \quad V_5 = (a_{s-2}, T) \\ & \cdots \\ & V_{2s-2} = (a_{s-1}, T) \quad V_{2s-1} = (a_1, T) \\ & v_{2s} = (a_s, F) \end{aligned} \quad (3)$$

Then it is obvious that mapping f $f : \{V_1, V_2, \cdots, V_n\} \to N : V_i \mapsto i$ is a 1-1 mapping. For these n alternatives, the k^{th} valuator (expert) give the following sub-criterion evaluation value matrix for each sub-criterion $\{C_{j1}, C_{j2}, \cdots, C_{jp_j}\}$ of the j^{th} criterion C_j as in (1), there are $m \times r$ sub-criterion evaluation value matrices. The aggregation is describes as follows.

$$(e_{(j)}^{(k)}\omega_{j1}^{(k)},\ e_{(j)}^{(k)}\omega_{j2}^{(k)},\ \cdots,\ e_{(j)}^{(k)}\omega_{jp_j}^{(k)})\circ\begin{pmatrix} v_{j1}^{(1)(k)} & v_{j1}^{(2)(k)} & \cdots & v_{j1}^{(n)(k)} \\ v_{j2}^{(1)(k)} & v_{j2}^{(2)(k)} & \cdots & v_{j2}^{(n)(k)} \\ \vdots & \vdots & \vdots & \vdots \\ v_{jp_j}^{(1)(k)} & v_{jp_j}^{(2)(k)} & \cdots & v_{jp_j}^{(n)(k)} \end{pmatrix}$$

$$=\begin{pmatrix} e_{(j)}^{(k)}\omega_{j1}^{(k)}v_{j1}^{(1)(k)} & e_{(j)}^{(k)}\omega_{j1}^{(k)}v_{jp_j}^{(2)(k)} & \cdots & e_{(j)}^{(k)}\omega_{j1}^{(k)}v_{j1}^{(n)(k)} \\ e_{(j)}^{(k)}\omega_{j2}^{(k)}v_{j2}^{(1)(k)} & e_{(j)}^{(k)}\omega_{j2}^{(j)(k)}v_{jp_j}^{(2)(k)} & \cdots & e_{(j)}^{(k)}\omega_{j2}^{(k)}v_{j2}^{(n)(k)} \\ \vdots & \vdots & \vdots & \vdots \\ e_{(j)}^{(k)}\omega_{jp_j}^{(k)}v_{jp_j}^{(1)(k)} & e_{(j)}^{(k)}\omega_{jp_j}^{(k)}v_{jp_j}^{(2)(k)} & \cdots & e_{(j)}^{(k)}\omega_{jp_j}^{(k)}v_{jp_j}^{(n)(k)} \end{pmatrix} \quad (4)$$

where j = 1, 2, …, m, i =1, 2, …, n, k =1, 2, …, r, the matrices in (4) is called sub-criterion evaluation value matrices with double importance coefficient.

Then aggregate these $m\times r$ sub-criterion evaluation value matrices with double importance coefficient as the follows:

Define $sum_j^{(i)(k)}(q)$ as a real number and value $sum_j^{(i)(k)}(q)=0.0$ first and follow up the process to aggregate as follows:

For j = 1, 2, …, m;

For i =1, 2, …, n;

For k =1, 2, …, r;

{For q = 1, 2, …, 2s;

{For t = 1, 2, …, p_j;

{

$sum_j^{(i)(k)}(q)=0;$

If $v_{jt}^{(i)(k)}=V_q$,

$sum_j^{(i)(k)}(q)=sum_j^{(i)(k)}(q)+e_{(j)}^{(k)}\omega_{jt}^{(k)}$;

else

$sum_j^{(i)(k)}(q)=sum_j^{(i)(k)}(q)$;

return $sum_j^{(i)(k)}(q)$.

}

}

}

and for the j^{th} criterion C_j, construct the modificatory linguistic truth-value distribution vector of the i^{th} alternative based on the value given by the k^{th} valuator (expert) as in (5):

$$(sum_j^{(i)(k)}(1), sum_j^{(i)(k)}(2), \cdots, sum_j^{(i)(k)}(2s)) \quad (5)$$

then sum all the r vectors as in (6) and get the modificatiory linguistic truth-value set distribution vector of the i^{th} alternative for the j^{th} criterion C_j as in (6),

$$(\sum_{k=1}^{r} sum_j^{(i)(k)}(1), \sum_{k=1}^{r} sum_j^{(i)(k)}(2), \cdots, \sum_{k=1}^{r} sum_j^{(i)(k)}(2s)) \quad (6)$$

construct the modificatory linguistic truth-value distribution matrices based on lower-layer aggregation as in (7),

$$\begin{pmatrix} \sum_{k=1}^{r} sum_j^{(1)(k)}(1) & \sum_{k=1}^{r} sum_j^{(2)(k)}(1) & \cdots & \sum_{k=1}^{r} sum_j^{(n)(k)}(1) \\ \sum_{k=1}^{r} sum_j^{(1)(k)}(2) & \sum_{k=1}^{r} sum_j^{(2)(k)}(2) & \cdots & \sum_{k=1}^{r} sum_j^{(n)(k)}(2) \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{k=1}^{r} sum_j^{(1)(k)}(2s) & \sum_{k=1}^{r} sum_j^{(2)(k)}(2s) & \cdots & \sum_{k=1}^{r} sum_j^{(n)(k)}(2s) \end{pmatrix}, \quad (7)$$

where each column

$$\begin{pmatrix} \sum_{k=1}^{r} sum_j^{(i)(k)}(1) \\ \sum_{k=1}^{r} sum_j^{(i)(k)}(2) \\ \vdots \\ \sum_{k=1}^{r} sum_j^{(i)(k)}(2s) \end{pmatrix} \quad (8)$$

of (7) is the modificatory linguistic truth-value distribution vector of the i^{th} alternative for the j^{th} criterion C_j.

III. Aggregate the upper-layer linguistic truth-value distribution matrices and get final evaluation result

There are m modificatory linguistic truth-value distribution matrices based on lower-layer aggregation as in (7), and the modificatory linguistic distribution vector of the i^{th} alternative for the j^{th} criterion C_j is as in (8), then we should aggregate the upper-layer information based on lower-layer aggregation result as in (7) and importance coefficient of each criteria $\omega_j, j=1,2,\cdots,m$ as follows:

Aggregate the important coefficient of each criterion as in (9) and (10)

$$\omega_j\begin{pmatrix} \sum_{k=1}^{r} sum_j^{(1)(k)}(1) & \sum_{k=1}^{r} sum_j^{(2)(k)}(1) & \cdots & \sum_{k=1}^{r} sum_j^{(n)(k)}(1) \\ \sum_{k=1}^{r} sum_j^{(1)(k)}(2) & \sum_{k=1}^{r} sum_j^{(2)(k)}(2) & \cdots & \sum_{k=1}^{r} sum_j^{(n)(k)}(2) \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{k=1}^{r} sum_j^{(1)(k)}(2s) & \sum_{k=1}^{r} sum_j^{(2)(k)}(2s) & \cdots & \sum_{k=1}^{r} sum_j^{(n)(k)}(2s) \end{pmatrix} \quad (9)$$

$$=\begin{pmatrix} \omega_j\sum_{k=1}^{r} sum_j^{(1)(k)}(1) & \omega_j\sum_{k=1}^{r} sum_j^{(2)(k)}(1) & \cdots & \omega_j\sum_{k=1}^{r} sum_j^{(n)(k)}(1) \\ \omega_j\sum_{k=1}^{r} sum_j^{(1)(k)}(2) & \omega_j\sum_{k=1}^{r} sum_j^{(2)(k)}(2) & \cdots & \omega_j\sum_{k=1}^{r} sum_j^{(n)(k)}(2) \\ \vdots & \vdots & \vdots & \vdots \\ \omega_j\sum_{k=1}^{r} sum_j^{(1)(k)}(2s) & \omega_j\sum_{k=1}^{r} sum_j^{(2)(k)}(2s) & \cdots & \omega_j\sum_{k=1}^{r} sum_j^{(n)(k)}(2s) \end{pmatrix} \quad (10)$$

then get the final evaluation result as in (11)

$$\sum_{j=1}^{m}\begin{pmatrix} \omega_j\sum_{k=1}^{r} sum_j^{(1)(k)}(1) & \omega_j\sum_{k=1}^{r} sum_j^{(2)(k)}(1) & \cdots & \omega_j\sum_{k=1}^{r} sum_j^{(n)(k)}(1) \\ \omega_j\sum_{k=1}^{r} sum_j^{(1)(k)}(2) & \omega_j\sum_{k=1}^{r} sum_j^{(2)(k)}(2) & \cdots & \omega_j\sum_{k=1}^{r} sum_j^{(n)(k)}(2) \\ \vdots & \vdots & \vdots & \vdots \\ \omega_j\sum_{k=1}^{r} sum_j^{(1)(k)}(2s) & \omega_j\sum_{k=1}^{r} sum_j^{(2)(k)}(2s) & \cdots & \omega_j\sum_{k=1}^{r} sum_j^{(n)(k)}(2s) \end{pmatrix} \quad (11)$$

$$= \begin{pmatrix} \sum_{j=1}^{m}\omega_j\sum_{k=1}^{r} sum_j^{(1)(k)}(1) & \sum_{j=1}^{m}\omega_j\sum_{k=1}^{r} sum_j^{(2)(k)}(1) & \cdots & \sum_{j=1}^{m}\omega_j\sum_{k=1}^{r} sum_j^{(m)(k)}(1) \\ \sum_{j=1}^{m}\omega_j\sum_{k=1}^{r} sum_j^{(1)(k)}(2) & \sum_{j=1}^{m}\omega_j\sum_{k=1}^{r} sum_j^{(2)(k)}(2) & \cdots & \sum_{j=1}^{m}\omega_j\sum_{k=1}^{r} sum_j^{(m)(k)}(2) \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{j=1}^{m}\omega_j\sum_{k=1}^{r} sum_j^{(1)(k)}(2s) & \sum_{j=1}^{m}\omega_j\sum_{k=1}^{r} sum_j^{(2)(k)}(2s) & \cdots & \sum_{j=1}^{m}\omega_j\sum_{k=1}^{r} sum_j^{(m)(k)}(2s) \end{pmatrix}$$

each column in (11) is the final evaluation modificatory linguistic truth-valued distribution vector. Based on the previous index of modificatory linguistic truth-value, we can describe the final evaluation value by using the value in matrix (11) and the corresponding linguistic truth-value to denote the final evaluation value vector as in (12).

For the i^{th} alternative, its final evaluation value is:

$$\left(\omega_j\sum_{k=1}^{r} sum_j^{(i)(k)}(1)V_1,\ \ \omega_j\sum_{k=1}^{r} sum_j^{(i)(k)}(2)V_2,\ \ \cdots,\ \ \omega_j\sum_{k=1}^{r} sum_j^{(i)(k)}(2s)V_{2s}\right) \quad (12)$$

The final evaluation result is a vector. Each term in the vector is a constant element in operator modificatory linguistic truth-value lattice-valued logic system.

In addition, we know that the mapping relation defined in (3) between the index term set and the modificatory linguistic truth-value lattice implication algebra is a 1-1 mapping and it is isomorphic mapping. Moreover, the modificatory linguistic truth-value lattice implication algebra has a partial order as defined in fig. 1. So the partial order can be induced to the index term set similarly, the induced partial order is noted as *R* here.

Base on *R*, we can define a partial relation "≤"on final evaluation value vector set $\{(\lambda_1V_1,\lambda_2V_2,\cdots,\lambda_{2s}V_{2s}) \mid \lambda\in[0,1]\}$ as follows:

For (k=2s; k ≥1; k=k-1)

{J: if $a_k < b_k$, then

$(a_1V_1,a_2V_2,\cdots,a_{2s-1}V_{2s-1},a_{2s}V_{2s}) \le (b_1V_1,b_2V_2,\cdots,b_{2s-1}V_{2s-1},b_{2s}V_{2s})$;
else if

if $a_k > b_k$, then

$(b_1V_1,b_2V_2,\cdots,b_{2s-1}V_{2s-1},b_{2s}V_{2s}) \le (a_1V_1,a_2V_2,\cdots,a_{2s-1}V_{2s-1},a_{2s}V_{2s})$;
else if

$a_k = b_k$ and k > 1, go to J;

if $a_k = b_k$ and k = 1, then

$(a_1V_1,a_2V_2,\cdots,a_{2s-1}V_{2s-1},a_{2s}V_{2s}) = (b_1V_1,b_2V_2,\cdots,b_{2s-1}V_{2s-1},b_{2s}V_{2s})$;
}.

By using the partial relationship "≤" defined as the above, we can get a partial order for every final evaluation value vector set $\{(\lambda_1V_1,\lambda_2V_2,\cdots,\lambda_{2s}V_{2s}) \mid \lambda\in[0,1]\}$ described in (12).

5. Conclusions

In this paper, a double-layer modifactory linguistic truth-value lattice-valued evaluation method is presented. It is an extension of our previous research result in [3]. The method is based on modificatory linguistic truth-value lattice-valued proposition logic system *MLTVLP(X)* and operator modificatory linguistic truth-value lattice-valued proposition logic *OMLTVLP(X)*. It is an attempt to solve the evaluation problem with incomparable linguistic qualitative information. In this evaluation framework, every valuator can specify a modificatory linguistic truth-value for each sub-criterion. Moreover, all different weights, which come from valuator (expert) importance, criterion importance and sub-criterion importance are synthesized into the final evaluation results. Similarly, this method can be extended to multi-layer or mixed-layer evaluation system with incomparable information.

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