Satellite Interference Locating Method based on Support Vector Regression

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Summary
In this paper, a new satellite interference locating method based on support vector regression and the satellite multi-beam antennas (MBA) model is presented. The proposed method can achieve higher accurate estimates for direction of arrival (DOA) while avoiding the all-direction peak value searching technique used in other traditional DOA estimation methods. Meanwhile, the performance of this method in high SNR situations is better than that of RBF neutral network approach. Computer simulation results show the effectiveness of the proposed method.

Key words: interference location, satellite MBA, DOA estimation, RBF neutral network, support vector regression

1. Introduction
Satellite system is an open communication system which always suffers from intentional or unintentional interferences. Once the satellite system is interfered by radio frequency interference (RFI), the normal communication traffic performance is degraded. To cope with RFI, it is necessary to estimate the position of the source radiating interference wave, or interference location, with the information the satellite receives when interference happens.

Interference location techniques using communication satellite are rather difficult because satellite communication system is not a location-oriented system. A single satellite in the non-geostationary orbit can measure the incoming direction of the interference wave using the movement information between the satellite and the interference source. However, this method can not be applied for the communication satellite in the geostationary orbit.

Time Difference of Arrival (TDOA) location technique [1, 2] is the most popular interference location method that needs two satellites in different positions on the geostationary orbit. In this kind of systems, it is necessary that two satellites must receive the same interference wave. A single satellite interference location system based on angle measurement using on-board MBA is presented in [3]. However, it is difficult to obtain high accuracy by this method.

Other methods [4] of DOA estimation techniques in the MBA model based on spatial spectrum estimation have high accuracy performance but with expensive computation time.

A method based on RBF neural network for estimation of DOA in MBA model has been proposed in [5], which is a new means for single geostationary satellite interference location. The neural network approach is simple and computationally effective owing to its superior ability of parallel processing and learning ability.

This paper presents a support vector regression method (v-SVR) for interference location in the case of single geostationary satellite system. It considers the DOA problem as a mapping from the space of the MBA output to the space of DOA, and the support vector regression instead of RBF network is used to form the mapping function. Simulation results show that when the SNR is under 40dB, the performances of two methods are similar, while when SNR is above 40dB, our method based on v-SVR is better in estimate accuracy.

2. Model for Satellite Multiple Beam Antennas
In order to compare our method with the method in [5], the same MBA antennas model is adopted. The structure of MBA is completely different from the traditional phased array antenna, which consists of multiple elements with different directional characteristics, while MBAs consist of multiple feeds that illuminate a single reflector or lens aperture and produce narrow spot beams focusing at different spatial locations. Besides that, they are generally constructed so that path length differences between the elements are equalized. Therefore, for MBA, a propagating plane wave generates element responses that differ in amplitude but have the same phase.

We now consider an array composed of M elements (i.e. there are M spot beams) and assume that d narrowband sources, around with a known frequency \( w_0 \), impinging on the array from directions \( \Theta = [\Theta_1, \Theta_2, \ldots, \Theta_d]^T \).

Here \( \Theta_k \) is composed of an azimuth angle and elevation angle component, \( \Theta_k = [\phi_k, \theta_k]^T \).
The signal received by the ith element of the MBA at time t can be expressed as:

$$x_i = \sum_{k=1}^{d} a_i(\Theta_k) s_k(t) + n_i(t), i = 1, 2, \cdots, M$$  \hspace{1cm} (1)$$

where $$x_i(t)$$ is the composite signal at ith element, $$s_k(t)$$ is the signal generated by kth source, $$n_i(t)$$ is the noise at ith element, and $$a_i(\Theta_k)$$ is the response of ith element to signal arriving from direction $$\Theta_k$$. The form of $$a_i(\Theta_k)$$ has the form:

$$a_i(\Theta_k) = \sqrt{\frac{\pi D}{\lambda}} J_1(u)$$ \hspace{1cm} (2)$$

where: $$u = \frac{\pi D}{\lambda} \sin(\sqrt{(\theta_k - \theta_i)^2 + (\phi_k - \phi_i)^2})$$ and $$J_1(u)$$ is the first order Bessel function, $$\eta$$ is antenna efficiency, $$D$$ is the aperture of antenna; $$\lambda = c / 2 \pi \nu_0$$ is the wavelength of signal; $$[\theta_i, \phi_i]$$ are the azimuth and elevation of the ith elements' beam center respectively.

Expressed in vector notation, the composite signal vector received by the MBA at time t is:

$$\mathbf{x}(t) = A(\Theta) \mathbf{s}(t) + \mathbf{n}(t)$$ \hspace{1cm} (3)$$

with

$$\mathbf{x}(t) = [x_1(t), x_2(t), \cdots, x_M(t)]^T$$

$$\mathbf{s}(t) = [s_1(t), s_2(t), \cdots, s_d(t)]^T$$

$$\mathbf{n}(t) = [n_1(t), n_2(t), \cdots, n_M(t)]^T$$

$$A(\Theta) = [a(\Theta_1), a(\Theta_2), \cdots, a(\Theta_d)]^T$$

The d columns of $$A(\Theta)$$ are the $$M \times 1$$ response vectors of the array to signals from d different directions:

$$a(\Theta_k) = [a_1(\Theta_k), a_2(\Theta_k), \cdots, a_M(\Theta_k)]^T$$ \hspace{1cm} (4)$$

The objective of DOA estimation technique is to find the d directions of arrival denoted by $$\Theta = [\Theta_1, \Theta_2, \cdots, \Theta_d]^T$$ from of the received signal vectors $$\mathbf{x}(t_1), \mathbf{x}(t_2), \cdots, \mathbf{x}(t_N)$$.

3. DOA Estimation Using SVR

3.1 Support Vector Regression

Given a set of data points $$(x_i, y_i), i = 1, 2, \cdots, l$$, where $$x_i \in \mathbb{R}^n$$ is the input and $$y_i \in \mathbb{R}^l$$ is the target output. In order to generate the relationship between input and output, conventional regression learning method (such as neural network) applies ERM (Empirical Risk Minimization) principle as following:

$$R_{\text{emp}}[f] = \frac{1}{l} \sum_{i=1}^{l} L(f(x_i) - y_i)$$ \hspace{1cm} (5)$$

The regression function $$y = f(x)$$ is generated by minimizing (5), where $$L(f(x_i) - y_i)$$ represents the difference between real value $$y_i$$ and regression function $$f(x_i)$$.

The minimization of ERM does not assure the minimization of real risk with limited samples because of over fitting. Support vector regression is a learning method based on SRM (Structural Risk Minimization principle), which minimizes the empirical risk and model complexity to ensure the best generating ability under the condition of limited samples.

Support vector regression could be expressed as following:

$$f(x) = \sum_{i=1}^{l} (\alpha_i^* a_i - b) \mathbf{K}(\mathbf{x}_i, \mathbf{x}) + b$$ \hspace{1cm} (6)$$

$$= \sum_{i=1}^{l} (\alpha_i^* - \alpha_i) \phi(\mathbf{x}_i) \phi(\mathbf{x}) + b = \mathbf{W} \phi(\mathbf{x}) + b$$

where $$\alpha_i^* \geq 0, \alpha_i \geq 0 \ (i = 1, 2, \cdots, l)$$, $$\mathbf{x}_i$$ is the support vector when $$a_i \neq 0$$, $$\mathbf{K}(\mathbf{x}_i, \mathbf{x}) = \phi(\mathbf{x}_i) \phi(\mathbf{x})$$ is the kernel.

The computation process of $$\alpha_i^*$$, $$\alpha_i$$ and b is in fact the minimization process of the following primal problem:

$$\frac{1}{2} \|\mathbf{w}\|^2 + C \cdot R_{\text{emp}}[f]$$ \hspace{1cm} (7)$$

$$\|\mathbf{w}\|^2$$ defines the model complexity; $$R_{\text{emp}}[f]$$ represents the empirical loss function, C is a balance constant. After applying $$\varepsilon$$,
\[ L(f(x_i) - y_i) = \left| y - f(x) \right|_{\varepsilon} = \max \{ 0, \left| y - f(x) \right| - \varepsilon \} \] (8)
and introducing reflex variables \( \xi \) and \( \xi^* \), (7) could be changed to minimize

\[
\tau(w, \xi, \xi^*) = \frac{1}{2} \| w \|^2 + C \sum_{i=1}^{l} (\xi_i + \xi_i^*) \] (9)

Subject to

\[
y_j - (w \cdot x_i) - b - y_i \leq \varepsilon + \xi_i,
\]
\[
y_j - (w \cdot x_i) + b - y_i \leq \varepsilon + \xi_i^*,
\]
\[
\xi_i > 0, \xi_i^* > 0, \varepsilon > 0
\]

After introducing Lagrange multiplier \( a \) and \( a^* \), according to the Wolfe duality principle, (9) equals to maximize the following:

\[
W(a, a^*) = -\varepsilon \sum_{i=1}^{l} (a_i + a_i^*) + \sum_{i=1}^{l} (a_i^* - a_i) y_i
\]
\[
- \frac{1}{2} \sum_{i,j=1}^{l} (a_i^* - a_i)(a_j^* - a_j)K(x_i, x_j)
\] (10)

subject to \( \sum_{i=1}^{l} (a_i^* - a_i) = 0, a_i \in [0, \frac{C}{l}], a_i^* \in [0, \frac{C}{l}] \)

The optimal \( a_i^* \) and \( a_i \) (\( i = 1, 2, \ldots, l \)) is generated by maximization of (10). After that, by using any training data to \( y_i = w \phi(x_i) + b \), the parameter \( b \) is generated. Therefore, equation (6) is used for prediction with the new input data.

This paper uses an improved \( \nu - SVR \) method [4] which improves minimization (9) to be (11).

\[
\tau(w, \xi, \xi^*, \varepsilon) = \frac{1}{2} \| w \|^2 + C(\mu \varepsilon + \frac{1}{l} \sum_{i=1}^{l} (\xi_i + \xi_i^*)) \] (11)

It uses a new constant \( \mu \) and a new variable \( \varepsilon \) to adjust the complexity and reflex variable of the model. Therefore the process is changed to maximize

\[
W(a, a^*) = \sum_{i=1}^{l} (a_i^* - a_i) y_i
\]
\[
- \frac{1}{2} \sum_{i,j=1}^{l} (a_i^* - a_i)(a_j^* - a_j)K(x_i, x_j)
\] (12)

subject to \( \sum_{i=1}^{l} (a_i^* - a_i) = 0, a_i \in [0, \frac{C}{l}], a_i^* \in [0, \frac{C}{l}] \)
\[
\sum_{i=1}^{l} (a_i^* + a_i) \leq C \cdot \mu
\]

Schölkopf in [4] proved that this improved support vector regression method can automatically minimize \( \varepsilon \).

3.2 Support Vector Regression Learning

To make it easier to learn and estimation, a preprocessing procedure is used to normalize the raw data \( \mathbf{x} \) of MBA output.

\[
r(i) = \frac{\mathbf{x}(\Theta_i)}{\|\mathbf{x}(\Theta_i)\| i = 1, 2, \ldots, N \quad (13)}
\]

Meanwhile, this paper chooses Gaussian Radial Function as kernel function:

\[
k(\mathbf{r}_i, \mathbf{r}) = \exp(-\frac{\|\mathbf{r}_i - \mathbf{r}\|^2}{2\sigma^2}) \quad (14)
\]

Based on (6), we can get

\[
\theta = \sum_{i=1}^{l} (\alpha_i^* - \alpha_i) k(\mathbf{r}_1, \mathbf{r}) + b_1 \quad (15)
\]
\[
\varphi = \sum_{i=1}^{l} (\alpha_i^* - \alpha_i) k(\mathbf{r}_2, \mathbf{r}) + b_2 \quad (16)
\]

By maximizing (12) to generate optimal parameters \( \alpha_i^* \), \( \alpha_i^* \), \( \alpha_i^* \), \( \alpha_i^* \) \( (i = 1, 2, \ldots, l) \), using any training data, \( b_1 \) and \( b_2 \) are computed from (15) and (16).

3.3 DOA Estimation

In the estimation phase, to reduce the effects of noise, a preprocessing procedure is used to normalize and time averaged the observation \( \mathbf{x}(t) \).

\[
\hat{\mathbf{r}}(i) = \left\langle \frac{\mathbf{x}(\Theta_i)}{\|\mathbf{x}(\Theta_i)\|} \right\rangle \quad (17)
\]

where \( \left\langle \cdot \right\rangle \) denotes time average with \( N_s \) snapshots. If the input vector is corrupted with noise, which is also the real
case of satellite interference, the timed averaged preprocessing can reduce the effects of noise, and if the span (i.e. the number of samples for time average) goes to infinity, the noise effect is removed and the estimation performance is perfect.

4. Simulation Result

Computer simulations are carried out to verify the efficiency of the DOA approach used in conjunction with satellite MBA architecture based on support vector regression. The MBA configuration available for simulation is 7 feeds arranged in a hexagonal lattice, with 3dB beam width (BW) of individual spot beam about 1.5°. The MBA scenario is shown in Figure 1.

![MBA Scenario](image)

Fig 1 An illustration of an MBA Scenario

For accuracy performance measure, we use the normalized RMS for angular estimation error:

\[
\Delta \epsilon = \sqrt{\frac{1}{N_f} \sum_{i=1}^{N_f} (\Delta \epsilon(i))^2}
\]

(18)

where \(N_f\) is the number of ensembles to be averaged (set to 10 in the experiment), and \(\Delta \epsilon(i)\) is the angular error of \(i\)th estimation.

We trains support vector regression with discrete samples at BM/4 and BM/8 increments from a visible square \(\pm 2.5°\), denotes SVR1 and SVR2 respectively in the following figures. The number \(N_s\) of samples for time average is 10, 100 and 1000. The azimuth/elevation of the interference radio source is set to be \((-1.0°, 0°)\). To build a SVR model, \(\sigma^2\) in (14) is 0.2; C is 100 and \(\mu\) is 0.5 in (11) for this experiment.

![RMS angular error of SVR1 vs. SNR](image)

Fig 2 RMS angular error of SVR1 vs. SNR

![RMS angular error of SVR2 vs. SNR](image)

Fig 3 RMS angular error of SVR2 vs.

Fig 2 and Fig 3 depict the SVR1 and SVR2 support vector regression interference location with the MBA model. It is obvious that when SNR goes higher, the RMS goes down. Another trend, that for larger values \(N_s\), the RMS of angular error is smaller, can be seen from the two pictures. This is because the equation (17) with large \(N_s\) decreases the impact of noise. When SNR goes higher than 40dB, the differences between all the RMS with different \(N_s\) are very small. All of values of RMS are below 0.0001 when SNR is larger than 30 dB. The SVR2 performs better than SVR1 under 40dB in accuracy. This phenomenon proves that SVR2 with more training samples points is more effective in accuracy than SVR1 as we expected. However, SVR1 with less training samples consumes less time than SVR2 in the training process which occupies more than 99% of all the estimation time. The estimation performance includes accuracy and consuming time, so that a suitable number of training samples should be balanced between the two standards.
We also have tested the whole performance of SVR on simulated measurements at SNR ranging from 0dB to 60dB with an increment of 5dB. At each SNR, the nets were tested on samples from $-2.5^\circ$ to $+2.5^\circ$ at $0.05^\circ$ increments. The averaged RMS angular error at various SNR is shown in Fig 4.

4. Conclusion

This paper investigates the interference location techniques for communication satellites. To estimate the interference location of the single geostationary satellite, an interference location method based on support vector regression is proposed and comparisons with the method based on RBF neural network in conjunction with satellite on-board MBA are made. Simulation results show that the proposed method is a potential solution for single geostationary satellite interference location with better performance in accuracy compared with the RBF method.

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References