On Variable Precision Limited Tolerance Based Dependency in IIS

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Summary
The paper first analyses the expanded rough set models in [1,2], proposes a new variable precision limited tolerance model to deal with incomplete information systems based on variable precision limited tolerance relation, that is a more restrictive condition imposed on the similarity between objects by attribute values. It then introduces new definitions of object dependency, knowledge dependency and dependency degree. Using them as basic concepts, it does reductions. Through an example, it shows that this new reduction approach is rational and efficient.
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Key words:
incomplete information system, variable precision limited tolerance relation rough set model, knowledge dependency.

Introduction

Rough set theory, proposed by Z. Pawlak in the early 1980s, is serving as a new mathematical soft computing tool to deal with vagueness and uncertainty[2]. It has been widely applied in fields such as machine learning, knowledge discovery, decision making, expert system, intelligent control and etc. and has attracted a great deal of interest among researchers in many areas. The traditional rough set theory can only process complete information system, but there are rather more incomplete information systems in applications due to errors in data measurements or weak power of data acquisition, the classical rough set can not handle them, therefore, the Pawlak's rough set theory must be extended in order to overcome the problem and to facilitate knowledge acquisitions from incomplete information systems. At present, there are primarily two ways of extending the classical rough set theory from complete incomplete information system to incomplete one. The first is indirect approach that transfers an incomplete information system to a complete one by substituting high frequent occurrence attribute value for missing values. The second is direct approach that expands related concepts in classical rough set theory to those in incomplete system, which is currently studied by many experts all over the world with great concentrations.

In incomplete information systems, missing values existing makes it impossible generating an indiscernibility relation, i.e. an equivalence relation which is a basic relation traditional rough set depends on. So people relax the condition to form the relation such as that M. Kryszkiewicz suggested tolerance relation[4], J. Stefanowski and A. Tsoukias introduced non-symmetric similarity relation[5], J. Stefanowski and A. Tsoukias generalized valued tolerance relation[6], G.Y.Wang extended limited tolerance relation[1] and so forth[7].

Tolerance relation likely partitions objects without identical attribute values such as (1, *, *, *) , (*, 2, *, *) into one tolerance class. Partitioned granules are coarser, make number of objects in lower approximation set is lesser and probably form many inconsistent rules. Non-symmetric similarity relation likely classifies objects with many identical known attribute values such as (1, 2, *, 2, 0, 3), (1, 2, 3, *, 0, 3) into different similarity classes. Partitioned granules are finer and lead to that the number of objects in lower approximation set is bigger and the number of objects supporting each rule is less, resulting in over simulating phenomenon. Valued tolerance relation depends much on probability distribution of data in previous statistics and is hard to analyze. Limited tolerance relation lays on the middle of two extremes made by tolerance relation and non-symmetric similarity relation, getting rid of shortages caused by tolerance relation and non-symmetric similarity relation, but it also clusters two low equivalence possibility objects such as (3, 2, 1, 0, 2), (*, 2, *, *, *) into the same limited tolerance class for the restriction of its definition is still relax.

For the reason of the above, the present paper introduces a new approach using variable precision limited tolerance relation in incomplete information system to form an extended rough set model. It inherits the merits of limited tolerance relation and erases radical flaws of loosely constructing similarity between two objects. It benefits from setting appropriate precision value to meet the concrete requirements of data processing.

In order to process knowledge reduction and knowledge reasoning, study dependency among knowledge should be

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studied. Although many knowledge dependencies and their measurement methods has already been widely used in attribute significance measurement, rule extraction and so on, but it is still a pity that they established on the assumption of being complete information system. In IIS, original knowledge dependency and measurement methods has no ability to reflect the real situation actually.

For the sake of extending knowledge dependency and measurement methods to incomplete information system, the present paper uses variable precisions limited tolerance relation to establish a new rough set and puts forward several newly defined concepts such as object complete dependency, knowledge complete dependency, object partial dependency, knowledge partial dependency and dependency degree in incomplete information system by analyzing the expanded rough set model in [1]. It uses them as bases for object reduction and attribute reduction. Combining an example, it finds out dependency relationships among objects (or among object subsets) and among attributes (or among attribute subsets) and then reduce redundant object(s) and attribute(s), with good effects and validity for our proposed approach.

2. Variable Precision Tolerance Relation

Definition 1. An incomplete information system (IIS for short) is a quadruple $S=(U,AT,V,f)$, where U is the finite non-empty universe, AT is the finite non-empty attribute set. $AT=C\cup D$ and $C\cap D=\varnothing$, where C represents condition attribute set, D represents decision attribute set. For any $a\in AT$, V_a represents the value domain of attribute a. $V=\cup_{a\in AT}V_a$ is called attribute domain. f is called information function. $f(a,x)=v$ means the value of attribute *a* on object $x\in U$ is $v\in V_a$. If there at least one attribute $a\in C$ such that V_a contains null value, then S is called an IIS, otherwise complete information system. Null value (applicable) is denoted by "*". We always assume that $V_d(d\in D)$ does not contain null value "*".

Definition 2. Let $S=(U,AT,V,f)$ be an IIS, If $L_A\subseteq U\times U$ satisfying

$$L_A=\{(x,y)\in U\times U\mid \forall a\in A(a(x)=a(y)=*)\vee(P_A(x)\cap P_A(y)\neq\varnothing\wedge\forall a\in A(a(x)\neq *\wedge a(y)\neq *\rightarrow a(x)=a(y))\},\quad (1)$$

then L_A is called a limited tolerance relation[1],where $P_A(x)=\{a\in A\mid a(x)\neq *\}$.

Definition 3. For $\forall x\in U$, $A\subseteq AT$, we denote the limited tolerance class of x with respect to attribute subset A by

$$L_A(x)=\{y\in U\mid (x,y)\in L_A\}\quad (2)$$

Definition 4. For $\forall X\subseteq U$, $A\subseteq AT$, we denote the limited tolerance class[1] of object subset X with respect to attribute subset A by

$$L_A(X)=\cup_{x\in X}L_A(x)\quad (3)$$

Especially, when $A=AT$, $L_{AT}(x)$ is abbreviated by $L(x)$, $L_{AT}(X)$ by $L(X)$.

Because the above defined limited tolerance relation has the high possibility to classify two objects with low equivalence possibility such as (3, 2, 1, 0, 2), (*, 2, *, *, *) into an identical limited tolerance class for its relax restriction to its definition, we propose an improved one affecting more restriction on the definition of the relation with a variable precision as a first step in the follows.

Definition 5. Let $S=(U,AT,V,f)$, $A\subseteq AT$ $0\le\alpha\le 1$. $\mu(x,y)=|P_A(x)\cap P_A(y)|/|P_A(x)|$, where |X| denotes the cardinality of X and $x,y\in U$. We call the following binary relation L_A^α a variable limited tolerance relation in S:

$$L_A^\alpha=\{(x,y)\mid L_A(x,y)\wedge\mu(x,y)>\alpha\}\quad (4)$$

L_A^α is reflexive, but maybe not symmetric and transitive.

Definition 6. For $\forall x\in U$, $A\subseteq AT$, $0\le\alpha\le 1$, we denote the variable limited tolerance class of object x with respect to attribute subset A by

$$L_A^\alpha(x)=\{y\in U\mid (x,y)\in L_A^\alpha\}\quad (5)$$

Definition 7. For $X\subseteq U$, $A\subseteq AT$, the limited tolerance class of object subset X with respect to attribute subset A is denoted by

$$L_A^\alpha(X)=\cup_{x\in X}L_A^\alpha(x)\quad (6)$$

Especially, when $A=AT$, $L_{AT}^\alpha(x)$ is abbreviated by $L^\alpha(x)$, $L_{AT}^\alpha(X)$ by $L^\alpha(X)$.

Definition 8. Under the condition of given variable precision limited tolerance relation in incomplete information system $S=(U,AT,V,f)$, for $\forall X\subseteq U$, $A\subseteq AT$, $0\le\alpha\le1$, the lower and upper approximations of object subset X are defined respectively as follows:

$$\underline{A}_{\alpha}^{L}(X)=\{x\in U \mid L_{A}^{\alpha}(x)\subseteq X\} \qquad (7)$$

$$\overline{A}_{\alpha}^{L}(X)=\{x\in U \mid L_{A}^{\alpha}(x)\cap X\neq\varnothing\} \qquad (8)$$

3. Object Dependency and Its Measurement

In this section, we discuss about object dependency and its measurement in our expanded rough set model based on variable precision limited tolerance relation. We pay our attentions on object dependency of the entire attribute set. Partial object dependency can be studied as the same as on the entire attribute set. To save the space, the later discussion is omitted.

Definition 9. For $\forall x,y\in U$, $0\le\alpha\le1$, if $L^{\alpha}(x)\subseteq L^{\alpha}(y)$, then object y is said to be completely dependent on object x with variable precision α, denoted by $x\xrightarrow[\alpha]{}y$. Especially, if $x\xrightarrow[\alpha]{}y$ and $y\xrightarrow[\alpha]{}x$ hold at the same time, then x and y are said to be equivalent with α, denoted by $x\underset{\alpha}{\leftrightarrow}y$.

Definition 10. Let $X,Y\subseteq U$, $0\le\alpha\le1$. If $L^{\alpha}(X)\subseteq L^{\alpha}(Y)$ then object set Y is said to be completely dependent on object set X with variable precision α, denoted by $X\xrightarrow[\alpha]{}Y$. Especially, if $X\xrightarrow[\alpha]{}Y$ and $Y\xrightarrow[\alpha]{}X$, then X and Y are equivalent with precision α, denoted by $X\underset{\alpha}{\leftrightarrow}Y$.

Definition 11. For $\forall x,y\in U$, $0\le\alpha\le1$, if $L^{\alpha}(x)\cap L^{\alpha}(y)\neq\varnothing$ then object y is said to be partially dependent on object x with variable precision α and dependency degree m, denoted by $x\xrightarrow[\alpha]{m}y$, or x is said to be partially dependent on object y with variable precision α and dependency degree n, denoted by $y\xrightarrow[\alpha]{n}x$, where

$$\begin{cases} m=\left|L^{\alpha}(x)\cap L^{\alpha}(y)\right| \big/ \left|L^{\alpha}(x)\right| \\ n=\left|L^{\alpha}(x)\cap L^{\alpha}(y)\right| \big/ \left|L^{\alpha}(y)\right| \end{cases} \qquad (9)$$

Definition 12. Let $X,Y\subseteq U$, $0\le\alpha\le1$, if $L^{\alpha}(X)\cap L^{\alpha}(Y)\neq\varnothing$ then object set Y is said to be partially dependent on object set X with variable precision α and dependency degree m, denoted by $X\xrightarrow[\alpha]{m}Y$, or object set X is said to be partially dependent on object set Y with variable precision α and dependency degree n, denoted by $Y\xrightarrow[\alpha]{n}X$, where

$$\begin{cases} m=\left|L^{\alpha}(X)\cap L^{\alpha}(Y)\right| \big/ \left|L^{\alpha}(X)\right| \\ n=\left|L^{\alpha}(X)\cap L^{\alpha}(Y)\right| \big/ \left|L^{\alpha}(Y)\right| \end{cases} \qquad (10)$$

From the statements of the above four definition, we can see that if α=0, then object dependency relationship of variable precision limited tolerance relation becomes that of limited tolerance relation. From definitions 11 and 12, it is known that m and n satisfy $0\le m,n\le1$. If $m=0$,then Y does not depend on X,that is, X and Y are independent; if $m=1$, then, Y completely depends on X with variable precision α, that is, $X\xrightarrow[\alpha]{}Y$.Similarly, if $n=0$, then X does not depend on Y, that is, X and Y are independent; if $n=1$, then X totally depends on Y with variable precision α, that is, $Y\xrightarrow[\alpha]{}X$.

When AT is replaced by a attribute subset $A(\subseteq AT)$ in the related definitions in the above, we can also obtain complete/partial dependency between objects or object subsets with respect to attribute subset A.

4. Knowledge Dependency and Measurement

Now we discuss about knowledge dependency and its measurement in our expanded rough set model based on variable precision limited tolerance relation.

Definition 13. Let $S=(U,AT,V,f)$ be an incomplete information system, $A_1,A_2\subseteq AT$, $0\le\alpha\le1$. If $\forall x\in U(L_{A_1}^{\alpha}(x)\subseteq L_{A_2}^{\alpha}(x))$ is always held, then attribute subset A_2 is said to be completely dependent on attribute subset A_1 with variable precision α, denoted by $A_1\xrightarrow[\alpha]{}A_2$.

Definition 14. Let $S=(U,AT,V,f)$ be an incomplete information system, $A_1,A_2\subseteq AT$, $0\le\alpha\le1$. If

$\forall x \in U(L^{\alpha}_{A_1}(x) \cap L^{\alpha}_{A_2}(x) \neq \emptyset)$, then attribute subset A_2 is said to be partially dependent on attribute subset A_1 with variable precision α and dependency degree m , denoted by $A_1 \xrightarrow[\alpha]{m} A_2$ (or attribute subset A_1 is said to be partially dependent on attribute subset A_2 with variable precision α and dependency degree n , denoted by $A_2 \xrightarrow[\alpha]{n} A_1$,where

$$\begin{cases} m = \left|L^{\alpha}_{A_1}(x) \cap L^{\alpha}_{A2}(x)\right| \Big/ \left|L^{\alpha}_{A_1}(x)\right| \\ n = \left|L^{\alpha}_{A_1}(x) \cap L^{\alpha}_{A2}(x)\right| \Big/ \left|L^{\alpha}_{A_2}(x)\right| \end{cases} \quad (11)$$

It is obvious that definitions of complete/partial dependency with variable precision between attribute subset defined at the above are also suitable for the situation that attribute subsets are reduced to be consisted by only single attribute.

According to the above definitions, we can obtain a simple property as follows.

Property. Let $S = (U, AT, V, f)$ be an incomplete information system, $A_1, A_2 \subseteq AT$, $0 \le \alpha \le 1$. If $A_2 \subseteq A_1 \subseteq AT$, then $A_1 \xrightarrow[\alpha]{} A_2$.

5. Example

In the following, we illustrate how to compute object or knowledge dependency in concrete. Table 1 is An incomplete information system $S = (U, AT, V, f)$ where $U = \{x_1, x_2, \cdots, x_{10}\}$, $AT = \{a_1, a_2, \cdots, a_{10}\}$.

Table 1: An incomplete information system

U	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
x_1	3	2	1	0	0	*	2	1	*	3
x_2	*	2	*	0	0	2	*	1	1	3
x_3	2	*	*	3	0	*	2	3	1	*
x_4	*	2	1	*	0	2	*	*	2	3
x_5	3	2	*	0	*	2	*	*	2	3
x_6	2	2	*	*	*	*	2	*	*	*
x_7	*	2	*	*	0	*	2	2	2	2
x_8	3	2	*	0	0	*	*	1	*	3
x_9	*	2	*	3	*	*	*	*	*	*
x_{10}	*	*	*	*	*	*	*	*	*	*

In Table 1, according to definition 2 and 3,we can obtain: L(x_1)={x_1,x_2,x_4,x_5,x_8},L(x_2)={x_1,x_2,x_6,x_8},L(x_3)={x_3,x_6, x_9},L(x_4)={x_1,x_4,x_5,x_6,x_8,x_9}, L(x_5)={x_1,x_4,x_5,x_8}, L(x_6)= {x_2,x_3,x_4,x_6,x_7,x_9},L(x_7)={x_6,x_7,x_9},L(x_8)={x_1, x_2,x_4,x_5,x_8}, L(x_9)={x_3,x_4,x_6,x_7,x_9 }, L(x_{10})={x_{10}}.

If α =1/2, then according to definition 5, we have L^{α} (x_1)={x_1,x_2,x_8}, L^{α} (x_2)={x_1,x_2,x_8}, L^{α} (x_3)={x_3}, L^{α} (x_4) ={x_1,x_4,x_5}, L^{α} (x_5)={x_1,x_4,x_5,x_8}, L^{α} (x_6)= {x_3,x_6}, L^{α} (x_7)={x_7}, L^{α} (x_8)={x_1,x_2,x_5,x_8}, L^{α} (x_9) ={x_9}, L^{α} (x_{10})={x_{10}}.

So variable precision limited tolerance relation is more restrict that limited tolerance relation. When α =0, variable precision limited tolerance relation will become limited tolerance relation.

(1) If α =1/2, according to Definition 9, we obtain complete dependency between objects: $x_1 \xrightarrow[\alpha]{} x_8$, $x_2 \xrightarrow[\alpha]{} x_8$, $x_3 \xrightarrow[\alpha]{} x_6$, $x_4 \xrightarrow[\alpha]{} x_5$, $x_7 \xrightarrow[\alpha]{} x_6$. In addition, $x_1 \xleftrightarrow[\alpha]{} x_2$.

By definition 11, we can get partial dependency between objects with variable precision α =1/2 (list some of them) : $x_4 \xrightarrow[\alpha]{k_1} x_8$, k_1= 2/3, $x_5 \xrightarrow[\alpha]{k_2} x_8$, k_2= 3/4.

From object dependency relationship in the above, we can find that object x_8 is dependent on x_1 or x_2 , x_6 on x_3, x_5 on x_4 with the variable precision α . Moreover, $x_1 \xleftrightarrow[\alpha]{} x_2$. Therefore, objects x_2, x_5, x_6 and x_8 are not necessary and can be reduced. After remaining objects x_1, x_3, x_4, x_7, x_9, x_{10}, we gain an object reduced incomplete information system shown in Table 2.

Table 2. Objects reduced IIS from Table 1 with α =1/2

U	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
x_1	3	2	1	0	0	*	2	1	*	3
x_3	2	*	*	3	0	*	2	3	1	*
x_4	*	2	1	*	0	2	*	*	2	3
x_7	*	2	*	*	0	*	2	2	2	2
x_9	*	2	*	3	*	*	*	*	*	*
x_{10}	*	*	*	*	*	*	*	*	*	*

(2) When $\alpha = 1/3$, we obtain: $x_2 \xrightarrow[\alpha]{} x_1$, $x_2 \xrightarrow[\alpha]{} x_8$, $x_3 \xrightarrow[\alpha]{} x_6$, $x_3 \xrightarrow[\alpha]{} x_9$, $x_4 \xrightarrow[\alpha]{} x_1$, $x_4 \xrightarrow[\alpha]{} x_8$, $x_5 \xrightarrow[\alpha]{} x_1$, $x_5 \xrightarrow[\alpha]{} x_8$, $x_6 \xrightarrow[\alpha]{} x_9$, $x_7 \xrightarrow[\alpha]{} x_6$, $x_7 \xrightarrow[\alpha]{} x_9$. At the same time, $x_1 \xleftrightarrow[\alpha]{} x_8$, $x_4 \xleftrightarrow[\alpha]{} x_5$.

By definition 1, we can see that partial dependency between any two objects with α =1/3 and dependency degree k=1 validates. Thus, when α =1/3, all object dependency relationships are complete dependency.

On the base of the above object complete dependency, we can find objects x_1, x_5, x_6, x_8,x_9 are not necessary and can be reduced. The resulted incomplete information system keeping objects x_2,x_3,x_4, x_7,x_{10} is demonstrated in Table 3.

From the above two cases, we may say that different reductions can be obtained after different variable precisions are given. The most proper reduction can be reached by setting related precision in accordance with different data requirements. This is just the advantage of variable limited tolerance relation over limited one.

Table 3. Object reduced IIS from Table 1 with α =1/3

U	***a₁***	***a₂***	***a₃***	***a₄***	***a₅***	***a₆***	***a₇***	***a₈***	***a₉***	***a₁₀***
x_2	*	2	*	0	0	2	*	1	1	3
x_3	2	*	*	3	0	*	2	3	1	*
x_4	*	2	1	*	0	2	*	*	2	3
x_7	*	2	*	*	0	*	2	2	2	2
x_{10}	*	*	*	*	*	*	*	*	*	*

Suppose A_1=AT={a_1,a_2,⋯,a_{10}}, A_2={a_2,a_3,⋯,a_{10}} $\subseteq A_1$. If we take α =1/3 as precision, then from Table 2, we can obtain: $L_{A_2}^{\alpha}$ (x_1)={x_1,x_4}, $L_{A_2}^{\alpha}$ (x_3)={x_3}, $L_{A_2}^{\alpha}$ (x_4) ={x_1,x_4}, $L_{A_2}^{\alpha}$ (x_7)={x_7}, $L_{A_2}^{\alpha}$ (x_9)={x_3,x_4,x_7,x_9}, $L_{A_2}^{\alpha}$ (x_{10})={x_{10}}; and $L_{A_1}^{\alpha}$ (x_1)={x_1,x_4}, $L_{A_1}^{\alpha}$ (x_3)={x_3}, $L_{A_1}^{\alpha}$ (x_4)={x_1,x_4}, $L_{A_1}^{\alpha}(x_7) = \{x_7\}$, $L_{A_1}^{\alpha}(x_9)$ ={x_3,x_4,x_7,x_9}, $L_{A_1}^{\alpha}$ (x_{10}) ={x_{10}}. At this time, it satisfies that for $\forall x \in U$, $L_{A_1}^{\alpha}$ (x) $\subseteq$ $L_{A_2}^{\alpha}(x)$ holds. So by Definition 13, we obtain $A_1 \xrightarrow[\alpha]{} A_2$. It also verifies the correctness of the property proposes in the above.

Suppose again that A_3 ={a_4, a_8, a_9, a_{10}}, A_4={a_1, a_2, a_3, a_4, a_5, a_6, a_7}. If we still take α =1/3 as precision, from Table 2 we can obtain: $L_{A_3}^{\alpha}$ (x_1)={x_1}, $L_{A_3}^{\alpha}$ (x_3)={x_3}, $L_{A_3}^{\alpha}$ (x_4)={x_1,x_4}, $L_{A_3}^{\alpha}$ (x_7)={x_7}, $L_{A_3}^{\alpha}$ (x_9)={x_3,x_9}, $L_{A_3}^{\alpha}$ (x_{10}) ={x_{10}}; and $L_{A_4}^{\alpha}$ (x_1)={x_1,x_4,x_7}, $L_{A_4}^{\alpha}$ (x_3)={x_3,x_7}, $L_{A_4}^{\alpha}$ (x_4)={x_1,x_4,x_7}, $L_{A_4}^{\alpha}$ (x_7) ={x_1,x_3,x_4,x_7}, $L_{A_4}^{\alpha}$ (x_9)= {x_3,x_4,x_7,x_9}, $L_{A_4}^{\alpha}$ (x_{10}) ={x_{10}}. It satisfies that for $\forall x \in U$, $L_{A_3}^{\alpha}$ (x) $\subseteq$ $L_{A_4}^{\alpha}(x)$. So from Definition 13, we get $A_3 \xrightarrow[\alpha]{} A_4$.

When $B_1 \xrightarrow[\alpha]{} B_2$, if $B = B_1 \cap B_2 = \varnothing$, then attribute subset B_2 can be reduced from the whole attribute set AT , keeping remaining attributes; If $B = B_1 \cap B_2 \neq \varnothing$, then attribute subset $B_2 - B$ can be reduced from the whole attribute set AT, keeping remainders in AT . In our example, $A_3 \cap A_4 = \{a_4\}$ $\neq \varnothing$, so we keep a_4 in attribute subset A_4 and remove $A_4 - A_3 \cap A_4 = \{a_1, a_2, a_3, a_5, a_6, a_7\}$ from AT . Please see Table 4. Table 4 is much simple than Table 1.

Table 4. Attributes remained (α =1/3)

U	***a₄***	***a₈***	***a₉***	***a₁₀***
x_1	0	1	*	3
x_3	3	3	1	*
x_4	*	*	2	3
x_7	*	2	2	2
x_9	3	*	*	*
x_{10}	*	*	*	*

Through analyzing object and attribute dependency relationship and realizing object and attribute reduction, the original incomplete information system is greatly simplified without loss of system information quantity according to reference[2]. The size of the final obtained table is much better for preparation to further acquire rules as knowledge by data mining approach [2]. To the limitation of space, we do not need to give details.

6. Conclusions

In the present paper, we first introduce some concepts in extended rough set model in incomplete information system proposed in [1] based on variable precision limited tolerance relation which not only takes advantages of expanded rough set model based on limited tolerance relation in[1], but also overcomes drawbacks of restriction being too loose in limited tolerance relation. It appropriates different demands of various data processes. Second, it gives out some new concepts concerning object dependency and knowledge dependency in our variable precision model and put them as references to reduce objects and attribute. Finally, combining a real example, it concretely solves its every object and object subset dependency relationships and attribute or attribute subset dependency relationships, finds out reductions through removing redundant objects and influent attributes. From its working efficiency, it also solidifies that our approach is effective and rational.

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