# An Approach to Determine the Attribute Weights Based on Different Formats of Evaluation Information

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**Summary**

This paper proposes an approach to determine attribute weights in the multiple attribute comprehensive evaluation (MACA) problem, in which the attribute values of alternatives are expressed in three formats of evaluation information such as real number, interval number and fuzzy linguistic term. In the approach, an optimization model is constructed to determine attribute weights, which makes the obtained attribute weights reflect the original evaluation information. An example is used to illustrate the applicability of the proposed approach.

***Key words***:
*attribute weight, multiple attribute comprehensive evaluation, optimization model*

## 1. Introduction

Multiple attribute comprehensive evaluation (MACE) refers to the problems of evaluating or ranking alternatives associated with multiple attributes [2, 5]. So far a lot of theoretical and applied research result on MACE problems has been obtained [1, 4, 10]. In the process of MACE analysis, one crucial problem is to assess the relative importance or weights of the attributes because attribute weights influence the results of rankings of alternatives [8]. Several approaches have been proposed to finish this work. They include subjective and objective integrated approach [8], eigenvector method [11] and mathematical programming model [12] etc.

In classical MACE problems, the attribute values of alternatives are usually known as only one format of evaluation information (e.g. numerical information or fuzzy linguistic information). However, there are many decision situations in which the evaluation information may be real numbers for some attributes, interval numbers for some attributes, and fuzzy linguistic terms for the other attributes. For instance, in the problem of evaluating air-conditioning systems, experts' evaluation information on attributes such as design and safety are often not precise enough to yield the precise numerical attribute values. They tend to use fuzzy linguistic terms instead of real numbers or interval numbers to describe their evaluation information on these attributes. But the attribute values with respect to attributes

such as service life of air-condition would be given as interval numbers while the attribute values with respect to attribute such as price would be given as precise real numbers. Therefore, MACE problems with different formats of evaluation information become new research topics in MACE and attract many scholars' attention.

This paper proposes a new approach to determine attribute weights in MACA problem, in which the attribute values of alternatives are expressed in three formats of evaluation information such as real number, interval number and fuzzy linguistic term. The approach is based on an optimization model.

The rest of this paper is arranged as follows: Section 2 presents the MACE problem with three kinds of attribute values investigated in this paper. Section 3 proposes an approach for determining attribute weights. In Section 4, an example is used to illustrate the proposed method. The last section concludes the work of this paper.

## 2. Representation of the problem

This section describes the MACE problem with numerical attribute values, interval attribute values and fuzzy linguistic attribute values. To facilitate representation and analysis, the following notations are used throughout the paper.

Let $X=\{x_1, \cdots, x_n\}$ be a finite set of alternatives, $C=\{C_1, \cdots, C_m\}$ be a set of attributes. Let set $C^1 = \{C_1, C_2, \cdots, C_{m_1}\}$, $C^2=\{C_{m_1+1}, C_{m_1+2}, \cdots, C_{m_2}\}$ and $C^3=\{C_{m_2+1}, C_{m_2+2}, \cdots, C_m\}$ be three subsets of set $C$ and $C^1 \cup C^2 \cup C^3 = C$. Let $P=[p_{ij}]_{n\times m}$ be the

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decision matrix, where $p_{ij}$ denotes the attribute value for alternative $x_i$ with respect to attribute $C_j$. For alternative $x_i$, expert provides the attribute value on the attribute $C_j \in C^1$ as a real number, i.e., $p_{ij} \in R$ for $j = 1,\cdots, m_1$, $R$ is the real line, and the attribute value on the attribute $C_j \in C^2$ as a interval number $\overline{a}_{ij} = [a_{ij}^L, a_{ij}^U]$, i.e., $p_{ij} = \overline{a}_{ij}$ for $j = m_1 + 1, \cdots, m_2$, where $a_{ij}^L, a_{ij}^U \in R$, $0 \le a_{ij}^L \le a_{ij}^U$. Let $\overline{a}_{ij} = [a_{ij}^L, a_{ij}^U]$ and $\overline{a}_{kj} = [a_{kj}^L, a_{kj}^U]$ be two interval numbers, the distance between $\overline{a}_{ij}$ and $\overline{a}_{kj}$ [9] can be given by

$$d(\overline{a}_{ij}, \overline{a}_{kj}) = | a_{ij}^L - a_{kj}^L | + | a_{ij}^U - a_{kj}^U |. \quad (1)$$

The expert provides the attribute value on the attribute $C_j \in C^3$ as a fuzzy linguistic term, i.e., $p_{ij} \in S$ for $j = m_2 + 1, \cdots, m$, where $S = \{s_0, \cdots, s_T\}$ be a pre-established finite and totally ordered linguistic term set with odd cardinals [6]. Usually, the vagueness of fuzzy linguistic information is adequate captured by the fuzzy numbers defined in the interval [0, 1]. These fuzzy numbers are described by membership function such as linear trapezoidal membership function [5], linear triangular membership function [2], Gaussian function [4] and so on. For simplicity, we assume that the fuzzy linguistic terms in this paper are represented in the triangular fuzzy numbers. For example, the fuzzy linguistic terms in the linguistic term set $S = \{s_0, \cdots, s_6\}$ are expressed in the following triangular fuzzy numbers [7]: $s_0$=*perfect*=(0.83, 1, 1), $s_1$=*Very_ High*=(0.67, 0.83, 1), $s_2$=*High*=(0.5, 0.67, 0.83), $s_3$=*Medium*= (0.33, 0.5, 0.67), $s_4$=*Low*=(0.17, 0.33, 0.5), $s_5$= *Very_Low*=(0, 0.17, 0.33), $s_6$=*None*=(0, 0, 0.17)}.

For a fuzzy linguistic term $p_{ij}$, let its associated triangle fuzzy number be $\hat{a}_{ij}$=( $a_{ij}^g$, $a_{ij}^f$, $a_{ij}^u$ ), i.e., $p_{ij} = \hat{a}_{ij}$ for $i = 1, \cdots, n$, $j = m_2 + 1, \cdots, m$. Let $p_{ij}$ and $p_{kj}$ be two fuzzy linguistic terms, their associated triangle fuzzy numbers be $\hat{a}_{ij}$=( $a_{ij}^g$, $a_{ij}^f$, $a_{ij}^u$ ) and $\hat{a}_{kj}$=( $a_{kj}^g$, $a_{kj}^f$, $a_{kj}^u$ ) respectively. The distance between $p_{ij}$ and $p_{kj}$ can be given by [3]

$$d(p_{ij}, p_{kj}) = d(\hat{a}_{ij}, \hat{a}_{kj}) = \sqrt{\frac{1}{3}[(a_{ij}^g - a_{kj}^g)^2 + (a_{ij}^f - a_{kj}^f)^2 + (a_{ij}^u - a_{kj}^u)^2]},$$

$$i = 1, \cdots, n, \; j = m_2 + 1, \cdots, m. \quad (2)$$

Following the above notations, the problem discussed in this paper is to assess the weights of attributes based on decision matrix $P = [p_{ij}]_{n\times m}$ with three formats of attribute values.

## 3. The proposed approach

Usually, there are two types of attributes in MACE problems, benefit attribute and cost attribute [5]. In order to measure all attributes in commensurate units and to facilitate inter- attribute comparisons, to normalize the decision matrix must be the first step of the resolution process of the MACE problem. Then an optimization model is constructed to assess the weights of attributes. In the following, we give the whole process for solving this problem.

Let set $\overline{C}^1$, $\overline{\overline{C}}^1$, $\overline{C}^2$ and $\overline{\overline{C}}^2$ be subsets of sets $C^1$ and $C^2$ respectively, where $\overline{C}^1$ and $\overline{C}^2$ be two sets of benefit attributes, $\overline{\overline{C}}^1$ and $\overline{\overline{C}}^2$ be two sets of cost attributes and $\overline{C}^1 \cup \overline{\overline{C}}^1 = C^1$, $\overline{C}^2 \cup \overline{\overline{C}}^2 = C^2$. Firstly, the following equations can be used to normalize the matrix $P = [p_{ij}]_{n\times m}$ into the matrix $\widetilde{P} = [\widetilde{p}_{ij}]_{n\times m}$ [5], where

$$\widetilde{p}_{ij} = b_{ij} = a_{ij} \Big/ \sum_{k=1}^{m} a_{kj} \text{ for benefit attribute } \overline{C}_j^1, \quad (3)$$

$$\widetilde{p}_{ij} = b_{ij} = (1/a_{ij}) \Big/ \sum_{k=1}^{m} (1/a_{kj}) \text{ for cost attribute } \overline{\overline{C}}_j^1, \quad (4)$$

and the following equations can be used to normalize the element $\overline{a}_{ij}$ into the element $\overline{b}_{ij} = [b_{ij}^L, b_{ij}^U]$, where

$$\widetilde{p}_{ij} = \overline{b}_{ij} = \overline{a}_{ij} \Big/ \sum_{k=1}^{m} \overline{a}_{kj} \text{ for benefit attribute } \overline{C}_j^2, \quad (5)$$

$$\widetilde{p}_{ij} = \overline{b}_{ij} = (1/\overline{a}_{ij}) \Big/ \sum_{k=1}^{m} (1/\overline{a}_{kj}) \text{ for cost attribute } \overline{\overline{C}}_j^2. \quad (6)$$

The equations (5) and (6) can be alternatively expressed as

$$\begin{cases} b_{ij}^L = a_{ij}^L \Big/ \sum_{k=1}^{m} a_{kj}^U \\ b_{ij}^U = a_{ij}^U \Big/ \sum_{k=1}^{m} a_{kj}^L \end{cases} \text{ for benefit attribute } \overline{C}_j^2, \quad (7)$$

$$\begin{cases} b_{ij}^L = (1/a_{ij}^U) \Big/ \sum_{k=1}^{m} (1/a_{kj}^L) \\ b_{ij}^U = (1/a_{ij}^L) \Big/ \sum_{k=1}^{m} (1/a_{kj}^U) \end{cases} \text{ for cost attribute } \overline{\overline{C}}_j^2. \quad (8)$$

Obviously, for any *i* and *j*, it holds $0 < b_{ij}^L, b_{ij}^U \le 1$. And $\tilde{p}_{ij} = p_{ij} = \hat{a}_{ij} = (a_{ij}^g, a_{ij}^f, a_{ij}^u)$ for attribute $C_j \in C^3$.

In the following, an approach is investigated to compute the weights of attributes. Let $w = (w_1, \cdots, w_m)^T$ be the weight vector of attributes, where $w_j \ge 0$, $\sum_{j=1}^{m} w_j^2 = 1$ and $w_j$ is the weight of attribute $C_j$. If the attribute values of all alternatives with respect to one attribute are same, this attribute has no influence on the alternative ranking. The greater is deviation between the attribute values of different alternatives with respect to same one attribute, the stronger is the influence of these attributes on alternative ranking, and the bigger are their weights [13]. In order to measure the deviation degree between attribute values $\tilde{p}_{ij}$ and $\tilde{p}_{kj}$, based on Eqs. (1) and (2), the distance $d(\tilde{p}_{ij}, \tilde{p}_{kj})$ between $\tilde{p}_{ij}$ and $\tilde{p}_{kj}$ can be given as follows:

$$d(\tilde{p}_{ij}, \tilde{p}_{kj}) = \begin{cases} d(b_{ij}, b_{kj}) = |b_{ij} - b_{kj}|, \quad j = 1, \cdots, m_1, \\ d(\overline{b}_{ij}, \overline{b}_{kj}) = |b_{ij}^L - b_{kj}^L| + |b_{ij}^U - b_{kj}^U|, \\ j = m_1 + 1, \cdots, m_2, \\ d(\hat{a}_{ij}, \hat{a}_{kj}) = \\ \sqrt{\frac{1}{3}[(a_{ij}^g - a_{kj}^g)^2 + (a_{ij}^f - a_{kj}^f)^2 + (a_{ij}^u - a_{kj}^u)^2]}, \\ j = m_2 + 1, \cdots, m, \end{cases}$$

$$\forall i, k = 1, \cdots, n. \quad (9)$$

In order to obtain the attribute weight $w_j$, an optimization model can be constructed as follows:

$$\text{maximize } D(w) = \sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{k=1}^{n} d(\tilde{p}_{ij}, \tilde{p}_{kj}) w_j, \quad (10a)$$

$$\text{subject to } \sum_{j=1}^{m} w_j^2 = 1, \quad (10b)$$

$$w_j \ge 0, \quad j = 1, \cdots, m. \quad (10c)$$

To obtain $w_j$, the following lagrangian function is set up:

$$L(w, \lambda) = D(w) + \lambda\Big(\sum_{j=1}^{m} w_j^2 - 1\Big), \quad (11)$$

where $\lambda$ is the Lagrangian multiplier. Differentiating Eq. (11) with respect to $w_j$ and let $\partial L / \partial w_j = 0$ and $\partial L / \partial \lambda = 0$, we can obtain

$$w_j = \frac{\sum_{i=1}^{n} \sum_{k=1}^{n} d(\tilde{p}_{ij}, \tilde{p}_{kj})}{\sqrt{\sum_{j=1}^{m} [\sum_{i=1}^{n} \sum_{k=1}^{n} d(\tilde{p}_{ij}, \tilde{p}_{kj})]^2}}. \quad (12)$$

By the following Eq.(13), we have the normalized attribute weights:

$$w_j^* = w_j \Big/ \sum_{j=1}^{m} w_j. \quad (13)$$

## 4. Illustrative example

In this section, a problem of evaluating air- conditioning systems is used to illustrate the approach proposed in this paper. Four types of air- conditioning systems (alternatives) $x_1$, $x_2$, $x_3$ and $x_4$ are available. Six attributes including price ($C_1$) (\$M/each), electricity economy ($C_2$) (\$M/hour), service life of air-condition ($C_3$) (year), design ($C_4$) and safety ($C_5$) are considered in the process of evaluating the four air- conditioning systems. Attribute values on $C_1$ are assessed in the format of real numbers, ones on $C_2$ and $C_3$ are assessed in the format of interval numbers, and ones on $C_4$ and $C_5$ are assessed in linguistic term set $S$ introduced in Section 2. Among the five attributes, $C_1$ and $C_2$ are of cost types and $C_3$ is of benefit type. It can be seen that $\overline{C}^1 = \phi$, $\overline{\overline{C}}^1 = \{C_1\}$, $\overline{C}^2 = \{C_3\}$ and $\overline{\overline{C}}^2 = \{C_2\}$. The producer provides the information on attributes $C_1$, $C_2$ and $C_3$ while the expert provides his evaluation information on attributes $C_4$ and $C_5$. The evaluation

information about air-conditioning systems is shown in Table 1.
Firstly, based on Eqs. (3)-(8), we can obtain the normalized decision matrix as follows,

$$\widetilde{P} = \begin{bmatrix} 0.27 & [0.14,0.33] & [0.17,0.32] & (0.67,0.83,1) & (0.50,0.67,0.83) \\ 0.16 & [0.14,0.40] & [0.19,0.43] & (0.50,0.67,0.83) & (0.33,0.50,0.67) \\ 0.33 & [0.13,0.50] & [0.14,0.36] & (0.33,0.50,0.67) & (0.50,0.67,0.83) \\ 0.23 & [0.17,0.50] & [0.17,0.39] & (0.50,0.67,0.83) & (0.67,0.83,1) \end{bmatrix}$$

**Table 1: The information of producer's and expert's**

| Alternatives | Attributes | | | | |
|---|---|---|---|---|---|
| | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
| $x_1$ | 3000 | [0.06,0.08] | [7,9] | V | H |
| $x_2$ | 5000 | [0.05,0.08] | [8,12] | H | M |
| $x_3$ | 2500 | [0.04,0.09] | [6,10] | M | H |
| $x_4$ | 3500 | [0.04,0.07] | [7,11] | H | V |

Then, by Eqs. (9) and (12), we have $w_1$ =0.31, $w_2$ = 0.41, $w_3$ =0.29, $w_4$ =0.57, $w_5$ =0.5. By Eq.(13), we have the normalized attribute weights $w_1^*$ =0.149, $w_2^*$ =0.197, $w_3^*$=0.139, $w_4^*$=0.274, $w_5^*$=0.240.

## 5. Conclusion

This paper presents a new approach to determine attribute weights in MACA problem with three formats of evaluation information such as real number, interval number and fuzzy linguistic term. The approach determines attribute weights by solving a mathematical programming model. The proposed approach enriches the theories and methods of MACA problems with different formats of evaluation information. It can also be extended to support the group multiple attribute comprehensive evaluation situation where the evaluation information be given by multiple experts.

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