A Framework for Single-Condition Approximate Reasoning

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Summary

Informally, approximate reasoning can be viewed as a process by which a possible imprecise conclusion is deduced from a collection of imprecise premises [9]. How to process the linguistic term used in approximate reasoning is the key problem for deducing an appropriately conclusion. In this paper, a generalized method for computing linguistic terms, says Linguistic Atom Model (LAM), is present; then a framework for singleton-conditional approximate reasoning which is the simplest model of approximate reasoning is proposed based on LAM; Finale, a simplified framework as an example is established. The conclusions of the approximate reasoning under this framework are more appropriately to human reasoning.

Key words:
approximate reasoning, linguistic atom, linguistic term, Linguistic Atom Model, Vector Compatibility

Introduction

The theory of approximate reasoning established in 1975 by L.A. Zadeh [1] claims to model the way of human reasoning. From then on, practitioners and researchers have drawn tremendous attention in the area of fuzzy approximate reasoning. Informally, approximate reasoning can be viewed as a process by which a possible imprecise conclusion is deduced from a collection of imprecise premises. For the single condition approximate reasoning which is the simplest model of approximate reasoning, one of the most widely used inference rule is the fuzzy compositional rule of inference which has the global scheme:

Relation: *If X is A then Y is B*

Premise: *X is A′*

$$\frac{\qquad}{\text{Conclusion:}\quad Y \text{ is } B'} \qquad (1)$$

Where X and Y are variables taking their values from fuzzy sets in classical sets U and V, respectively, A and B are unary fuzzy predicates, labeled by linguistic term, in U and V, respectively.

Traditionally, the if-then rule is represented by a fuzzy relation R (a fuzzy set in U ×V), and to obtain an inference B' about Y, The conclusion B' is computed in the mathematical apparatus as follow

$$B' = A' \bullet R(A \to B) \qquad (2)$$

Where the operator "•" is compositional calculation "*Max-Min*", and the $R(A \to B)$ is a binary fuzzy relation in $U \times V$, which is typically modeled by a t-norm T. Some implementing of fuzzy compositional rule of inference was proposed by L.A. Zadeh, M. Mizumoto and S. Zimmerman etc. such as:

In order to get a reasonable conclusion B', the fuzzy sets, A and B, must satisfy the follow conditions [S.Fulami etc.]:

$$\{\mu_A(x) \mid \forall x \in X\} \supseteq \{\mu_B(y) \mid \forall y \in Y\} \quad (4)$$

$$\exists x \in X, \mu_A(x) = 1 \wedge \exists x \in X, \mu_A(x) = 0 \quad (5)$$

$$R_s : a \to b = \begin{cases} 1, a \le b \\ 0, a > b \end{cases} \qquad (3)$$

$$\exists y \in Y, \mu_B(y) = 1 \wedge \exists y \in Y, \mu_B(y) = 0 \quad (6)$$

But in AI systems, especially for fuzzy expert systems, when we applied linguistic values to fuzzy compositional rule of inference, the linguistic approximation of the conclusion that is in the form of fuzzy set, sometimes, is unacceptable or is difficult. An example is shown as in figure 1.

When the support of A', such as A_1, is identical with the support of A, then conclusion, B_1, is valid. Otherwise if the support of A', such as A_2 and A_3, more then the support of A, the conclusion, B_2 and B_3 respectively, have a constant value in tail, and the constant value is the

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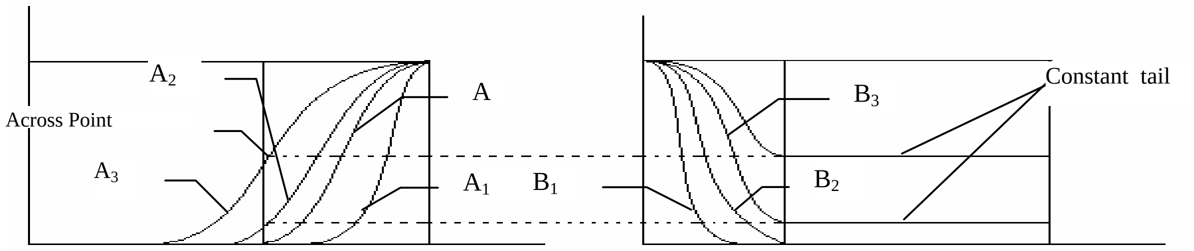

Figure 1. Fuzzy compositional rule of inference

membership value of the point that *A*' across *A*. In these cases, there are two basis issues to be concerned with:

(1) The information of *A*' which is out of supp(*A*) is lost; and

(2) The linguistic approximation of conclusion is very difficult.

In this paper, the linguistic atom model and vector compatibility is present in section 2 and section 3 respectively, then we give a generalized computable inference algorithm based on linguistic atom model [4] firstly, and a general approximate reasoning framework is proposed. It is suitable for linguistic variables and can avoid the question mention above.

2. Linguistic Atom Model

A linguistic variable can be regarded as a variable whose values are defined in linguistic terms. Every linguistic term is characterized by a fuzzy set *F* in a universe of discourse *U*, and Each element $u \in U$ belongs to a fuzzy set *F* with a degree of membership $\mu_F(u) \in [0, 1]$.

The linguistic terms of a linguistic variable are classified into atomic term, linguistic hedge and composite term. The atomic term is primary term in nature language; linguistic hedge, such as *very, approximate, extremely*, etc., is a modifier for atomic term, and the composite term is consist of one or more hedges and an atomic term. The linguistic term is corresponding to the most elementary concept. Basically, natural language serves to describe complicated concept with most elementary ones, and their kinds of combination.

We assume the quantity of atomic terms to each linguistic variable is finite but the quantity of composite term is infinite. For example of the linguistic variable "age", the atomic terms are *young*, *middle-aged* and *old*, while the composite terms are *very old*, *very very old*, *extremely young*, etc. In practice the number of atomic terms of a linguistic variable is not more than 7.

Definition1: *A linguistic atom is an abstract concept of atomic term of the linguistic variables; it is a normalized fuzzy set defined in the universe of discourse [0,1].*

The same as linguistic value, linguistic atoms are classified into meta-atom and composite-atom too. The meta-atom is corresponding to atomic term of linguistic variables and the composite atom is corresponding to composite term of linguistic variables

Definition2: *The Linguistic Atom Model, or LAM for short, is an abstract model of the linguistic variables based on linguistic atoms, denoted as a quadruple of the form x(n, G, R, H). It mapping the universe of discourse of a linguistic variable into the interval [0,1]. n is the number of linguistic meta-atoms, which mapped from the atomic terms of a linguistic variable, totally covered the interval [0,1]; G is a grammar that used to produce a composite linguistic atom; R is a bijective mapping rule between linguistic atom and atomic term; H is definitions of linguistic hedges which modifies the membership function of linguistic atoms.*

Discussion1: In the definition2, *linguistic atoms totally covered the interval [0,1]* means:

$$\forall x \in [0,1], \exists Y \in \{O\}, \mu_Y(x) > 0$$

Discussion2: The number *n* in LMA determined the precision of the system; it is selected based on the system demands. But we must be on to the follow facts:

(1) The more small *n* is, the more plentiful information the linguistic atom contains, then the difference of linguistic atoms is obliterate, and results in a conclusion may be no meaning;

(2) If *n* is large enough, LMA emphasize difference excessively, the result that is in some conclusions is too accurate to deviate from nature language

characteristic.

We denote linguistic atoms as [O] in practice, for n = 3, the linguistic atoms is represented as [O-], [O] and [O+]; while n = 5, it represented as [O--], [O-], [O], [O+], [O++].

Discussion3: The membership function of the linguistic meta-atoms can be defined as any shape such as trapezoidal, Gaussian, etc. But it is uniform when the Linguistic Atom Model is "instance" and begins to be calculated. In fact, what the most important is the distribution of the linguistic atoms, but not the definitions of the membership functions of the linguistic atoms.

What the linguistic meta-atoms are distributed in LAM as an example is depicted in figure 2. There are three linguistic meta-atoms in the linguistic atom model, and the other linguistic atom can be gained by compound operation according to grammar *G* and hedge definition *H*.

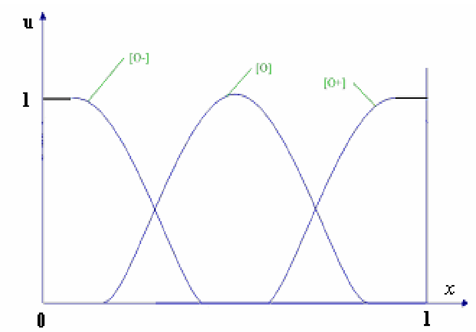

Figure 2. A linguistic Atom Model with 3 linguistic meta-atoms

LAM is instanced by the linguistic variable in calculation. When the linguistic variable *"age"* takes part in calculation, for example, a LAM is instanced by it. In this "instanced" process, x is *"age"*, n is 3, and mapping relation R mapped the atomic terms, *"young"*, *"middle-aged"* and *"old"*, in $T(x)$ into meta-atoms [O-], [O] and [O+], respectively, and other values of the linguistic variable "*age*" can be mapped onto LAM based on the composition of meta-atom and hedges in *H*.

3. Inference Based on Vector Compatibility

3.1 The concept of Vector Compatibility

In a rule, it is well know that there is a causal relationship between the condition and the consequence, which is means: because of the existence of the condition, we can get the consequence. But on the other hand, there exist an implicit relation which is that the tendency of the conclusion is made out. For instance, the rule "if X is tall, then X is heaven", it supports the relation not only "Because of one is tall, he is heaven", but also "The taller one is, the heavier he is".

The importance of the implicit relation in the rules has not been noticed so far. In the present way, "Compatibility" just means the degree of similarity between two linguistic values. In order to show the tendency of the consequence, we introduce a new concept named "Vector Compatibility (VC)".

For two values of a linguistic variable, say A and B, the Vector Compatibility of the value B to the value A, say $T(A,B)$, is a real numerical pair (α , β), which defines as:

$$\alpha = S(A \cap B)/S(B) \qquad (7)$$

$$\beta = Sgn(A)\times(D(B) - D(A)) / D(A) \qquad (8)$$

Here S(·) is the area covered by possibility distribution curve of a linguistic value. D(·) is the typical value of a linguistic value, Sgn(·) is sign function of linguistic value defined in [4].

3.2 Algorithm of Inference Using VC

Now we illustrate the computing process for approximate reasoning such as formula (1):

1) Get the vector compatibility $T(A,A')$ of fuzzy set A to fuzzy set A';

$$T(A, A') = (\alpha, \beta)$$

2) Get the fuzzy set $\tilde{B}$ by modifying the consequence B with β

$$\delta = Sgn(B)\times \beta \times D(B) \qquad (9)$$

$$\mu_{\tilde{B}}(x) = \mu_B(x - \delta) \qquad (10)$$

3) Get middle-conclusion $\bar{B}$ as conclusion by modifying the fuzzy set $\tilde{B}$ with α

$$\mu_{\bar{B}}(x) = (\mu_{\tilde{B}}(x))^{\alpha} \qquad (11)$$

4) Mapping back the middle-conclusion $\bar{B}$ to universe of discourse of linguistic term B, we get the linguistic conclusion B'.

This method take account of three facts: (1) a causal relationship between the condition and the consequence in the rule; (2) an implicit relationship which produces the tendency of the consequence; and (3) all information include in premise is represented in conclusion.

4. A Framework for Approximate Reasoning

So, we can establish a general framework for approximate reasoning by using the linguistic atom model (LMA) and the vector compatibility (VC).

The main ideal of the general framework for the approximate reasoning is:

(1) Mapping all linguistic terms in the rule and the premise onto linguistic atom model according to linguistic variable;

(2) Using VC to figure out the conclusion in the form of fuzzy set;

(3) Using linguistic approximate to get the linguistic atom;

(4) Re-mapping linguistic atoms back onto the universe of linguistic variable and output a linguistic conclusion.

The framework is depicting as figure 3

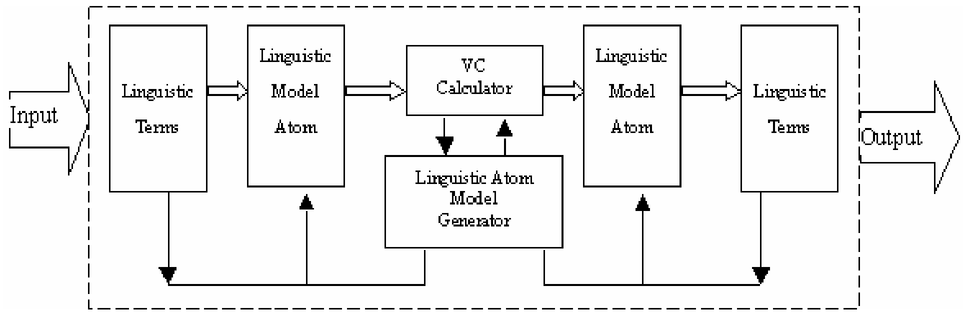

Figure 3. The framework of approximate reasoning using LMA

Here we construct an example of the generalized framework. In the framework, some hypotheses as follows are given for simplification:

***Hypothesis 1*:** The example LAM is consist of 3 linguistic meta-atoms, say [O-], [O], [O+], shown as Figure 4. And the definitions of the membership functions of meta-atoms are linear functions defined as follow：

$$[O-] = \begin{cases} \int_0^{0.1} 1 / x \\ \int_{0.1}^{0.4} (1 - \frac{10}{3}(x - 0.1)) / x \end{cases}$$

$$[O] = \begin{cases} \int_{0.1}^{0.4} \frac{10}{3}(x - 0.1) / x \\ \int_{0.4}^{0.6} 1/x \\ \int_{0.6}^{0.9} (1 - \frac{10}{3}(x - 0.6)) / x \end{cases}$$

$$[O+] = \begin{cases} \int_{0.6}^{0.9} \frac{10}{3}(x - 0.6) / x \\ \int_{0.9}^{1} 1/x \end{cases}$$

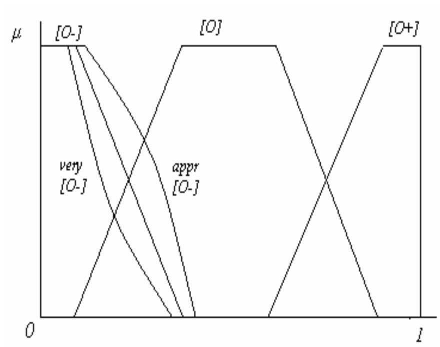

Figure 4. A simple LAM with 3 linguistic meta-atoms

Hypothesis 2: There are 5 hedges, *"Very"*, *"Approximately"*, *"Not so"*, *"Certainly"*, *"Perhaps"*, the denotation of them are *"V"*, *"Appr"*, *"Ns"*, *"Minus"* and *"Plus"*. And the hedges *"Minus"* and *"Plus"* is used as the output only. The definitions of hedges are:

$$\mu_{VeryO}(x) = \mu_O(x)^2$$
$$\mu_{ApproO}(x) = \mu_O(x)^{0.5}$$
$$\mu_{MinusO}(x) = \mu_O(x)^{0.75}$$
$$\mu_{PlusO}(x) = \mu_O(x)^{1.75}$$
$$\mu_{NsO}(x) = \mu_{MinusO}(x) + (1 - \mu_{VeryO}(x))$$

Example 1: The approximate reasoning as follow:

Rule: if *x* is *tall* then *x* is *heaven*

Premise: John is *very tall* （*Approximately tall*, *Not so tall*）

John is?

Where linguistic variables *height(x)* and *weight(x)* are used to instanced two linguistic atom models (LAM), *height*(3,G,R,H) and *weight*(3,G,R,H), respectively. And linguistic terms, *tall* and *heaven,* are mapping onto linguistic meta-atom, [O+], of the LAMs, respectively; Linguistic term *very tall* is mapping onto linguistic atom *Very*-[O+]. The VC of *Very*-[*O*+] to [*O*+] is

T(*very*-[O+],[O+]) = (α, β) = (*1.0, 0.05*)

Then using α, β to modify the conclusion represented by linguistic meta-atom [O+] by formulae (9) - (11), we get a new linguistic atom *plus*-[O+]. Mapping back this linguistic atom to universe of discourse of *weight*, we get the linguistic term, say *Certainly heaven*.

Where linguistic term *approximately tall* and *Not so tall* is mapping to linguistic atom *appr-[O+]* and *Ns-[O+]*, respectively, and the linguistic conclusion calculated is *perhaps heaven* and *approximately heaven* respectively.

In this case, we can construct a linguistic atom logical table for the approximate reasoning with a single condition when the linguistic atom is determined. For the simplified LMA above, the linguistic atom logical table is established as table 1.

Table 1. An linguistic atom logical table for linguistic atom model

		[O-]	[O]	[O+]
[O-]	V	Plus B	U	U
	O	B	U	U
	A	Minu B	U	U
	N	Appr B	Appr B	U
[O]	V	U	Plus B	U
	O	U	B	U
	A	U	Appr B	U
	N	U	Appr B	Appr B
[O+]	V	U	Plus B	Plus B
	O	U	U	B
	A	U	U	Minu B
	N	U	Appr B	Appr B

In the table 1, 'U' represents linguistic term *unknown*; 'B' represents linguistic atom which corresponded to linguistic term in the consequence of the relation. The atoms in row correspond to the linguistic term in the relation; the atoms in column correspond to the linguistic term in premise.

So we can calculate the conclusions of the approximate reasoning by lookup the linguistic atom logical table just like Boolean logic. For example as above, we can get the conclusions of the approximate reasoning as below:

Linguistic terms in premise	Conclusions in linguistic atom	Conclusions in nature language
John is *very tall*	Plus [O+]	John is *certainly heaven'*
John is *approximately tall*	Minus [O+]	John is *perhaps heaven*
John is *not so tall*	Appr [O+]	John is *approximately heaven*

5. Conclusion

We have proposed the concepts of linguistic atom model and vector compatibility, and then a generalized framework for fuzzy approximate reasoning is presented. A characteristic feature of this framework is extensive use of linguistic variables. The model give a unified method for constructing membership function of a linguistic term used in nature language and a unified calculation process for linguistic values. We demonstrated how the framework is "instance" for approximate reasoning, and a linguistic logical table is further constructed, it is simplified the calculation for approximate reasoning.

The further work is focus on the approximate reasoning with multi-conditional in a relation. In this case, the question is how to get the vector compatibility of linguistic terms in premise, $A_1', A_2', \ldots A_n'$, to linguistic terms in consequence, $A_1, A_2, \ldots, A_n$. The other work is focus on the instancing of linguistic atom model with linguistic terms.

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