Exponential Stability of BAM Neural Networks with Delays and Impulses

Deqin Chen and Kelin Li

Department of Mathematics, Sichuan University of Sience & Engineering, Sichuan 643000, P.R. China

Summary
In this paper, a generalized model of bi-directional associative memory (BAM) neural networks delays and impulses is investigated. By constructing suitable Lyapunov functional, Halanaly differential inequality and M -matrix theory, some sufficient conditions for global exponential stability of generalized BAM neural networks with delays and impulses are obtained. An examples are given to show the effectiveness of the obtained results.

Key words:
BAM neural networks; delays;impulses; global exponential stability.

1. Introduction

The bidirectional associative memory (BAM) neural network models were first introduced by Kosko [1-3]. It is a special class of recurrent neural networks that can store bipolar vector pairs. The BAM neural network is composed of neurons arranged in two layers, the X -layer and Y -layer. The neurons in one layer are fully interconnected to the neurons in the other layer, while there are no interconnections among neurons in the same layer. Through iterations of forward and backward information flows between the two layer, it performs a two-way associative search for stored bipolar vector pairs and generalize the single-layer autoassociative Hebbian correlation to a two-layer pattern-matched hetero-associative circuits. Therefore, this class of networks possesses good applications prospects in the areas of pattern recognition, signal and image process, artificial intelligence [1]. It is well known, in both biological and man-made neural networks, the delays arise because of the processing of information [4-11]. Time delays may lead to oscillation, divergence, or instability which may be harmful to a system. Therefore, study of neural dynamics with consideration of the delayed problem becomes extremely important to manufacture high-quality neural networks.

However, besides delay effect, impulsive effect likewise exists in a wide variety of evolutionary processes in which states are changed abruptly at certain moments of time, involving such fields as medicine and biology, economics, mechanics, electronics and telecommunications, etc. Many interesting results on impulsive effect have been gained, e.g., Refs. [8-20]. As artificial electronic systems, neural networks such as CNN, bidirectional neural networks and recurrent neural networks often are subject to impulsive perturbations which can affect dynamical behaviors of the systems just as time delays. Therefore, it is necessary to consider both impulsive effect and delay effect on the stability of neural networks.

Motivated by the above discussions, we intend to study the global exponential stability of generalized BAM neural networks with delays and impulses in a very general setting. Under quite general conditions, we apply the idea of vector function, M-matrix theory and Hananaly differential inequality, by constructing suitable Lyapunov functional, several new sufficient conditions are obtained to ensure the existence, uniqueness, and global exponential stability of equilibrium point for the generalized BAM neural network with delays and impulsese. These results generalize and improve the earlier publications.

The paper is organized as follows. Model description and preliminaries are given in section 2. In section 3, main results and their proofs are presented. Some remarks are given to compare with the earlier references, and an example are given to illustrate our theory in section 4. Finally, in section 5 we give the conclusion.

2. Model description and preliminaries

Consider the generalized BAM neural networks with delays and impulses:

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$$\begin{cases} \dfrac{dx_i(t)}{dt} = -a_i x_i(t) + \sum_{j=1}^{m} c_{ij} f_j(y_j(t)) \\ \qquad - \sum_{j=1}^{m} \tilde{c}_{ij} f_j(y_j(t-\tau_{ij})) + I_i, \quad t>0, t \neq t_k, \\ \Delta x_i(t_k) = P_k(x_i(t_k)), \quad i=1,2,\cdots,n, \ k=1,2,\cdots \\ \dfrac{dy_j(t)}{dt} = -b_j y_j(t) + \sum_{i=1}^{n} d_{ji} g_i(x_i(t)) \\ \qquad + \sum_{i=1}^{n} \tilde{d}_{ji} g_i(x_i(t-\sigma_{ji})) + J_j], \ t>0, t \neq t_k, \\ \Delta y_j(t_k) = Q_k(y_j(t_k)), \quad j=1,2,\cdots,m, \ k=1,2,\cdots \end{cases} \tag{1}$$

where $x_i(t)$ and $y_j(t)$ are the state variable of the i th neuron and the j th neuron at time t, respectively; $f_j(x_j(t))$ and $g_i(y_i(t))$ denote the signal functions of the j th neuron and the i th neuron at time, respectively; $a_i > 0, b_j > 0$, $c_{ij}, \tilde{c}_{ij}, d_{ij}, \tilde{d}_{ij}$ are constants, a_i and b_j represent the rate with which the i th neurons and the j th neurons will reset their potential to the resting state in isolation when disconnected from the networks and external inputs, respectively; $c_{ji}, \tilde{c}_{ji}, d_{ij}$ and $\tilde{d}_{ij}$ denote the connection weights. I_i and J_j denote the external bias on the ith unit and j th unit, respectively; constants τ_{ji} and σ_{ij} represent transmission delay, respectively. $\Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-)$, $\Delta y_j(t_k) = y_j(t_k^+) - y_j(t_k^-)$ are the impulses at moment t_k and $t_1 < t_2 < \cdots$ is strictly increasing sequence such that $\lim_{k\to\infty} t_k = +\infty$.

The initial conditions associated with system (1) are of the form:

$$\begin{cases} x_i(s) = \phi_i(s), & s \in [-\tau, 0], \ i=1,2,\cdots,n, \\ y_j(s) = \varphi_j(s), & s \in [-\tau, 0], \ j=1,2,\cdots,m, \end{cases} \tag{2}$$

where $\phi_i(s), \varphi_i(s)$ are bounded continuous functions on $[-\tau, 0]$, $\tau = \max_{1\le i\le n, 1\le j\le m} \{\tau_{ji}, \sigma_{ij}\}$. Denote

$$\phi = (\phi_1, \phi_2, \cdots, \phi_n)^T, \ \varphi = (\varphi_1, \varphi_2, \cdots, \varphi_m)^T,$$

Let (x^*, y^*) be the equilibrium point of the system (1), for $r \ge 1$, define the norm as the following:

$$\left\| \phi - x^* \right\| = \sup_{-\tau\le s\le 0} \sum_{i=1}^{n} \left| \phi_i(s) - x_i^* \right|^r,$$

$$\left\| \varphi - y^* \right\| = \sup_{-\tau\le s\le 0} \sum_{j=1}^{m} \left| \varphi_j(s) - y_j^* \right|^r.$$

To obtain our results, we give the following assumption:

(**H1**) There exist positive constants F_j and G_i such that

$$| f_j(u) - f_j(v) | \le F_j \, | u - v |, \quad j=1,2,\cdots,m,$$

$$| g_i(u) - g_i(v) | \le G_i \, | u - v |, \quad i=1,2,\cdots n,$$

for any $u, v \in R$.

(**H2**) The impulsive operators $P_k(x_i(t_k))$, $Q_k(y_j(t_k))$ satisfy

$$\begin{cases} P_k(x_i(t_k)) = -\delta_{ik}(x_i(t_k) - x_i^*), \ 0 < \delta_{ik} < 2, \\ Q_k(y_j(t_k)) = -\eta_{jk}(y_j(t_k) - y_j^*), \ 0 < \eta_{jk} < 2, \end{cases}$$

for $i=1,2,\cdots,n$, $j=1,2,\cdots,m$, $k=1,2,\cdots$.

For convenience, we introduce several notations. For a $n \times n$ matrix A, $|A|$ denotes the absolute value matrix given by $|A| = (|a_{ij}|)_{n\times n}$, For the system (1), we denote

$$z = (z_1, z_2, \cdots, z_{n+m})^T = (x_1, \cdots, x_n, y_1, \cdots, y_m)^T,$$

$$h = (h_1, h_2, \cdots, h_{n+m})^T = (g_1, \cdots, g_n, f_1, \cdots, f_m)^T,$$

$$I = (I_1, I_2, \cdots, I_{n+m})^T = (I_1, \cdots, I_n, J_1, \cdots, J_m)^T,$$

$$\psi = (\psi_1, \psi_2, \cdots, \psi_n)^T = (\phi_1, \cdots, \phi_n, \varphi_1, \cdots, \varphi_m)^T,$$

$$\alpha = diag(a_1, a_2, \cdots, a_n, b_1, b_2, \cdots, b_m),$$

$$C = (c_{ij})_{n\times m}, \tilde{C} = (\tilde{c}_{ij})_{n\times m}, D = (d_{ji})_{m\times n},$$

$$\tilde{D} = (\tilde{d}_{ji})_{m\times n}, \ A = \begin{pmatrix} 0 & C \\ D & 0 \end{pmatrix} = (a_{ij})_{(n+m)\times(n+m)},$$

$$B = \begin{pmatrix} 0 & \tilde{C} \\ \tilde{D} & 0 \end{pmatrix} = (b_{ij})_{(n+m)\times(n+m)}$$

Definition 1. $z(t) = (x(t), \ y(t))^T = (x_1(t), \cdots, x_n(t), y_1(t), \cdots, y_m(t))^T$ is said to be a solution of (1), if it satisfies the following conditions:

(i) $x_i(t)$ and $y_j(t)$ are piecewise continuous functions with first-discontinuity at t_k ,and at each discontinuity point they are continuous on the left, i.e. $x_i(t_k^-) = x_i(t_k)$, $y_j(t_k^-) = y_j(t_k)$, for $i=1,\cdots n, j=1,\cdots,m, k=1,2,\cdots$.

(ii) $x_i(t)$ and $y_j(t)$ satisfy (1) and (2).

In particular, a constant solution $z^* = (x^*, y^*)^T$ of (1) is called an equilibrium point of model (1).

Definition 2. The equilibrium point $z^* = (x^*, y^*)^T$ of model (1) is said to be globally exponentially stable if there exist two positive constants $M \ge 1$ and λ such that

$$\sum_{i-1}^{n}\left|x_i(t)-x_i^*\right|^r + \sum_{j=1}^{m}\left|y_j(t)-y_j^*\right|^r \le Me^{-\lambda t}\left(\left\|\phi - x^*\right\| + \left\|\varphi - y^*\right\|\right), \quad t \ge 0.$$

Definition 3. ([21]) A real matrix $A = (a_{ij})_{n\times n}$ is said to be a nonsingular M -matrix if $a_{ij} \le 0, i, j = 1,2,\cdots,n, \quad i \ne j$, and all successive principal minors of A are positive.

Lemma 1. ([21,22]) D is a nonsingular M -matrix if and only if the diagonal elements of D are all positive and there exists a positive vector l such that $Dl > 0$ or $D^T l > 0$.

Lemma 2. ([20])Let $C = (c_{ij})_{n\times n}$ and $D = (d_{ij})_{n\times n}$ be two real matrices, $y(t)$ is the solution of differential inequality

$$\frac{dy(t)}{dt} \le Cy(t) + D\overline{y}(t),$$

where $\overline{y}(t) = (\overline{y}_1(t),\cdots,\overline{y}_n(t))^T, \overline{y}_i(t) = \sup_{t-\tau\le s\le 0}\{y_i(s)\}$.
If $c_{ij} \ge 0 (i \ne j), d_{ij} \ge 0$, and $-(C + D)$ is a nonsingular M -matrix, then there exist a positive constant $\lambda > 0$ and a positive vector $k \ge \overline{y}(0)$ such that

$$y(t) \le ke^{-\lambda t}$$

for $t \ge 0$.

3. Main results

As $\Delta x_i(t_k) = 0, \Delta y_j(t_k) = 0$, ($i = 1,\cdots,n, j = 1,\cdots,m$), in model (1), then model (1) reduce to the usual BAM neural networks with dalays:

$$\begin{cases} \dfrac{dx_i(t)}{dt} = -a_i x_i(t) + \displaystyle\sum_{j=1}^{m} c_{ij} f_j(y_j(t)) \\ \qquad - \displaystyle\sum_{j=1}^{m} \tilde{c}_{ij} f_j(y_j(t-\tau_{ij})) + I_i, \quad t > 0, \\ \dfrac{dy_j(t)}{dt} = -b_j y_j(t) + \displaystyle\sum_{i=1}^{n} d_{ji} g_i(x_i(t)) \\ \qquad + \displaystyle\sum_{i=1}^{n} \tilde{d}_{ji} g_i(x_i(t-\sigma_{ji})) + J_j], \ t > 0, \end{cases} \tag{3}$$

for $i = 1,\cdots,n, j = 1,\cdots,m$. For (3), we have

Lemma 3. [9] Under assumption (H), the system (3) has at least one equilibrium point, if

$$\alpha - (|A| + |B|)L$$

is a nonsingular M -matrix. Where

$$L = diag(G_1, G_2, \cdots, G_n, F_1, F_2, \cdots F_n).$$

Let $z^* = (x^*, y^*)$ be the equilibrium point of model (3), then we hav

Theorem 1. Under assumptions (**H1**) and (**H2**), model (1) has a unique equilibrium point, which is globally exponentially stable if

$$\alpha - (|A| + |B|)L$$

is a nonsingular M -matrix, where

$$L = diag(G_1, G_2, \cdots, G_n, F_1, F_2, \cdots F_n).$$

Proof. Since

$$\begin{cases} P_k(x_i^*) = 0, i = 1,2,\cdots,n, k = 1,2,\cdots, \\ Q_k(y_j^*) = 0, \ j = 1,2,\cdots,m, k = 1,2,\cdots, \end{cases}$$

$z^* = (x^*, y^*)$ is also the equilibrium point of the system (1).

Case 1. when $t > 0, \ t \ne t_k, \ k = 1,2,\cdots$.

Let $z(t) = (x(t), y(t))^T$ be any solution of the system (1), then we have

$$\frac{d(z_i(t)-z^*)}{dt} = -\alpha_i(z_i(t)-z^*) + \sum_{j=1}^{n+m} a_{ij}[h_j(z_j(t)) - h_j(z_j^*)] + \sum_{j=1}^{n+m} b_{ij}[h_j(z_j(t-\rho_{ij})) - h_j(z_j^*)] \tag{4}$$

Multiplying both side equation (4) by $\operatorname{sgn}(z_i(t) - z_i^*)$, we get

$$D^+\left|z_i(t)-z_i^*\right| \le -c_i\left|z_i(t)-z_i^*\right| + \sum_{j=1}^{n+m} l_j\left|a_{ij}\right|\cdot\left|z_j(t)-z_j^*\right| + \sum_{j=1}^{n+m} l_j\left|b_{ij}\right|\cdot\left|z_j(t-\tau_{ij})-z_j^*\right| \quad (5)$$

Set $\omega(t) = (\omega_1(t), \omega_2(t), \cdots, \omega_{n+m}(t))^T$, where

$$\omega_i(t) = \begin{cases} \left|z_i(t)-z_i^*\right|, & t>0 \\ \left|\psi_i(t)-z_i^*\right|, & -\tau \le t \le 0 \end{cases}$$

then we have

$$\dot{\omega}(t) \le -C\omega(t) + |A|L\omega(t) + |B|L\overline{\omega}(t), \quad (6)$$

where $\overline{\omega}(t) = (\overline{\omega}_1(t), \overline{\omega}_2(t), \cdots, \overline{\omega}_{n+m}(t))^T$, $\overline{\omega}_i(t) = \sup_{t-\tau\le s\le 0}\{\omega_i(s)\}$.

Since $\alpha - (|A|+|B|)L$ is a nonsingular M -matrix, from Lemma 2, we know that there exist a constant $\lambda > 0$ and a positive vector $k = (k_1, k_2, \cdots, k_{n+m})^T$ (where $k_i \ge \overline{\omega}_i(0), i = 1,2,\cdots,n+m$) such that

$$\omega_i(t) \le k_i e^{-\lambda t}, \quad i = 1,2,\cdots,n+m.$$

Set $M_i = \dfrac{k_i}{\overline{\omega}_i(0)} \ge 1$, then

$$\omega_i(t) \le M_i \overline{\omega}_i(0) e^{-\lambda t}, \quad i = 1,2,\cdots,n+m.$$

That is,

$$\left|x_i(t)-x_i^*\right| \le M_i \sup_{-\tau\le t\le 0}\left|\phi_i(t)-x_i^*\right| e^{-\lambda t},$$

$$\left|y_j(t)-y_j^*\right| \le M_j \sup_{-\tau\le t\le 0}\left|\varphi_j(t)-y_j^*\right| e^{-\lambda t}$$

for $i = 1,2,\cdots,n$, $j = 1,2,\cdots,m$.

It follows that we have

$$\left|x_i(t)-x_i^*\right|^r \le M_i^r \sup_{-\tau\le s\le 0}\left|\phi_i(s)-x_i^*\right|^r e^{-\lambda r t}, \quad (7)$$

$$\left|y_j(t)-y_j^*\right|^r \le M_j^r \sup_{-\tau\le s\le 0}\left|\varphi_j(s)-y_j^*\right|^r e^{-\lambda r t} \quad (8)$$

for $r \ge 1$, $i = 1,2,\cdots,n$, $j = 1,2,\cdots,m$.

Hence

$$\sum_{i=1}^{n}\left|x_i(t)-x_i^*\right|^r + \sum_{j=1}^{m}\left|y_j(t)-y_j^*\right|^r \le M\left(\left\|\phi - x^*\right\| + \left\|\varphi - y^*\right\|\right) e^{-\lambda p t},$$

where $M = \sup_{1\le i\le n+m} M_i^r \ge 1$.

Case 2. when $t > 0$, $t = t_k$, $k = 1,2,\cdots$.

From (**H2**), we have

$$\begin{aligned} |x_i(t_k^+)-x_i^*| &= |\Delta x_i(t_k) + x_i(t_k^-) - x_i^*| \\ &= |P_k(x_i(t_k)) + x_i(t_k) - x_i^*| \\ &= |-\delta_{ik}(x_i(t_k)-x_i^*) + x_i(t_k) - x_i^*| \\ &= |(1-\delta_{ik})(x_i(t_k)-x_i^*)| \\ &\le |x_i(t_k)-x_i^*|, \end{aligned}$$

from (7), it is noted that $e^{-\lambda r t}$ is a continuous function on R, we get

$$\begin{aligned} \left|x_i(t_k^+)-x_i^*\right|^r &\le |(x_i(t_k)-x_i^*|^r \\ &\le M_i^r \sup_{-\tau\le s\le 0}\left|\phi_i(s)-x_i^*\right|^r e^{-\lambda r t_k} \\ &= M_i^r \sup_{-\tau\le s\le 0}\left|\phi_i(s)-x_i^*\right|^r e^{-\lambda r t_k^+}. \end{aligned} \quad (9)$$

For the same reason, we have

$$\left|y_j(t_k^+)-y_j^*\right|^r \le M_j^r \sup_{-\tau\le s\le 0}\left|\varphi_j(s)-y_j^*\right| e^{-\lambda r t_k^+} \quad (10)$$

From (9) and (10), we have

$$\sum_{i=1}^{n}\left|x_i(t_k^+)-x_i^*\right|^r + \sum_{j=1}^{m}\left|y_j(t_k^+)-y_j^*\right|^r \le M\left(\left\|\phi - x^*\right\| + \left\|\varphi - y^*\right\|\right) e^{-\lambda p t_k^+}.$$

where $M = \sup_{1\le i\le n+m} M_i^r \ge 1$.

Summarizing Case 1 and Case 2, we obtain

$$\sum_{i=1}^{n}\left|x_i(t)-x_i^*\right|^r + \sum_{j=1}^{m}\left|y_j(t)-y_j^*\right|^r \le M\left(\left\|\phi - x^*\right\| + \left\|\varphi - y^*\right\|\right) e^{-\lambda p t},$$

for all $t \ge 0$, where $M = \sup_{1\le i\le n+m} M_i^r \ge 1$. The proof is completed.

Corollary 1. Under hypothesis (**H1**) and (**H2**), model (3) has one unique equilibrium point, which is globally exponentially stable if any one of the following conditions is true:

(i) $$\begin{cases} a_i > \sum_{j=1}^{m} F_j \left[| c_{ij} | + | \tilde{c}_{ij} | \right], & i = 1,2,\cdots,n, \\ b_j > \sum_{i=1}^{n} G_i \left[| d_{ji} | + | \tilde{d}_{ji} | \right], & j = 1,2,\cdots,m. \end{cases}$$

(ii) $$\begin{cases} a_i > G_i \sum_{j=1}^{m} \left(| c_{ji} | + | \tilde{c}_{ji} | \right), & i = 1,2,\cdots,n, \\ b_j > F_j \sum_{i=1}^{m} \left(| d_{ij} | + | \tilde{d}_{ij} | \right), & j = 1,2,\cdots,m. \end{cases}$$

(iii) There exists a positive vector $\xi = (\xi_1, \cdots, \xi_{n+m})^T > 0$ such that

$$\begin{cases} a_i \xi_i > \sum_{j=1}^{m} \xi_{n+j} F_j \left[| c_{ij} | + | \tilde{c}_{ij} | \right] & i = 1,2,\cdots,n, \\ b_j \xi_{n+j} > \sum_{i=1}^{n} \xi_i G_i \left[| d_{ji} | + | \tilde{d}_{ji} | \right] & i = 1,2,\cdots,m. \end{cases}$$

Proof. In fact, any one of the conditions (i)-(iii) in Corollary 1 can assure $\alpha - (| A | + | B |)L$ is a nonsingular M-matrix.

4. Remark and example

Remark. In [8,9], authors considered a special case of model (1) as $c_{ij} = 0, d_{ji} = 0 (i = 1,2,\cdots,n, j = 1,2,\cdots,m)$. It is easy to check that our results conclude those in [10,11], therefore we improve some previous results.

Example. Consider the following model

$$\begin{cases} \dfrac{dx_i(t)}{dt} = -a_i x_i(t) + \sum_{j=1}^{2} c_{ij} f_j(y_j(t)) \\ \qquad - \sum_{j=1}^{2} \tilde{c}_{ij} f_j(y_j(t - \tau_{ij})) + I_i, \quad t > 0, t \neq t_k, \\ \Delta x_i(t_k) = -\delta_{ik}(x_i(t_k) - 1), \quad i = 1,2, \; k = 1,2,\cdots \\ \dfrac{dy_j(t)}{dt} = -b_j y_j(t) + \sum_{i=1}^{2} d_{ji} g_i(x_i(t)) \\ \qquad + \sum_{i=1}^{2} \tilde{d}_{ji} g_i(x_i(t - \sigma_{ji})) + J_j], \; t > 0, t \neq t_k, \\ \Delta y_j(t_k) = \eta_{jk}(y_j(t_k) - 1), \quad j = 1,2, \; k = 1,2,\cdots \end{cases} \tag{11}$$

where $t_1 < t_2 < \cdots$ is a strictly increasing sequence such that $\lim_{k \to \infty} t_k = +\infty$, and

$h(z) = f_1(z) = f_2(z) = g_1(z) = g_2(z) = | z + 1 | - | z - 1 |$.

Since $\forall z_1, z_2 \in R$, $| h(z_1) - h(z_2) | \leq 2 | z_1 - z_2 |$, $F_1 = F_2 = G_1 = G_2 = 2$. Let

$$\delta_{ik} = 1 - \frac{1}{2}\sin(1 + k), \; \eta_{jk} = 1 - \frac{1}{3}\cos(k^3), \; k = 1,2,\cdots,$$

$$a_1 = 7, \; a_2 = 12, \; b_1 = 4, \; b_2 = 36,$$

$$C = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}, \; \tilde{C} = \begin{pmatrix} 1 & 1 \\ 0.5 & 1.5 \end{pmatrix}, \; \tau = (\tau_{ij}) = \begin{pmatrix} 0.1 & 02 \\ 0.2 & 0.1 \end{pmatrix}$$

$$D = \begin{pmatrix} 1 & \frac{3}{2} \\ \frac{3}{2} & 2 \end{pmatrix}, \tilde{D} = \begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{3}{2} \end{pmatrix}, \sigma = (\sigma_{ji}) = \begin{pmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{pmatrix},$$

$$I_1 = -3, \; I_2 = 4, \; J_1 = -4, \; J_2 = 25.$$

It follows that we have

$$\alpha = \mathrm{diag}(7,12,4,36), \; L = \mathrm{diag}(2,2,2,2),$$

$$A = \begin{pmatrix} 0 & C \\ D & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 1 & 1.5 & 0 & 0 \\ 1.5 & 2 & 0 & 0 \end{pmatrix},$$

$$B = \begin{pmatrix} 0 & \tilde{C} \\ \tilde{D} & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0.5 & 1.5 \\ 1 & 0.5 & 0 & 0 \\ 0.5 & 1.5 & 0 & 0 \end{pmatrix}.$$

Obviously, $(1,1,1,1)^T$ is the unique equilibrium point of the system (11). However, it is easy to check that assumptions (**H1**) and (**H2**) are satisfied, and

$$\alpha - (| A | + | B |)L = \begin{pmatrix} 7 & 0 & -4 & -6 \\ 0 & 12 & -3 & -5 \\ -4 & -4 & 4 & 0 \\ -4 & -7 & 0 & 36 \end{pmatrix}$$

is a nonsingular M-matrix. Hence, from Theorem 1, we know that the unique equilibrium point $(1,1,1,1)^T$ of the system (11) is globally exponentially stable.

5. Conclusions

In this paper, a generalized model of bi-directional associative memory (BAM) neural networks delays and impulses have been studied. Some sufficient conditions for the existence and exponential stability of the equilibrium point have been established. These obtained results are new and they complement previously known results. Moreover, an example are given to illustrate the effectiveness of the new results.

References

[1] B. Kosto, Neural Networks and Fuzzy Systems-A Dynamical System Approach Machine Intelligence[M].Prentice-Hall, Englewood Cliffs, NJ, 1992, 38-108..

[2] B. Kosto Bi-directional associative memories, IEEE Trans. Systems Man. Cybernet.,1998,18(1):49-60.

[3] B. Kosto,Adaptive bi-directional associative memories. Appl. Optim., 1987, 26(23):4947-4960.

[4] K. Gopalsamy, X. Z. He, Delay-independent stability in bi-directional associative memory networks. IEEE Trans. Neural Networks, 1994, 14(5): 998-1002.

[5] J.D. Cao, L. Wang, Periodic oscillatory solution of bi-directional associative memory networks with delays, Phys. Rev. E, 2000,61(2):1825-1828.

[6] X. F. Liao, J. B. Yu, Qualitative analysis of bi-directional associative memory with time delay, Int.J.Circuit Theory and Applications, 1998, 26 (3): 219-229.

[7] K. L. Li, Global exponential stability of bi-directional associative memory cellular neural networks with time delays, Journal of Sichuan University of Science & Engineering (in Chinese), 18 (4) (2005) 3-8.

[8] Y.K. Li, Global exponential stability of BAM neural networks with delays and impulses, Chaos, Solitons & Fractals 24 (2005) 279-285.

[9] X.Y. Lou, B. T. Cui, Global exponential stability of delay BAM neural networks with impulses, Chaos, Solitons & Fractals, 29 (2006) 1023-1031.

[10] Y. T. Li, C. B. Yang, Global exponential stability analysis on impulsive BAM neural networks with distributed delays, J. Math.l Anal. . Appl., 324 (2006) 1125-1139.

[11] Y. K. Li, L. H. Lu, Global exponential stability and existence of periodic solution of Hopfield-type neural networks with impulses, Physics letters A, 33 (2004) 62-71.

[12] Y. K. Li, W. Y. Xing, L. H. Lu, Existence and global exponential stability of periodic solution of a class of neural networks with impulses, Chaos, Solitons & Fractals, 27 (2006) 437-445.

[13] D.D. Bainov, P.S. Simenov, Systems with Impulse Effect: Stability Theory and Applications, Ellis Horwood, Chichester, 1989.

[14] V. Lakshmikantham, D.D. Bainov, P.S. Simeonov, Theory of Impulsive Differential Equations, World Scientific, Singapore, 1989.

[15] K. Gopalsamy, B.G. Zhang, On delay differential equation with impulses, J. Math. Anal. Appl. 139 (1989) 110-122.

[16] J. Yan, Stability theorems of perturbed linear system with impulse effect, Portugal. Math. 53 (1996) 43-51.

[17] Z.H. Guan, G. Chen, On delayed impulsive Hopfield neural networks, Neural Networks 12 (1999) 273-280.

[18] M.U. Akhmetov, A. Zafer, Stability of the zero solution of impulsive differential equations by the Lyapunov second method, J. Math. Anal.Appl. 248 (2000) 69-82.

[19] X. Liu, G. Ballinger, Uniform asymptotic stability of impulsive delay differential equations, Comput. Math. Appl. 41 (2001) 903-915.

[20] D. Y. Xu, Z.C. Yang, Impulsive delay differential inequality and stability of neural networks, J. Math. Anal. Appl. 305 (2005): 107-120.

[21] A. Berman, R.J. Plemmons, Nonnegative Matrices in Mathematical Sciences, Academic Press, New York, 1979.

[22] R.A. Horn, C.R. Johnson, Matrix Analysis, World Publishing, Beijing, 1991.