Curve fitting in terms N-Control points on 2D using fuzzy set and rough set theory

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Summary
In this paper, we develop a new hybrid curve fitting model using fuzzy set and rough set theory. The developed method differs from classical curve fitting techniques and algorithms. For produced rule sets of model to create by using fuzzy logic technique and the membership functions of input and output functions, the range of membership functions and the relations between membership functions is utilized from the bezier curve algorithm because of it was base one of parametric curve fitting algorithms in computer graphics and computer aided geometric design. One major disadvantage in the production of parametric curves such as Bezier curve or B-spline curve is the update of the whole formula while adding new control points. For avoided from that, our work is only updating between the two control points that are close to each other. In this paper, we give as comparative results to obtained by using the proposed model and results to produced with Bezier curve algorithm

Key words:
Fuzzy Logic, Rough Sets, Curve Fitting, Bezier Curve, Data Mining.

1. Introduction

The curve and surface modeling is one of the basic subjects in computer graphics. Two main methods used for curve and surface modeling in computer graphics are approximation and interpolation. Interpolation models use control points to produce the curve. Second of these techniques are producing the curve via control points. Spline and Bezier curves are two such examples. In literature on Spline curves, Bezier curves etc are available too many [2-11].

First literature on fuzzy logic which is introduced as aim for control can be found in Zadeh and then it was used in different areas such as sound detection, pattern recognition, etc [1]. The works about computer graphics are relation to curve fitting [7-11], approximation curve modeling [2], [3] and surface fitting [5], [6]. At this study, we will be focus on the curve producing technique by n control points. The basis of work is that, in case of knowing the start and end points and least more two points, is produced approximation curve by means of n control points by using rough set theory and fuzzy logic technique. The reason of using fuzzy logic technique is try to decrease computational complexity as create to curve, have control points such as Bezier curve. As for the aim of used rough set theory, is obtain of a minimal sufficient set of rules necessary for the description of all rules. While a curve is produced, curve to be satisfied *C1* continuous at starting and ending points and curve to be closer to another control points are supposed. By the help of the control points have to be known, the compute of distance *Y*-axis which is location of point far away distance *X*-axis from the starting control point is to make. If noticed the work, the main focus of this research is on the using of a exist algorithm for feature reduction and rule generation based on rough set theory and on the curve fitting by fuzzy logic technique based on set of rules, obtained from the rough set-based algorithm. That is, Let a set of points (control points) be given on the plane. Our purpose is to fit (by interpolation or approximation using new approach) a curve according to them. By using purposed method, each curve is modeled and then a surface can be generated by joining the created curves.

This work; for lower the calculation complexity, alternate approach to problem of curve fitting with rough set theory and fuzzy logic is presents. We begin our discussion with the short definition of Bezier curve, the Rough set theory and fuzzy logic.

2. The Bezier Curve

The following describes the mathematics for the so called Bézier curve. It is attributed and named after a French engineer, Pierre Bézier, who used them for the body design of the Renault car in the 1970's. They have since obtained dominance in the typesetting industry and in particular with the Adobe Postscript and font products [19].

Consider $n+1$ control points P_k ($k=0,..,n$) in 3D-space. The Bézier parametric curve function is of the form

$$B(u) = \sum_{k=0}^{n} p_k \frac{n!}{k!(n-k)!} u^k (1-u)^{n-k} \quad for\ 0 \le u \le 1 \tag{1}$$

Where $B(u)$ is a continuous function in 3D space defining the curve with N discrete control points P_k. $u=0$ at the first control point ($k=0$) and $u=1$ at the last control point ($k=n$). They have following several properties: The curve in general does not pass through any of the control points except the first and last; The curve is always contained

Manuscript received November 5, 2006.
Manuscript revised November 25, 2006.

within the convex hull of the control points, it never oscillates wildly away from the control points; If there is only one control point P_0, i.e.: $n=0$ then $B(u) = P_0$ for all u; If there are only two control points P_0 and P_1, i.e.: $N=1$ then the formula reduces to a line segment between the two control points; the term

$$\frac{n!}{k!(n-k)!} u^k (1-u)^{n-k} \quad (2)$$

is called a *blending function* since it blends the control points to form the Bézier curve; The blending function is always a polynomial one degree less than the number of control points. Thus three control points results in a parabola, four control points a cubic curve etc.; Closed curves can be generated by making the last control point the same as the first control point. First order continuity can be achieved by ensuring the tangent between the first two points and the last two points are the same; Adding multiple control points at a single position in space will add more weight to that point "pulling" the Bézier curve towards it. Bézier curves have wide applications because they are easy to compute and very stable. There are similar formulations which are also called Bézier curves which behave differently; in particular it is possible to create a similar curve except that it passes through the control points.

From this, it can be seen that, in generated the Bézier curve, mathematical formulas are too many. Hence the complexity of our approach less then the formula of Bézier curves (for detailed see section 5). Also our approach has the most of Bézier curves features.

3. Rough Set Theory

The rough sets theory approach is a formal construction to transform data into knowledge. A set of data is generally complicated and disorganized but knowledge is not. The framework of rough sets is chiefly considered more directly related to data reduction to diminish the level of redundancy or noise, and facts discovery to make the data pattern more observable. The appearance of knowledge can be presented as classification or decision rules [17].

The rough set methodology is a series of logic reasoning procedures by analyzing decision tables, which are flat tables containing a finite set of objects as rows and a set of attributes as columns. The set of attributes include several condition attributes and one or more decision attributes. From the decision table, the rough set theory utilizes the concept of indiscernibility to clarify equivalent classes and to compute a minimal set of significant attributes (reduct) [17].

Attribute reduction (feature selection) is a process of finding an optimal subset of all attributes according to some criterion so that the attribute subset are good enough to represent the classification relation of data. A good choice of attribute subset provided to a classifier can increase its accuracy, save the computational time, and simplify its results [12],[18].

Attributes deleted in attribute reduction can be classified into two categories. One category contains irrelevant and redundant attributes that have no any classification ability. An irrelevant attribute does not affect classification in any way and a redundant feature does not add anything new to classification [13] [19]. The other category is noisy attributes. These attributes represent some classification ability, but this ability will disturb the mining of true classification relation due to the effect of noise.

Many researchers have brought forward their attribute reduction algorithms, a comprehensive overview can be found in [4], [18]. Recently, many methods based on rough sets [14], 15], [16] and [[18].

In general, rough set theory provides useful techniques to reduce irrelevant and redundant attributes from a large database with a lot of attributes [17]. However, it is not so satisfactory for the reduction of noisy attributes [11] [19].

In this work, we used the Rough Set Theory for reduction the rules in rule database to producing the curve corresponding to given control points. This paper not explains used method for obtaining the reducts of the information tables constructed in the rough set theory. But used the Rough Set Theory based reduction algorithm uses the binary nature of the relations existing between the control points and their attributes.

In developed program, the reduction algorithm is executed as recursive function and all of elementary set elements is give to this algorithm and the rule database is obtained. In first, approximate 100.000 rule are generated in rule database, after the same rules are eliminated, the purposed rule reduction algorithm is called and set of rules is reduced to approximate 400 rules.

4. Fuzzy Logic

The concept of ‘fuzzy logic’ was introduced by Zadeh [1] as an extension of Boolean logic to enable modeling of uncertainty. Fuzzy logic introduces a concept of partial truth-values that lie in between “completely true” and “completely false”.

The basic structure of a fuzzy system consists of four main components (1) a knowledge base, which contains both an ensemble of fuzzy rules, known as the rule base, and an ensemble of membership functions known as the database; (2) a fuzzifier, which translates crisp inputs into fuzzy values; (3) an inference engine that applies a fuzzy reasoning mechanism to obtain a fuzzy output; and (4) a defuzzifier, which translates this latter output into a crisp value. These components perform the aforementioned processes necessary to fuzzy inference [9]. In here, it is supposed that reader is known the basic information's about fuzzy logic and fuzzy sets. In the next section, fuzzy method to use is explained.

5. Proposed Hybrid Curve Fitting Method

The set-up of a fuzzy logic model will depend on exactly the definition of the target function and parameters that is to be the affect the problem. The factors to which a component was affected the modeled problem is called the input parameters and the parameters to which a component was formed the target function is called the output parameters.

The numbers, name's, lower and upper limits of the membership functions all of parameters with respect to activity on want to modeled problem of input and output parameters was determined. For example, for "the distance X", the distance between first control point and placed point and the measured over the x axis, to be one of the input parameters, lower limit value is zero, upper limit value is the relative length of between P_0 and P_3, membership functions are "closer to P_0" and "distant to P_0" as a linguistic variable, where P_i are control points, last (4th) control point is P_3 while fist control point is P_0, and P_i-P_k is the chordal distance from P_i to P_k.

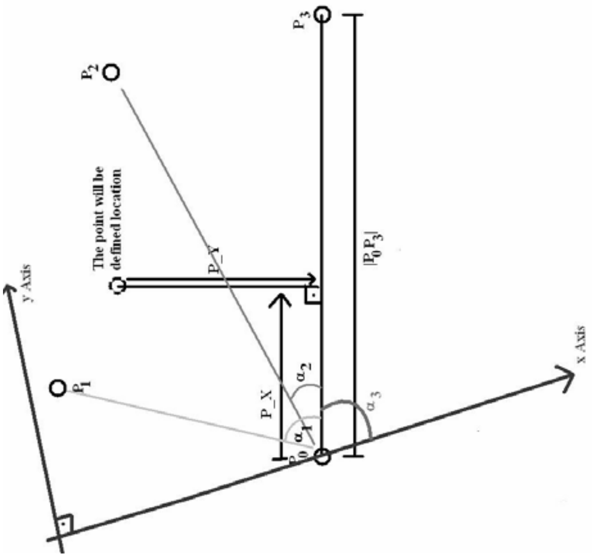

Fig. 1 Input membership function.

Since generation to the solve model of curve fitting problem by fuzzy logic, the lower and upper limit value of parameters were determined according to the aim of problem, the expert knowledge and the previous experiments (based on several Bezier curves). There are five membership functions (four input parameters and one output parameter). The first is chordal distance between first control point P_0 and point that will be defined location but this distance is on $|P_0P_3|$, and it is relative to the P_0P_3, so called *P_X*. The second is the angle of between first control point P_0 and second control point $P1$, called α_1. The third is the angle of between second control point P_1 and third control point P_2, called α_2. And the last input parameter is called α_3 and it is the angle of between first control point P_0 and last control point P_3. The angels of between the control points are the (degree) range of *[0,360]* and fall into six categories (A: is degree from 0 to 30, and as a similar; B:31-60, C:61-90, D:91:120, E:121-150, F:151-180, etc). The membership functions, lower limit value and upper limit value for each of input parameters (*P_X*, α_1, α_2, and α_3) are in presented Fig. 2 and Fig. 3, while the definition of problem in presented Fig. 1. The membership functions, lower limit value and upper limit values of the output parameter corresponding to these input parameters are presented in Fig. 4. The problem is curve fitting, the best curve to approximate as near it or passing through control points, in terms of control points P_i are given. In Fig. 1, the control points P_0, P_1... P_n and the distance of *P_X* are known and the distance of *P_Y* (perpendicular to *P_X* and $|P_0P_3|$) will be found.

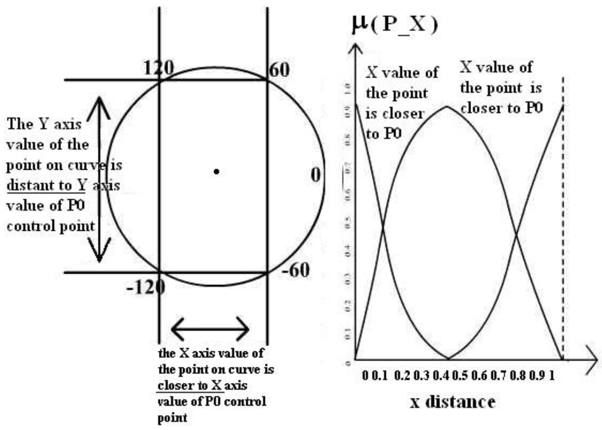

Fig. 2 The *P_X* membership function for the distance *X* according to the line of P_0-P_3 (The length of *X* axis is angular).

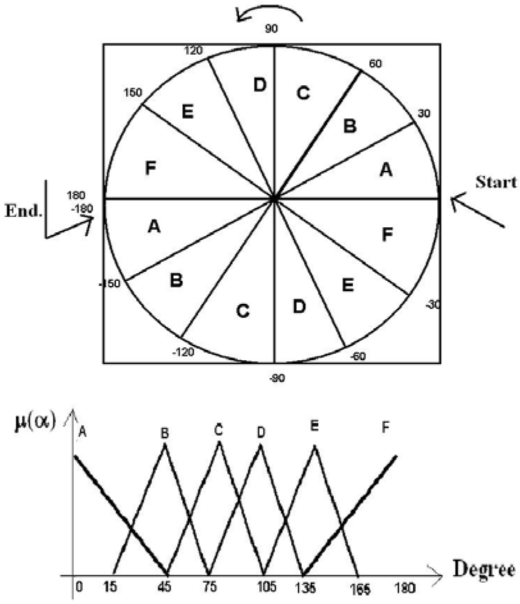

Fig. 3 $\mu(\alpha_1)$; the membership function for the angle of P_i control point according to the line of P_0P_3 (α_2 is the angle according to second control point and the P_0P_3 is angle of first and last control points)

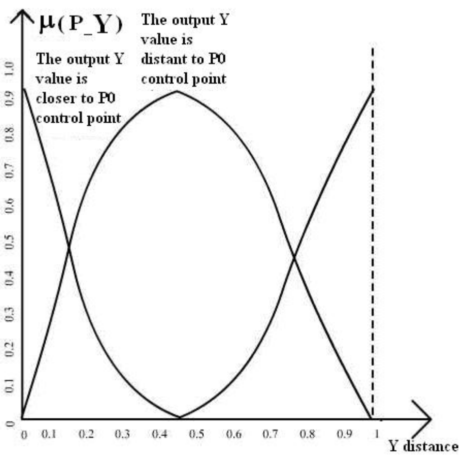

Fig. 4 The membership function for feature of output. (*P_Y*) (The length of Y axis is angular: since 0 is degree of zero, 1 is degree of 180).

For "the distance Y" to be output parameters, lower limit value is zero, upper limit value is 1, membership functions are "distant to P_0" and "closer to P_0" as a linguistic variable. It is called *P_Y*.

Inputs				***Output***	Rule
P_X	α_1	α_2	α_3	*P_Y*	
2	A	C	D	1	If *P_X*=2 and α_1=A and α_2=C α_3=D then *P_Y*=1.
1	B	C	D	1	If *P_X*=1 and α_1=B and α_2=C α_3=D then *P_Y*=1.
2	D	A	D	2	…
1	C	C	A	2	…
2	A	A	D	1	…
1	C	A	C	2	…
1	C	F	D	2	…
2	B	C	D	1	If *P_X*=2 and α_1=B and α_2=C α_3=D then *P_Y*=1.

Fig. 5 Fuzzy control rules (Inputs and Output). Note (1) For *P_X*; the 1, 2 values show "closer the P_0" and "distant to P_0", respectively. (2) For *P_Y*; the 1, 2 values show "distant to P_0" and "closer the P_0", respectively.

After described that it is necessary to the membership functions and lower and upper limit of membership functions for set-up of fuzzy curve model, A part of rules that to be generated according to Bezier Curve have four control point for description of relations between parameters in the conceptual model in terms of knowledge rules is shown as follows (see Fig. 5). Here, if the value of feature P_X is 1 then the point on curve is closer to first control point otherwise (namely value is 2) the point on curve is distant to first control point. The representations of A, B, C, D, E and F letters for feature of α_1, α_2 and α_3 is angular values that shown in Fig. 3. Finally, if the value of feature P_Y is 1 then the point on curve is distant to first control point otherwise (namely value is 2) the point on curve is closer to first control point. In special case, when the values of feature P_X and P_Y are 99 (or *), this feature doesn't affect the decision feature and in any case the value of decision feature was fixed. A part of the generated rules is shown in Fig. 5;

Based on producing rules according to four control points, by decreasing the number of control points, the curves are generated with developed program. In case of the number of control points is more than four; the first and the last control points to given doesn't changed and they accepted as P_0 and P_3 (extreme) control points. Also the control points to interval in P_0 and P_3 are respectively according to sequent arrangement were accepted as P_1 and P_2. When the value X of request point is arrive to value X of the one of interval control points, the information of control points of P_1 and P_2 are updated to new values. For instance, let five control points Q_0, …, Q_4 be given. In this case, for first step, P_0 and P_3 control points respectively are Q_0 and Q_4 and P_1 and P_2 control points respectively are Q_1 and Q_2. These aren't changed until the distance of P_X is less or equal to the distance of Q_1. When distance of P_X is great to the distance of Q_1, P_0 and P_3 control points respectively are still Q_0 and Q_4 and P_1 and P_2 control points respectively are Q_2 and Q_3.

In this work that to be aim of curve fitting by fuzzy logic and rough set theory according to the distance X and control points in curve fitting processes, for a testing curve, approximate 400 rules is written with reducing of exist too many rules (approximate 100.000 and generated by using a special program). For one point (x, y), The image of input and output parameters and the calculation of the distance Y is depicted in Fig. 1.

In this work, the disadvantage of generated of parametric curves such as bezier curve or spline curve which the process is updated of all of algorithm steps needed, it is thrown and only process of update of two interval control points become closer added control points to be enough is provided. The Bezier curve was selected for generated rules because of it was base of NURBS and B-Spline curves and it was generalized.

6. Experimental Results

By doing given values to P_X input parameter in between 1 and the distance of $|P_0P_3|$ (that is from 150 to 650 for this example) for a problem of control points is to be (150,170), (180,500), (200,800) and (650,170), the create of curve is tried.

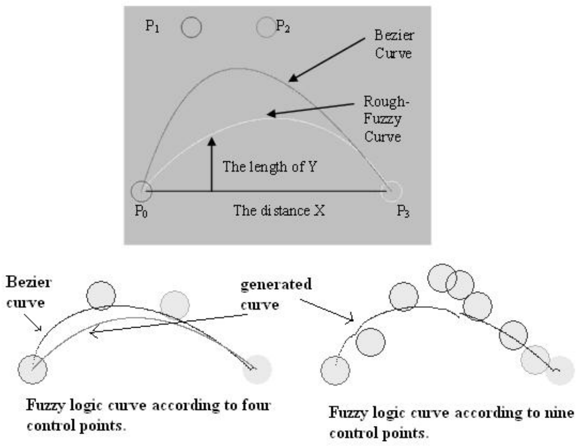

Fig. 6 (a) A image of curve from developed program, (b) The curve which has nine control points.

The visualization of rule table using for the solving of this problem is presented in Fig. 5. According to this rule database, the length value of Y for the increase of X is arrived by using rough set theory. The developed program was listed and showed the value of Y appropriate to all of values X. For the simplicity and understanding, the image of curves generated by rough-fuzzy curve developed program and Bezier curve appropriate to this values is presented in Fig. 6.a. For two new examples similar to above sample, first have to be four control points and next eight to be done, the producing of curve is tried by doing given values to X input parameter in between 1 to the distance of $|P_0P_3|$. The visualization of the curve model of the rough-fuzzy processes corresponding to present in Fig. 5 according to this control points is presented in Fig. 6.b.

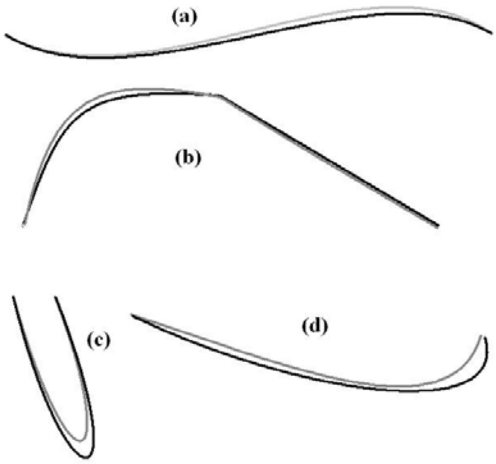

Fig. 7 Several curves generated by proposed hybrid algorithm and Bezier or Spline algorithm (Black lines are Bezier Curves; Gray Lines are curves of Hybrid Approach.

In point of comparing and underlying, the values of Square of the Pearson product moment correlation coefficient through the examples of 4, 5, 6, 7, 8, 9 and 10 control points (according to Fig. 6) is also shown in table 1. Moreover, the curves generated by proposed approach for several general types of Bezier curves can be meeting are presented in Fig. 7 (A number of control points in examples are interval in [8, 28]). For the smooth visualize of curves, some points of the curves are soften.

Table 1: The R2 values according to N control points.

A number of control points are given	The R2 value (in between Bezier and hybrid rough-fuzzy curves)	After from added control points (the first points; (150,170), (300,300) and (465,170).
4	0.567	(246,274)
5	0.600	(431,180)
6	0.635	(200,210)
7	0.712	(400,220)
8	0.798	(325,290)
9	0.870	(350,260)
10	0.970	(180,200)

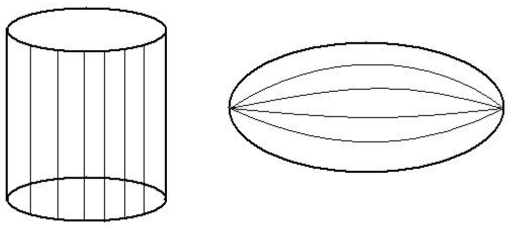

Fig. 8 The two simple 3-D shapes.

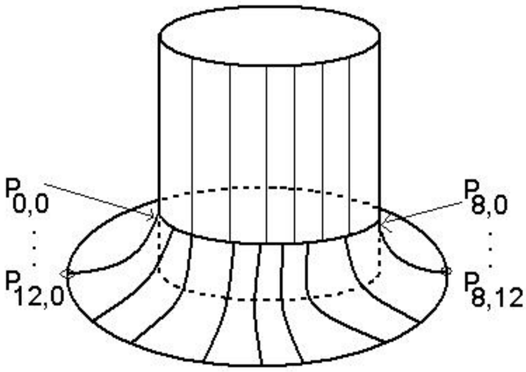

Fig. 9 The resulting blending shape for the simple shapes in Fig. 8.

Furthermore, proposed method has been applied to the three-dimensional object blending process. To make this, it is chosen that the two simple 3-D shapes shown in Fig. 8. The resulting blending shape in according to 12 control points is presented in Fig. 9. In here, the z values of each line segment are either fix or in the case of regular increments.

Form experimental results it can be seen that if the distance of the Y values (on Y axis) among of interval control points is a great many then similarity between produced curve and Bezier curve for the same control points is decreased, otherwise the produce curve is similar to Bezier curve. For obtained of curve more closer to control points, again look over of the α membership functions, that is used to create of decision table, by make scattered to a wide area as well as be possible will be good.

7. Conclusion and Discussion

In this study we proposed a general purpose curve fitting algorithm based on fuzzy logic model supported by rough sets. Form Fig. 6, it can be seen that if the P_2 control point angle increases and P control point angle decreases then the curvature reaches to minimal value and "length Y comes closer to P_0". On the other hand if "closeness of X to P_0" increases and the "closeness of X to P_0" in the representation of P_2 control parameters increases (given that the P_2 control point angle increases) then value Y becomes "distant to P_0". The different Y value that corresponds to X values of the curve fitting parameters is shown in Fig. 4.

With this study, finding the curve fitting parameters in 2-D will be simply and quickly determined. On the other hand, new conditions of curve fitting can be quickly determined by updating the rule base, the membership functions ranges according to changing control points. Our model can also be used to keep the control points under control in order to simultaneously inspect the curvatures during the curvature construction process.

The Rough set-based knowledge acquisition techniques have been widely developed and used for data mining. Many application areas, such as decision-making, medical diagnosis, classification and prediction, have taken the advantage of using the knowledge acquisition techniques. In this paper, we have utilized a methodology for the direct acquisition of knowledge base based on rough set theory. As with all methods based on rough set theory, the utilized method can deal with uncertainty and inconsistency in the data set quite easily and effectively.

The proposed method in this paper can easy be used to create 3D surfaces for surfaces in 3-D can be generated by joining the 2-D fitted curves. In further, the developments and efforts will be at this direction.

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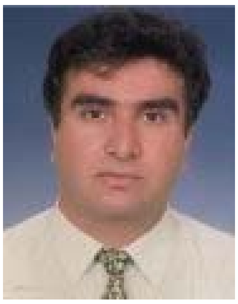

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