Photovoltaic Cell Battery Model for Wireless Sensor Networks

JongGyu Kim†, John Paul M. Torregoza††, InYeup Kong††† and WonJoo Hwang††††

†School of Computer Information Technology, Yeungjin College, Buk-ku, Taegu, Korea
††, ††††Department of Information and Communications Engineering, Inje University, Gimhae, Gyeognam, Korea
†††Department of Computer Engineering, Pusan National University, Busan, Korea

Summary
Power management is one of the criteria which characterize any existing wireless communications device in the market. It has been a concern of design engineers and has been on continual development for a number of years now. Modules with rechargeable power source has been seen in the market recently giving rise to what is so called as "perpetually" operating devices. This paper proposes a battery model for a solar cell replaceable power source taking account the time of day as well as the duty cycle of the device. With this model, routing protocols, clustering algorithms and other networking power saving methods can be devised.

Key words:
Battery Model, Photovoltaic Cell, Wireless Sensor Network

1. Introduction

A wireless sensor network is a large number of power-limited sensor nodes which are distributed over some specific area to gather information about a certain observable event. Due to the power limited nature of each sensor node, power efficiency is one of the major merit factors for commercially available devices as well as for wireless communication standards. Also for some applications, the location of the sensors, as well as the method of deployment, would make it hard to replace power sources each time a node loses power. Renewable sources are starting to gain popularity with regards to the powering nodes for various applications. Solar energy is a major source of energy especially for outdoor wireless sensor applications. This is primarily due to the periodicity and reliability of solar energy systems. Several years ago, photovoltaic cells (PV), as a means of an endless source of energy, are viewed as an expensive alternative to the customary and limited Lithium batteries. Due this cost issue, engineers opted to use the limited battery source arguing that it would be cheaper to let the nodes "die" than to employ photovoltaic cells to recharge the power source. Recent developments have changed this tough scenario into a favorable one for solar energy. With the increase in the need for longer lasting nodes, solar energy is one of the alternatives.

2. Background and Related Works

Power efficiency has been one of the vast areas of research when one talks about wireless sensor networks. Numerous MAC and routing protocol standards have been researched and implemented to solve the problem of power efficiency. Some of them is discussed in [1], [2], [3], [4], [5] and [6]. However, in the end, power efficiency solutions only solve problem on how to make power last given a specific amount of energy. Majority of papers that were published deals with customary batteries which cannot be sustained. This is under the assumption that the utilization of perpetual energy sources such as solar energy would prove to be more expensive as compared to letting the nodes die. Standard batteries can be modeled in several ways according to a number of parameter i.e. temperature, capacity, rate discharge among others. [2] discusses some models as well as classifies them according to the method in arriving at a model. This paper, as well as [5], also discusses about several battery properties that describes battery operation during discharge as well as during idle times. Charge recovery effect is one of the battery properties most often considered in battery modeling of standard batteries as in [3], [4], [5] and [6]. Charge recovery can be measured using experimental means and was modeled using stochastic equations by [3] and [4] using the discharge profile provided by battery manufacturers. In [1] and [6] the battery is modeled to serve as a transmission cost in building a framework for a sensor network in [1] or applying it to an existing protocol in [6].

With developments of technology as well as necessity, designs and theory of utilizing an endless energy source are being produced. Despite currently being disadvantaged in terms of cost, photovoltaic cell attributes and its great potential to reduce its current price gives it advantage, motivating manufacturers and engineers to design sensor

†††† Corresponding author
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nodes with renewable power source. In addition to being an unlimited source, solar energy from photovoltaic cells is a environmentally clean and safe source of energy. Prometheus in [7] is one of the researches and implementation of a sustainable energy source for wireless sensor nodes. This project utilizes photovoltaic cells to sustain energy buffers in the form of super capacitors and standard Li+, NiCad or NiMH rechargeable batteries. Combined with power efficient algorithms, we can increase the lifetime of sensor network nodes using charging mechanisms. To model the battery several factors, as well as concepts, are needed to ensure accuracy. The proceeding the discussions is divided into five parts. The first discussion would be regarding battery mechanisms and their effects during the discharge of battery. Solar energy fundamentals would be discussed in the second part of the discussion focusing on parameters affecting solar energy efficiency and energy delivery. The Markov model formulated would be discussed in the fourth part of the discussion. Conclusion would be brought out in the last section.

3. Solar Battery Modeling and Analysis

3.1 Battery Mechanism

To model battery accurately, one must understand the mechanisms that take effect during battery utilization. These mechanisms lets us know how batteries behave in a given conditions. Two main mechanisms of batteries to consider are rate capacity effect and recovery effect [2], [6]. These two mechanisms are dependent on the discharge profile of a specific battery. Discharge profile are usually provided by battery manufacturers with information on the amount of time the battery voltage fall to a certain threshold voltage, i.e. the amount of time the battery reaches a discharged or "empty" state.

To begin with, we must understand the nature at which the battery discharges its energy. The battery is an electrochemical device which allows storage of energy using the battery's chemical characteristics. During a discharge situation, the battery is attached to some load which provides a path for charges to follow. These charges are produced by chemical reactions in the battery's composition. In rechargeable type of batteries, an externally applied supply current can be applied to the battery to reverse the chemical process of discharging. Manufacturers usually classify batteries according to their rated capacity which is the amount of charge a battery can store. This amount of charge can be measured in A-hours (3600 Coulombs). This capacity is dependent on the amount of current being supplied as well as in the current charge state of the battery.

This manufacturer-provided capacity does not necessarily equal the amount of charge delivered to the load. Some battery characteristics influence the usable capacity, termed actual capacity [2], delivered by a battery at a given temperature. Self-discharge can occur which brings down the actual capacity of the battery. This property of batteries is heavily dependent on temperature causing higher self-discharge rate in tropical countries. Rechargeable batteries exhibit higher self-discharge rate than standard batteries (about 2-3% a day). Due to this, rechargeable batteries alone are not enough to replace a standard alkaline battery in a network environment. Sustainable recharging mechanism would provide a much needed leverage in terms of network lifetime. Rate Capacity Fading is also one the factors that affects how much energy a battery can provide. Rate Capacity Fading describes the observed data that the larger the discharge current drawn from the power source, the less the capacity delivered. This is also known as Peukert effect. The Peukert Effect can be described by Eq. (1).

$$C=I^{n}T \tag{1}$$

where,

C is the rated capacity of the battery
I is the discharge current
T is the runtime
n is Peukert's exponent
$C=I^{n}T$

In [9], a modification of Eq. (1) gives a more realistic way to use the Peukert formula according to the rated capacity given by battery manufacturer. The equation as used by Smart Gauge is

$$T = C(C/R)^{1-n}/I^{n} \tag{2}$$

where

R is the battery hour rate at which the rated capacity is taken

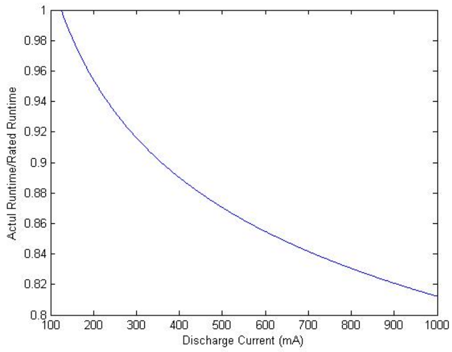

Fig. 1 Rate Capacity Fading

$$P_R = e^{-k(N-q/N)} \quad \text{for } q \neq N \qquad (3)$$
$$= 0 \quad \text{for } q = N, q=0$$

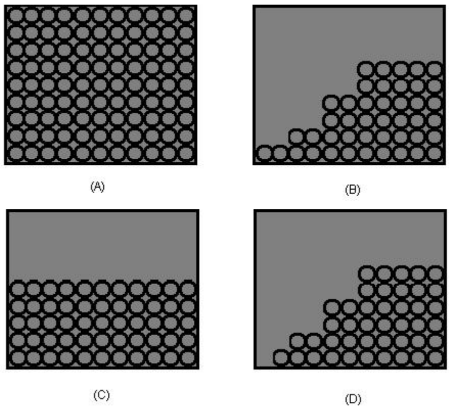

Fig. 2 Recovery effect visualization. (A) fully-charged, (B) discharging state, (C) recovered state, and (D) depleted state.

From the Fig. 1, we can observe that as the discharged current is increased, the difference between the actual runtime and rated runtime also increase. This can be explained in conjunction with another battery characteristic, Battery Recovery Effect. Battery effect occurs when the charge level inside battery composition balances out during diffusion. During the discharge process, charges near the anode flows out of the battery in a rate faster than the internal diffusion in the battery composition. The result is an internal imbalance of charges which can be graphically shown by Fig. 2B. When the battery is in the idle state, the electrons inside the electrolyte would diffuse to equalize the concentration of charges as in Fig. 2C. When the used continuously, the charges near the anode would be depleted thereby stopping the transfer of electrons from the battery. Thus the battery would be in discharge or 'empty" state (Fig. 2D). Even if there are still charges present, the battery cannot supply any charges anymore because there are no more charges near the anode. The rate of diffusion is much less than the rate at which charges are discharged. The probability that a recovery would occur is Eq. (3). *q* describes the amount of discharge of the battery. *N-q/N* describes what fraction of the capacity remains in the battery. The probability that a battery recovery process would occur during a certain idle time is less when the amount of charges in the battery is small. *k* defines a constant which is dependent on the battery used. Since the diffusion rate us less than the discharge rate, recovery effect can explain the reason of energy inefficiency during continuous utilization of the battery as described by the rate capacity fading.

3.2 Solar Energy

The amount of solar energy that a certain photovoltaic cell can provide is dependent on several factors. Since solar energy is a natural resource, it is heavily dependent on the environmental conditions which are generally random in nature. To simplify the model, some parameters or conditions were assumed. Firstly, the temperature dependence of the solar energy is assumed to be negligible. The parameter we would focus on is the relationship between time and the amount of energy provided by the photovoltaic cell. We begin with the equation that describes the flux intensity for a photovoltaic cell. The flux intensity equation [8] is defined by Eq. (4).

$$I(z) = I_O e^{-c(\sec z)^S} \qquad (4)$$

where

I(z) = Flux Intensity in kW/m2
I_o = Exoatmospheric solar flux (1.353kW/m2)
Z = zenith distance
C = 0.357
S = 0.678

Solar flux intensity is a measure of the energy which is absorbed by a photovoltaic cell. S and c are empirical data numerical constants while Io is the flux intensity outside the earth's atmosphere. The solar flux intensity data can be used to know how much energy is provided by the cells. From [8], at constant voltage, increasing the amount of solar flux intensity would also increase the amount of current supplied to the load. With this in mind, one can know the amount of current being supplied to charge a rechargeable battery.

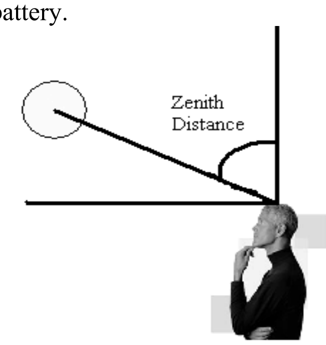

Fig. 3 Zenith distance.

The zenith distance is the angular distance from the position of the sun directly above a spectator in Fig. 3. This parameter is dependent on the time of day. Eq. (5) [8] describes the zenith distance *z*. λ is the latitude of the collector site and δ is the solar declination. Solar declination is angle between the earth-sun line and the equatorial plane in Fig. 4.

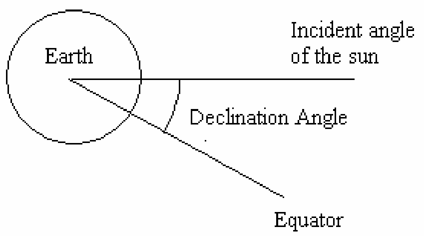

Fig. 4 Declination angle.

Due to a 23.45-degree tilt of the earth's equatorial line with respect to the earth's orbit, there would be variation of the solar declination throughout the year which causes seasons. The value of the declination angle can be approximated at about 23.50 during summer, about 00 during equinox, and -23.50 during winter. A graphical representation is provided by the University of Southern Mississippi in [8] showing the declination angle throughout the year. Day 0 is defined as January 1. *t* is defined as the hour- angle which defines the hour of day. To define this angle, we define *t* as Eq. (6) where *T* is the number of hours from solar noon(highest point of the sun).

$$\cos z = \sin\lambda\sin\delta + \cos\lambda\cos\delta\cos t \tag{5}$$

$$t = (360/24)T \tag{6}$$

The transition model for the battery is illustrated in Fig. 5.

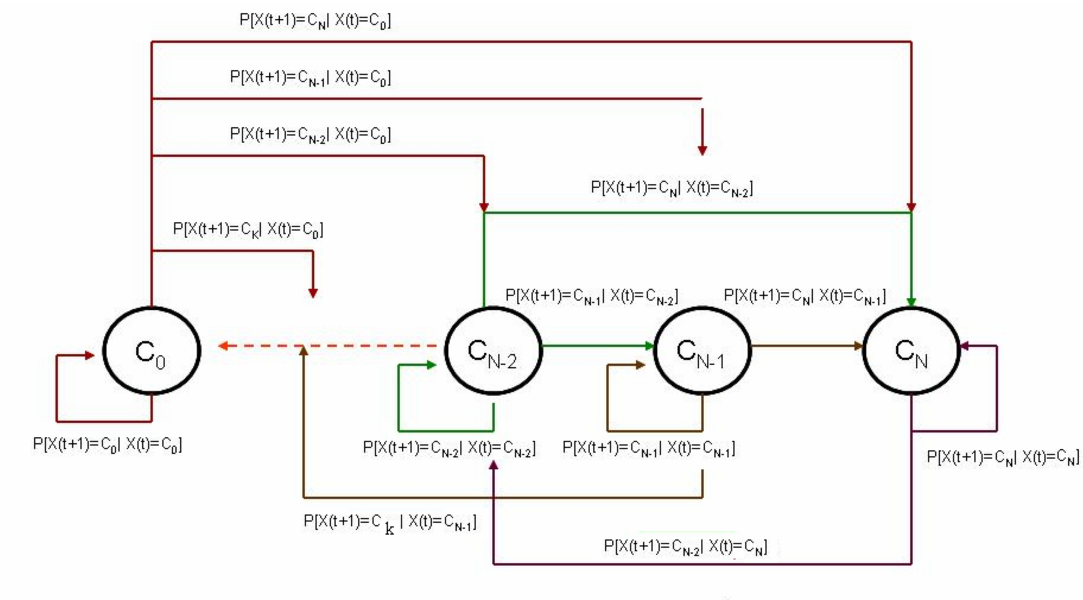

Fig. 5 Markov chain model of battery process

The states C_N, C_{N-1},C_{N-2},…C_0 represents the battery charge states with C_N denoting the fully-charged state while C_0 corresponds to the depleted or discharged state of the battery. The directional arrows represent the transition with the probability of making those transitions indicate above each arrow. To get these probabilities, equations defined above would be used. Some assumptions were made in formulating the model in Fig. 5. First, we assume that for every discharge process, two charge units are provided by the battery to the load. Second, the sensor network senses parameters in a periodic time, that is, it senses in a cycle. Third, the amount of charge recovered in a specific time is one charge unit. Lastly, we assume that for a certain season, the solar flux intensity behavior for is periodic for each day. During the whole process, two independent events are involved: a) battery is sourcing energy and b) solar energy charging. These events, *a* and *b*, are not mutually exclusive such that these events may happen simultaneously. The three sub-events of solar charging, namely recovery, discharge and no activity are mutually exclusive. A direct way to describe this is that while the battery is charging through the photovoltaic cells, one o the battery processes, namely discharging, battery recovery or no activity is occurring.

To model the battery, equation from discussion A and B are used. We begin with state C_N. *N* is the maximum amount of charge stored by a battery. Therefore, the battery cannot charge any more than the value of *N*. With this in mind, the probability that solar charge or battery recovery would occur is 0. At state C_N, the battery can either have a loss of charge or stay at the same state. For C_0, only solar charge or a no change can happen because this is the depleted state of the battery. To find each transition probability, we must determine each combination of events that can lead to each state. Also, we must initially find the probability of each component of the events.

For the case of solar charging, we need to use the solar flux intensity equation as given in Eq. (4). To get the probability, we need to express time in terms of the solar flux intensity. Re-arranging Eq. (4), we get Eq. (8). Combining this with Eq. (5), we can finally arrive at Eq. (9). From Eq. (6), we plot the solar flux intensity against the number hours from noon as shown in Fig. 6. The plot is symmetric about the x-axis which takes into account the energy supplied in the morning. The plot describes the solar flux intensity throughout the day. Looking at Fig. 6, we can observe that at different levels of solar intensity, the photovoltaic cell can provide an amount of current at constant voltage per time slot. This observation leads us to the idea that depending on the value of a charge unit, the photovoltaic cells can provide a discrete number of charge units. To illustrate the idea, suppose that one charge unit has a value of 10mAh and the time slot length is one hour. Looking at figure three, we can safely say that at the solar flux intensity range from 750W to 1250W, the PV cells can provide approximately 40mAh which is four charge units. In the same way, we can find the range from which the PV cells can provide 1, 2 or 3 charge units. The probability of the PV supplying a given number of charges (CH) given that the PV is supplying energy is $P_{CHARGE}(I)$ Eq. (10) which is the energy from a to b (the range at which the PV is supplying CH) divided by the total energy for the day. We get energy per day because we assumed that the solar flux intensity for a given season is periodic on a day to day basis. We need to find the probability that a solar charging process is occurring to determine the probability in a day that the battery is charging CH charges. Using Eq. (9) and Eq. (6), we can have Eq. (7) which defines the equation that describes how much time in a day the PV cell is supplying energy.

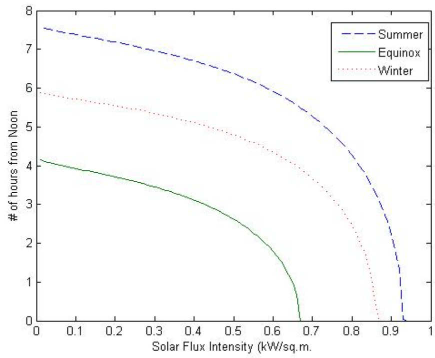

Fig. 6 Solar flux intensity vs the number of hours from noon

The sensor node duty cycle is $P_{discharge}$, the probability that the node is discharging the power of the battery. $1-P_{discharge}$ is the probability of having an idle time for the sensor node. At this idle time, the sensor node can either be recovering (if the concentration of charge is not uniform) or have no activity (uniform concentration). The probability of recovery and no activity are given by Eq. (10) and Eq. (11) respectively where P_R is given by Eq. (3). Now we have the values of the probabilities for all the events involved. The transition probability of arriving at a state C_k given that the current state is C_j is determined by finding the combinations of events to arrive at a *k* state.

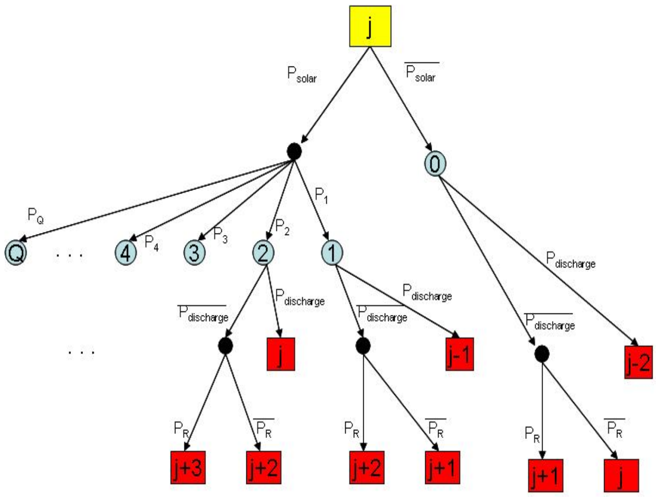

Fig. 7 Probability tree for the battery process

Fig. 7 shows a probability tree which depicting the combinations of these events as well as their probability. The yellow box with level *j* represents the current state of the battery. The red boxes in the lowest levels of the tree represents the *k* to which we could transition to. To get the probability of transitioning from C_j to C_k, we add the probabilities of the independent events that results from going from *j* charges to *k* charges. For example, if wants to find the probability of transition from C_j to C_{j+1}, one must find all the path in the tree from j to $j+1$ then sum the probabilities derived from these paths. This would give us Eq. (13).

4. Simulation Results

Simulations were conducted using Matlab as the calculation engine as well as the plotting software. Simulations were done to plot the change in capacity with respect to time during the operation of a rechargeable battery. Simulations were done where in solar charging, battery recovery and battery discharge is present. Some assumptions were made in the simulations for each scenario. A general assumption is the number of charge units utilized, recovered or regained by each of battery process. As we know, the amount of charge units gained or lost is random and is based on how much charge units are needed as well as how much charge is provided. In the simulation, we assume a constant range of the rate of recovery, discharge and solar charging. Also, the range of the rate has a linear relationship.

$$P_{solar} = t(0)/\pi \tag{7}$$

$$\sec z = \left[\frac{-\ln\left(\frac{I}{I_0}\right)}{c} \right]^{\frac{1}{S}} \tag{8}$$

$$t(I) = \begin{cases} \cos^{-1}\left[\dfrac{1 - \left[\dfrac{\left(-\ln\left[\frac{I}{I_0}\right]\right)}{c} \right]^{\frac{1}{S}} \sin\lambda \sin\delta}{\left[\dfrac{-\ln\left(\frac{I}{I_0}\right)}{c} \right]^{\frac{1}{S}} \cos\lambda \cos\delta} \right] & I < I_0 e^{-c/\left(\cos(\lambda-\delta)\right)^S}, I \neq 0 \\ \lim\limits_{I \to 0} \cos^{-1}\left[\dfrac{1 - \left[\dfrac{\left(-\ln\left[\frac{I}{I_0}\right]\right)}{c} \right]^{\frac{1}{S}} \sin\lambda \sin\delta}{\left[\dfrac{-\ln\left(\frac{I}{I_0}\right)}{c} \right]^{\frac{1}{S}} \cos\lambda \cos\delta} \right] & I = 0 \\ 0 \quad elsewhere \end{cases} \tag{9}$$

$$P_{CHARGE}(I) = \frac{\int_a^b t\,dI}{2\int_0^1 t\,dI} \tag{10}$$

$$P_{recovery(n)} = (1-P_{discharge})(P_R) \tag{11}$$

$$P_{no\ activity(n)} = (1-P_{discharge})(1-P_R) \tag{12}$$

$$P[X(t+1) = C_{j+1} \mid X(t) = C_j] = \overline{P_{solar}}\,\overline{P_{discharge}}\,P_R + P_{solar} P_1 \overline{P_R} + P_{solar} P_3 \overline{P_{discharge}} \tag{13}$$

The following is the general flow of the simulation process. The initial time state in simulation is 0.

1. Set constants and variables according to scenario.
2. Determine the next time interval where in discharge would occur. This is a randomized determination of interval. The range of the interval is also set.
3. Determine the solar flux absorbed by the photovoltaic cells each hour according to the value of the time interval from step 2. Get the amount of charge gained. Adjust the increment according to this amount.
4. Get a randomized probability of the amount of discharged units. This would determine the amount of current discharged for a certain discharge event. Adjust the increment according to this probability.
5. Get the probability that a recovery occurred during the idle state. Unlike the solar charging in step 3, this event should only occur once in every idle state and not in a per-hour basis. Adjust the increment according to this probability.
6. Update the current capacity according to this adjusted increment value.
7. Repeat process until capacity is equal to 0.

In the first scenario, we aim to see the performance of our model in terms of varying capacity and time of discharge. Fig. 8 shows the graph of the first scenario. In this scenario we vary the amount of initial charge unit capacity (full charge load). In addition, we can also look at this graph to see what time instant a specific state of capacity would occur given a certain full load capacity. As one can observe in Fig. 1, the data taken and plotted is raw data and not averaged. The reason for this is to observe the instantaneous events. The rough edges in the graph describe sudden changes in the charging and discharging of the battery. Although we have a linear range, we note that the combinations of these ranges as indicated by increment adjustments results in random amount of charge gained or lost. This produces spur like data to be plotted which represents the variation of charge gained and lost.

Note also that the capacity values used in the simulation are relatively small compared to actual battery capacity. Also, the increment adjustment is set such that the amount of charge gained is comparatively less than the amount of charge lost which is also what happens in reality on the average. Also, looking at the figure, one can see the linear nature of plots with varying values of capacity. This is due to the linear relation of the rate ranges.

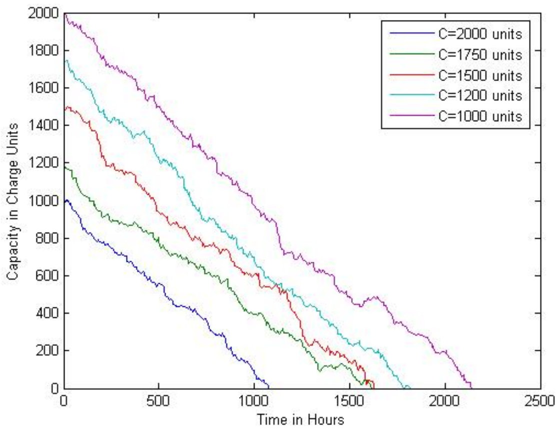

Fig. 8 Capacity vs time plot.

In the second scenario, we aim to compare the amount of time it would take for the battery to get fully discharged. As in the first scenario, we present raw data. Also, we set the full-charge capacity to 1000 charge units. As we can see from the Fig. 2, the runtime for simulation with solar charging is double that of the ordinary setup. This is based on a conservative increment where in the amount of discharged units are relatively larger than charges gained for each event.

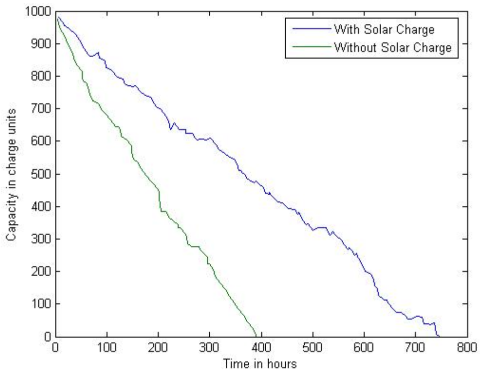

Fig. 9. Solar charging vs no solar charging

In the third scenario, we vary the frequency of the discharge. The frequency of a discharge event is determined by the time interval as determined by step 2 in the simulation process. In the previous scenarios, we have set up the simulation such that a discharge event occurs at least once every 12 hours. On this third scenario, we want to vary the discharge rate and see how it affects the time it takes to fully discharge the battery. Increasing the discharge rate yields an increase in the sloe of the time-capacity plot giving larger amounts of charge loss. Also, we need to note that due to increase in discharge rate, the rate of recovery also decreases.

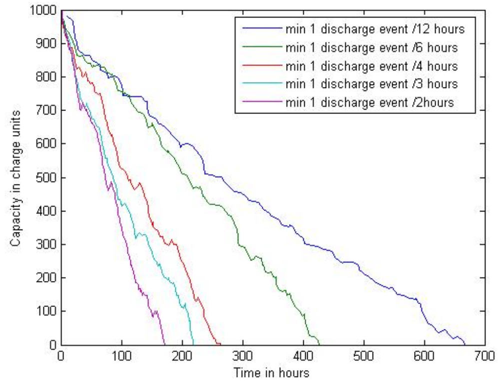

Fig. 10. Variable discharge rate.

Finally, we look at the effect on the time of year or season in the time-capacity plot. The previous scenarios use summer as the working scenario. Fig. 11 describes the behavior of the time-capacity plot in response to change in season. The change in season can be implemented by varying the declination angle δ in Eq. (5). Since photovoltaic cells can sustain more energy during the summer, the runtime in summer should be longer as compared to spring or winter. This is seen in the figure. However, this is assuming a fixed discharge rate throughout the year.

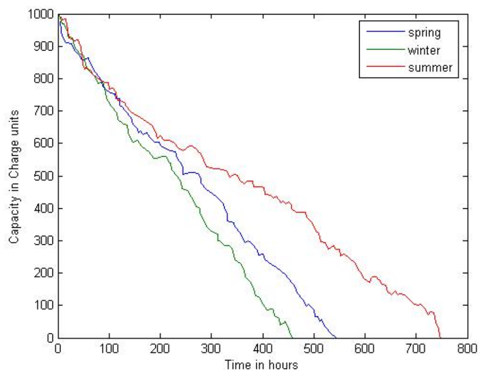

Fig. 11 Season variation.

5. Conclusion

In this paper, a mathematical model for a wireless sensor node with photovoltaic cell power source using Markov Chain model. The secondary buffer is an ordinary rechargeable battery (preferably Ni-MH). This model can be used to predict the state of the battery during running condition as well as determining the amount of charge the battery has stored. Formulation of the battery model a super capacitor primary buffer similar to [7] can be done for future work. Another development to be done is the formulation of a battery model which also takes note of the effect of ambient temperature and geographical height which also affects the amount of energy supplied by a photovoltaic cell array.

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JongGyu Kim received the B.S. and M.S. degrees in Computer Engineering from Pusan National University, Korea in 1992 and 1999, respectively and complete Ph.D. course in Computer Engineering from Pusan National University, Korea, in 2002. He is an Assistance Professor of Information Technology at Yeungjin College, Korea. He also is an IT director of Asian Summit College in the Philippines. He conducts research in the area of home networks, wireless sensor networks and optimization of communication systems.

John Paul Torregoza received his bachelor’s degree in Electronics and Communications Engineering the University of the Philippines, Diliman, republic of the Philippines, in 2002. He is now a MS student at Inje University, Gimhae, Korea. His research interests are in sensor networks and routing protocols.

In-Yeup Kong received her bachelor’s degree and M.S. degree in Computer Engineering from Pusan National University, Pusan, Republic of Korea, in 2000 and 2002. She is a Ph.D candidate in Pusan National University. Her research interests are in IPv6, IPv6/IPv4 translator, home network, and cross layer design. She is a member of KISS, KICS Korea.

Won Joo Hwang received his Ph.D Degree from Osaka University Japan in 2002. He received his bachelor’s degree and M.S. degree in Computer Engineering from Pusan National University, Pusan, Republic of Korea, in 1998 and 2000. He is the professor of Inje University in Korea. His research interests are in sensor networks and ubiquitous sensor networks. He is a member of KISS, KICS Korea, and a member of IEICE Japan.