Stability of Recurrent Neural Networks

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Summary
This article studies the existence, uniqueness and bounds almost periodic solution for recurrent neural networks with fractional distributed delays. We show that the zero solution is asymptotically and globally exponentially stable by using generalized Halanay inequality and Laypaunov functional method.

Key words:
Almost- periodic solution; Recurrent neural networks; Globally exponentially stable ;Fractional delays.

1. Introduction

It is well know that series of recurrent neural networks model have been proposed by many authors. Periodicity of the solutions involving almost-periodicity, pseudo-almost-periodicity, ω-periodicity and ω-periodicity with zero mean value arise in a wide variety off scientific and engineering applications including control systems, signal processing systems, dynamical systems ,identification systems and neural networks systems[1-3].This article deals with the neural network system of the form:

$$\begin{aligned} u'(t) &= -A(t)[u(t)+\int_0^t \frac{(t-\tau)^{\alpha-1}}{\Gamma(\alpha)} u(\tau)d\tau] \\ &+B(t)[f(u(t))+\int_0^t \frac{(t-\tau)^{\beta-1}}{\Gamma(\beta)} f(u(\tau))d\tau] \\ &+C(t)[f(u(t-\sigma(t))) \qquad (1) \\ &+\int_0^t \frac{(t-\tau)^{\gamma-1}}{\Gamma(\gamma)} f(u(\tau-\sigma(\tau))d\tau] \\ &+Q(t), \quad 0<\alpha,\beta,\gamma\le 1, J=[0,T] \end{aligned}$$

Subject to the initial condition u(0)= u_0,with the following assumption

(H1) A(t):=[a_{ij}(t)], B(t):=[b_{ij}(t)] and C(t):=[c_{ij}(t)] are continuous and nonnegative n x n connection matrices with

$$\overline{A} := \sup_{t\in J} A(t), \overline{B} := \sup_{t\in J} B(t), and$$

$$\overline{C} := \sup_{t\in J} C(t).$$

(H2) u(t) corresponds to the state of the *i-th* unit at time t.

(H3) Q(t) denotes the external bias on the *i-th* unit at time t, which is continuous and nonnegative on J with

$$\overline{Q} := \sup_{t\in J} Q(t).$$

(H4) f(u) denotes the n- dimensional activation function which is continuous and nonnegative on $\mathbf{R}^n_+$. Moreover, for a positive constant ℓ,satisfies |f(u)|≤ ℓ ||u|| and that f(0)=0.

Our plan is as follows :In section 2,we discuss the existence and uniqueness of almost-periodic solution for system (1) by using Schauder fixed point theorem and Banach fixed point theorem respectively[4].In section 3, we study the explicit bounds of the solution for system (1) by using the explicit bounds of some inequalities involving fractional order. In section 4 ,we discuss the global exponentially stability for the zero solution of (1) by using the generalized Halanay inequality and constructing a suitable Lyapunov function with Dini derivative and we illustrate our results with an example .The following definitions and results are used in the sequel.

Definition 1.1.[5]The fractional integral operator I^α of the continuous function f(t) is given by:

$$= f(t)*\psi_\alpha(t) \; I_0^\alpha f(t) := I^\alpha f(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1} f(\tau)d\tau$$

Where:
0< α ≤1 and

$$\psi_\alpha(t) = \frac{t^{\alpha-1}}{\Gamma(\alpha)}$$

for t>0 and $\psi_\alpha(t)$ =0 for t≤0 and $\psi_\alpha(t) \to \delta(t)$ (the delta function) as α→0.

Definition 1.2.A function fε**B** (**B** is Banach space) is called almost periodic in tε**R** uniformly in any K a bound subset of **B**,if for each ε>0,there exitsts δ_ε>0 such that every interval of length δ_ε>0 contains a number s with the following property:

$$\|f(t+s,u) - f(t,u)\| < \varepsilon,$$

$$t \in R, u \in K.$$

Definition 1.3.The network(1) is said to be globally exponentially stable ,if there are constants ε>0 and M≥1 such that for any two solutions x(t) and y(t) with the initial function Φ and φ respectively ,for all t≥$t_0$one has

$$|x(t)-y(t)| \le M\|\Phi-\varphi\| \exp^{-\varepsilon(t-t_0)}.$$

Definition 1.4.Let f:**R**→**R** be a continuous function. Then the upper right Dini derivative is:

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$$D^{+} f(t) = \overline{\lim}_{h\to 0} \frac{1}{h}[f(t+h) - f(t)].$$

Lemma 1.1.[6](Generalized Halanay inequality) Assume p(t) and q(t) be continuous with p(t)>q(t)>0 and

$$\inf_{t\ge t_0} \frac{p(t)-q(t)}{1+2\tau q(t)} \ge \eta > 0$$

for all t≥0,and y(t) is a continuous function on $t \ge t_0$ satisfying the following inequality for all

$$t \ge t_0 : D^{+}y(t) \le -p(t)y(t) + q(t)\overline{y}(t), \quad where$$
$$\overline{y}(t) = \sup_{t-\tau\le s\le t}\{|y(s)|\}.$$

Then for $t \ge t_0$,we have,

$$y(t) \le \overline{y}(t_0)\exp^{-\lambda*(t-t_0)} \quad in\ which\ \lambda^{*}$$

Is

$$\lambda^{*} := \inf_{t\ge t_0}\{\lambda(t) : \lambda(t) - p(t) + q(t)\exp^{\lambda(t)\tau} = 0\}. \tag{2}$$

Lemma1.2. [7] Let X(t) be a fundamental matrix of the nonlinear system x′(t)=f(t,x(t)).Assume further that there exists a constant K>O such that

$$\int_0^t \|X(s)\|ds \le K, t \ge 0.$$

Further let

$$\|f(t,x)\| \le \mu\|x\|$$

With 0≤μ≤1/K.
Then the zero solution is asymptotically stable.

2. The existence and uniqueness solution.

In this section we give conditions for the existence and uniqueness of almost-periodic solution for the modeling system(1).Define a Banach space
B :=C[J,$\mathbf{R}^n_+$] endow with the norm :

$$\|u\| = \sup_{t\in J}\{|u(t)|\}.$$

System (1) is equivalent to:

$$u(t) = u_o + \int_0^t \{-A(s)[u(s) + I^{\alpha}u(s)] + B(s)[f(u(s)) + I^{\beta}f(u(s))] + C(s)[f(u(s-\sigma(s))) + I^{\gamma}f(u(s-\sigma(s))] + Q(s)\}ds \tag{3}$$

Define a continuous operator P∈$\mathbf{R}^n_+$ as follows

$$Pu(t) = u_o + \int_0^t \{-A(s)[u(s) + I^{\alpha}u(s)] + B(s)[f(u(s)) + I^{\beta}f(u(s))] + C(s)[f(u(s-\sigma(s))) + I^{\gamma}f(u(s-\sigma(s))] + Q(s)\}ds$$

Let U be a convex close subset of **B** define by

$$U := \{u(t) : u(t) \in B, \|u\| \le r; r > o\},$$

Then we have

Theorem 2.1. Let assumptions (H1-H4) be hold. If

$$0 < \rho := \overline{A}T[1 + \frac{T^{\alpha}}{\Gamma(\alpha+1)}] + \overline{B}\ell T[1 + \frac{T^{\beta}}{\Gamma(\beta+1)}] + \overline{C}\ell T[1 + \frac{T^{\gamma}}{\Gamma(\gamma+1)}] < 1$$

Then the modeling system (1) has a solution.

Proof. To prove that (1) has a solution we only need to prove that P has a fixed point

$$|Pu(t)| \le |u_0| + \int_0^t \{|A(s)|[|u(s)| + I^{\alpha}|u(s)|] + |B(s)|[|f(u(s))| + I^{\beta}|f(u(s))|] + |C(s)|[|f(u(s-\sigma(s))| + I^{\gamma}|f(u(s-\sigma(s))|] + |Q(s)|\}ds$$
$$\le |u_0| + \overline{A}\|u\|T[1 + \frac{T^{\alpha}}{\Gamma(\alpha+1}] + \overline{B}\ell\|u\|T [1 + \frac{T^{\beta}}{\Gamma(\beta+1)}] + \overline{C}\ell\|u\|T[1 + \frac{T^{\gamma}}{\Gamma(\gamma+1)}] + T\overline{Q}$$
$$= |u_0| + \rho\|u\| + T\overline{Q}.$$

Hence we have

$$|Pu(t)| \le \frac{|u_0| + T\overline{Q}}{1-\rho} := r, \rho < 1.$$

Thus P: $B_r \to B_r$. Then P maps B_r into itself. And P maps the convex closure of P[B_r]into itself. Since f is bouned on B_r ,P[B_r] is equicotinuous and the Schauder fixed point theorem shows that P has a fixed point u(t) ∈ **B** such that P(u(t))=u(t),which is corresponding to the solution of (1)

Theorem 2.2. Let assumptions (H1-H3) be hold. Assume that there exist a positive constant L such that for

$$u, v \in B, |f(u) - f(v)| \le L\|u - v\|$$

Satisfies

$$\{\bar{A}T[1+\frac{T^{\alpha}}{\Gamma(\alpha+1)}]+\bar{B}LT[1+\frac{T^{\beta}}{\Gamma(\beta+1)}]$$
$$+\bar{C}LT[1+\frac{T^{\gamma}}{\Gamma(\gamma+1)}]\}<1$$

Then system (1) has a unique solution.

Proof. By the assumptions of the theorem ,we have

$$|P(u(t))-P(v(t))|\le\int_0^t\{|A(s)|[|u(s)-v(s)|$$
$$+I^{\alpha}|u(s)-v(s)|]+|B(s)|[|f(u(s))$$
$$-f(v(s))|+I^{\beta}|f(u(s))-f(v(s))|]$$
$$+|C(s)|[|f(u(s-\sigma(s))-f(v(s-\sigma(s))|$$
$$+I^{\gamma}|f(u(s-\sigma(s))-f(v(s-\sigma(s))|]\}ds$$
$$\le\bar{A}\|u-v\|T[1+\frac{T^{\alpha}}{\Gamma(\alpha+1)}]$$
$$+\bar{B}L\|u-v\|T[1+\frac{T^{\beta}}{\Gamma(\beta+1)}]$$
$$+\bar{C}L\|u-v\|T[1+\frac{T^{\gamma}}{\Gamma(\gamma+1)}]$$
$$=\{\bar{A}T[1+\frac{T^{\alpha}}{\Gamma(\alpha+1)}]+\bar{B}LT[1+\frac{T^{\beta}}{\Gamma(\beta+1)}]$$
$$+\bar{C}LT[1+\frac{T^{\gamma}}{\Gamma(\gamma+1)}]\}\|u-v\|$$

Implies that P is a contraction mapping then by Banach fixed point theorem has unique fixed point which corresponds to the solution of system (1).
The next result will discuss the conditions of almost – periodic for the solution (3).
(H5):For $\varepsilon>0$ and $\theta>0$ assume that A(t),B(t),C(t) and Q(t) are almost-periodic functions of period θ. And satisfy

$$|A(t+\theta)-A(t)|\le\frac{\varepsilon}{4T\|u\|[1+\frac{T^{\alpha}}{\Gamma(\alpha+1)}]},$$
$$|B(t+\theta)-B(t)|\le\frac{\varepsilon}{4T\|u\|\ell[1+\frac{T^{\beta}}{\Gamma(\beta+1)}]},$$
$$|C(t+\theta)-C(t)|\le\frac{\varepsilon}{4T\|u\|\ell[1+\frac{T^{\gamma}}{\Gamma(\gamma+1)}]},$$

And

$$|Q(t+\theta)-Q(t)|\le\frac{\varepsilon}{4T}.$$

Lemma 2.1. Let assumption (H5) be hold. Then operator P is almost periodic function.
Proof. By assumption (H5) we have
$\forall\varepsilon>0$
There exist δ_ε such that there exist
$s\in[\gamma,\gamma+\delta_\varepsilon]$
With the following properties

$$|P(u(t+\theta))-P(u(t)|\le\int_0^t\{|A(s+\theta)-A(s)|$$
$$[|u(s)|+I^{\alpha}|u(s)|]+|B(s+\theta)-B(s)|$$
$$[|f(u(s))|+I^{\beta}|f(u(s))|]+$$
$$|C(s+\theta)-C(s)|[|f(u(s-\sigma(s))|$$
$$+I^{\gamma}|f(u(s-\sigma(s))|]+|Q(s+\theta)-Q(s)|\}ds$$
$$\le\frac{\varepsilon}{4T\|u\|[1+\frac{T^{\alpha}}{\Gamma(\alpha+1)}]}\times T\|u\|[1+\frac{T^{\alpha}}{\Gamma(\alpha+1)}]$$
$$+\frac{\varepsilon}{4T\|u\|\ell[1+\frac{T^{\beta}}{\Gamma(\beta+1)}]}\times T\|u\|\ell[1+\frac{T^{\beta}}{\Gamma(\beta+1)}]$$
$$+\frac{\varepsilon}{4T\|u\|\ell[1+\frac{T^{\gamma}}{\Gamma(\gamma+1)}]}$$
$$\times T\|u\|\ell[1+\frac{T^{\gamma}}{\Gamma(\gamma+1)}]+\frac{\varepsilon}{4T}\times T=\varepsilon.$$

Implies that P is almost periodic function.
Theorem 2.3. Let assumptions of theorem 2.2 with (H5)be hold .Then system (1) has a unique almost periodic solution.
Proof. By theorem 2.2 and Lemma 2.1.

3. Explicit bounds for fractional integral inequalities.

In this section we establish the explicit bounds variants integral inequalities in the following results.
Theorem 3.1. Assume that u(t) is nondecreasing nonnegative and continuous on J. If for $0<\alpha,\beta,\gamma\le1$,$c_k\ge0$,for all k=1,….,m and $\mu\ge0$

$$u(t)\le\mu+\int_0^t[\sum_{k=1}^{m}c_k(u(s)+I^{\alpha_k}u(s))]ds \tag{4}$$

Then

$$u(t)\le\mu\{1+[\sum_{k=1}^{m}c_k]\exp\sum_{k=1}^{m}c_k[1+\frac{T^{\alpha_k-1}}{\Gamma(\alpha_k)}]t\} \tag{5}$$

Proof. Define a function z(t) by the right hand side of (4)

$$z(t) := \mu + \int_0^t [\sum_{k=1}^m c_k (u(s) + I^{\alpha_k} u(s))] ds$$

Then z(t) is nondecreasing nonnegative and continuous on J. Also z(0)=μ, μ(t)≤z(t) and

$$z'(t) = \sum_{k=1}^m c_k (u(t) + I^{\alpha_K} u(t))$$

$$\leq \sum_{k=1}^m c_k (z(t) + I^{\alpha_k} z(t)).$$

Suppose that

$$x(t) := \sum_{k=1}^m c_k (z(t) + I^{\alpha_k} z(t))$$

Then

$$x(0) = \mu[\sum_{k=1}^m c_k], z(t) \leq x(t), z'(t) \leq x(t),$$

Then by using some properties of the fractional calculus [5] we obtain

$$x'(t) \leq \sum_{k=1}^m c_k [z'(t) + \frac{d}{dt} I^{\alpha_k} z(t)$$

$$\leq \sum_{k=1}^m c_k [1 + \frac{T^{\alpha_K - 1}}{\Gamma(\alpha_K)}] z'(t)$$

$$\leq \sum_{k=1}^m c_k [1 + \frac{T^{\alpha_K - 1}}{\Gamma(\alpha_K)}] x(t) \Rightarrow$$

$$x(t) \leq \mu[\sum_{k=1}^m c_k] \exp \sum_{k=1}^m c_k [1 + \frac{T^{\alpha_k - 1}}{\Gamma(\alpha_k)}] t \Rightarrow$$

$$z(t) \leq \mu\{1 + [\sum_{k=1}^m c_k] \exp \sum_{k=1}^m c_k [1 + \frac{T^{\alpha_k - 1}}{\Gamma(\alpha_k)}] t\}$$

Consequently we obtain the result. Hence the proof.

Theorem 3.2. Assume that u(t) is nondecreasing nonnegative and continuous on J. If for t∈j=[0,T], 0<α,β,γ≤1,c_k≥0,for all k=1,….,m and continuous, nondecreasing and nonnegative function a(t) on J

$$u(t) \leq a(t) + \int_0^t [\sum_{k=1}^m c_k (u(s) + I^{\alpha_k} u(s))] ds \quad (6)$$

Then

$$u(t) \leq a(t) + e(t)$$

$$\{1 + [\sum_{k=1}^m c_k] \exp \sum_{k=1}^m c_k [1 + \frac{T^{\alpha_k - 1}}{\Gamma(\alpha_k)}] t\} \quad (7)$$

Where

$$e(t) := \int_0^t [\sum_{k=1}^m c_k [a(t) + I^{\alpha_k} a(t)] ds.$$

Proof. Define a function z(t)by

$$z(t) := \int_0^t [\sum_{k=1}^m c_k [u(s) + I^{\alpha_k} u(s)] ds,$$

Then z(t) is nondecreasing nonnegative and continuous on J. Also z(0)=0,u(t) <a(t)+z(t) and

$$z(t) \leq \int_0^t [\sum_{k=1}^m c_k [(a(t) + z(t))$$

$$+ I^{\alpha_k} (a(t) + z(t))]] ds$$

$$\leq \int_0^t [\sum_{k=1}^m c_k (a(t) + I^{\alpha_k} a(t))] ds$$

$$+ \int_0^t [\sum_{k=1}^m c_k [z(t) + I^{\alpha_k} z(t)]] ds$$

$$:= e(t) + \int_0^t [\sum_{k=1}^m c_k [z(t) + I^{\alpha_k} z(t)]] ds$$

$$\frac{z(t)}{e(t)} = 1 + \int_0^t [\sum_{k=1}^m c_k [\frac{z(t)}{e(t)} + I^{\alpha_k} \frac{z(t)}{e(t)}]] ds$$

Thus an application of theorem 3.1 ,with μ=1 we have

$$\frac{z(t)}{e(t)} \leq 1 + [\sum_{k=1}^m c_k] \exp \sum_{k=1}^m c_k [1 + \frac{T^{\alpha_K - 1}}{\Gamma(\alpha_K)}] t$$

$$z(t) \leq e(t)\{1 + [\sum_{k=1}^m c_k] \exp \sum_{k=1}^m c_k [1 + \frac{T^{\alpha_K - 1}}{\Gamma(\alpha_K)}] t\} \Rightarrow$$

$$u(t) \leq a(t) + e(t) \times$$

$$\{1 + [\sum_{k=1}^m c_k] \exp \sum_{k=1}^m c_k [1 + \frac{T^{\alpha_K - 1}}{\Gamma(\alpha_K)}] t\}.$$

Theorem 3.3. Let the assumptions (H1-H4) be hold. Then all solutions of system (1) are bounded.

Proof. Let u(t) be a nondecreasing nonnegative and continuous solution on J for the system (1). Denotes

$$a(t) := |u_0| + \int_0^t |Q(s)| ds.$$

Then in view of theorem 3.2,u(t) has a bound of the form (7):

$$|u(t)| \le |u_0| + \int_0^t \{|A(s)|[|u(s)| + I^\alpha |u(s)|]$$
$$+ |B(s)|[|f(u(s))| + I^\beta |f(u(s))|]$$
$$+ |C(s)|[|f(u(s-\sigma(s))| + I^\gamma |f(u(s-\sigma(s))|]$$
$$+ |Q(s)|\} ds$$
$$\le a(t) + \int_0^t \{\overline{A}[|u(s)| + I^\alpha |u(s)|]$$
$$+ \ell \overline{B}[|u(s)| + I^\beta |u(s)|]$$
$$+ \ell \overline{C}[|u(s)|] + I^\gamma |u(s)|]\} ds.$$

4. Asymptotic and global exponential stability.

The study of stability of system(1) is equivalent to study of stability for the following system

$$u'(t) = -A(t)[u(t) + \int_0^t \frac{(t-\tau)^{\alpha-1}}{\Gamma(\alpha)} u(\tau) d\tau]$$
$$+ B(t)[f(u(t)) + \int_0^t \frac{(t-\tau)^{\beta-1}}{\Gamma(\beta)} f(u(\tau)) d\tau]$$
$$+ C(t)[f(u(t-\sigma(t))$$
$$+ \int_0^t \frac{(t-\tau)^{\gamma-1}}{\Gamma(\gamma)} f(u(\tau - \sigma(\tau)) d\tau]. \quad (8)$$

On the asymptotic stability of the zero solution for system (8) we have the following result.

Theorem 4.1. Let the assumptions (H1-H4) be hold. Then the zero solution of (8) is asymptotically stable.

Proof. Directly from Lemma 1.2 with

$$\ell = \frac{1}{T\|u\|}$$

Now by using the generalized Halanay inequality and Lyapunov method to establish the global exponential stability. For this purpose we need to the following assumptions.

(H6) Denotes

$$\max_{1 \le i \le n} |u_i(t)| := |\overline{u}_i(t)|, \; p(t)$$
$$:= \min_{1 \le i \le n} \{\sum_{j=1}^n a_{ij}(t)[1 + \frac{t^{\alpha-1}}{\Gamma(\alpha)}]\}$$

And

$$q(t) := \max_{1 \le i \le n} \{\sum_{j=1}^n b_{ij}(t)\ell[1 + \frac{t^{\beta-1}}{\Gamma(\beta)}]$$
$$+ \sum_{j=1}^n c_{ij}(t)\ell[1 + \frac{t^{\gamma-1}}{\Gamma(\gamma)}]\}$$

Such that p(t)>q(t)>0.Moreover,

$$\inf_{t \ge t_0} \{p(t) - q(t)\} \ge \eta > 0, t \ge t_0 \ge 0.$$

(H7) For a positive constant ℓ,satisfies

$$|f_i(u_i)(t)| \le \ell |u_i(t)|$$

And that $f_i(0)=0$.

Theorem 4.2. Let assumptions (H6) and (H7) be hold. Then the zero solution of system (8) is globally exponential stable.

Proof. Set

$$u(t) := (u_1(t), ..., u_n(t))^T = \overline{u} - \underline{u}$$

where $\overline{u}, \underline{u}$

are two solutions for (8) with the initial conditions $\overline{u}(0) = \overline{u}_0$, $\underline{u}(0) = \underline{u}_0$ respectively.

Consider the Lyapunov function[6],

$$V(u) := \sum_{i=1}^n |u_i(t)| \text{ with } \overline{V}(u) := \sum_{i=1}^n |\overline{u}_i(t)|.$$

Calculating the Dini derivative D^+V along the solution of (8) and by using the assumptions (H6-H7),we get

$$D^+V = \sum_{i=1}^n \operatorname{sgn}(u_i(t)) u_i'(t)$$
$$= \sum_{i=1}^n \{\operatorname{sgn}(u_i(t))(-\sum_{j=1}^n a_{ij}(t)[u_j(t) + I^\alpha u_j(t)]$$
$$+ \sum_{j=1}^n b_{ij}(t)[f_j(u_j(t)) + I^\beta u_j(u_j t))]$$
$$+ \sum_{j=1}^n c_{ij}(t)[f_j(u_j(t - \sigma_j(t))$$
$$+ I^\gamma f_j(u_j(t - \sigma_j(t))])\}$$
$$\le \sum_{i=1}^n \{\operatorname{sgn}(u_i(t))(-\sum_{j=1}^n a_{ij}(t)[u_j(t)$$
$$+ \frac{t^{\alpha-1}}{\Gamma(\alpha)} u_j(t)] + \sum_{j=1}^n b_{ij}(t)\ell[|u_j(t)|$$
$$+ \frac{t^{\beta-1}}{\Gamma(\beta)} |u_j(t)|] + \sum_{j=1}^n c_{ij}(t)\ell[|u_j(t - \sigma_j(t))|$$
$$+ \frac{t^{\gamma-1}}{\Gamma(\gamma)} |u_j(t - \sigma_j(t))|]\}$$
$$\le \sum_{i=1}^n \{-\sum_{j=1}^n a_{ij}(t)[1 + \frac{t^{\alpha-1}}{\Gamma(\alpha)}]|u_j(t)|$$
$$+ \sum_{j=1}^n b_{ij}(t)\ell[1 + \frac{t^{\beta-1}}{\Gamma(\beta)}]|u_j(t)|$$
$$+ \sum_{j=1}^n c_{ij}(t)\ell[1 + \frac{t^{\gamma-1}}{\Gamma(\gamma)}]|u_j(t - \sigma_j(t))|\}$$

$$\le -\min_{1\le i\le n}\{\sum_{j=1}^{n} a_{ij}(t)[1+\frac{t^{\alpha-1}}{\Gamma(\alpha)}]\}$$
$$\times\{\sum_{i=1}^{n}|u_i(t)| + \max_{1\le i\le n}\{\sum_{j=1}^{n} b_{ij}(t)\ell[1+\frac{t^{\beta-1}}{\Gamma(\beta)}]$$
$$+\sum_{j=1}^{n} c_{ij}(t)\ell[1+\frac{t^{\gamma-1}}{\Gamma(\gamma)}]\}\times\sum_{i=1}^{n}|\overline{u}_i(t)|$$
$$= -p(t)V + q(t)\overline{V}.$$

According to Lemma 1.1 we obtain

$$\sum_{i=1}^{n} |u_i(t;t_0;u_0)|$$
$$= V(t) \le V(t_0)\exp^{-\lambda^*(t-t_0)}, t \ge t_0 \ge 0$$

Where λ^* is defined in equation (2).
Implies that the zero solution of the network (8) and consequently (1) is globally exponentially stable. This complete the proof.

An example. Consider the simple recurrent neural network of the form

$$\begin{pmatrix} u_1'(t) \\ u_2'(t) \end{pmatrix} = -\begin{pmatrix} \cos(t) & 1 \\ 1 & 1 \end{pmatrix}$$
$$[\begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} + \begin{pmatrix} I^{\alpha}u_1(t) \\ I^{\alpha}u_2(t) \end{pmatrix}]$$

$$\begin{pmatrix} \sin(t) & 0 \\ 0 & 1 \end{pmatrix}[\begin{pmatrix} 0.5u_1(t) \\ 0.5u_2(t) \end{pmatrix} + \begin{pmatrix} 0.5I^{\beta}u_1(t) \\ 0.5I^{\beta}u_2(t) \end{pmatrix}]$$
$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}[\begin{pmatrix} 0.5u_1(t) \\ 0.5u_2(t) \end{pmatrix} + \begin{pmatrix} 0.5I^{\gamma}u_1(t) \\ 0.5I^{\gamma}u_2(t) \end{pmatrix}] + \begin{pmatrix} 1 \\ 0.5 \end{pmatrix}.$$

Subject to the initial condition u(0)=0.Assume that $\alpha=\beta=\gamma=1$,tϵJ:=[0,0.25],ℓ=L=0.5 and $\sigma(t)=0$.From above information , we have

$$\rho = \frac{20}{32} < 1 \to r = \frac{2}{3}.$$

Thus in view of theorems 2.3 and 3.3 the neural system has a unique ,almost-periodic and bounded solution ,also we can find that

$$p(t) = \min\{2|\cos(t)| + 2{,}4\} \quad and$$
$$q(t) = \max\{|\sin(t)|, 2\}$$

Then we have p(t)>q(t)>0.
Moreover,

$$\inf_{t\ge t_0}\{p(t) - q(t)\} \ge 2 := \eta > 0,$$

Thus in view of theorems 4.1 and 4.2,the solution of the system is asymptotically stable and globally exponentially stable.

5. Conclusion.

This paper studies the existence, uniqueness and bounds almost periodic solution for recurrent neural networks with fractional distributed delays.
The main point of this study is to apply the Dini derivative combined with Lyapunov function method to obtain exponentially stable solution for recurrent neural networks with fractional distributed delays.

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