Applying Fuzzy Logic to Estimate Setup Times in Sequence Dependent Single Machine Scheduling Problems

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Summary

Single machine scheduling problems with earliness and tardiness penalty are well known, but in the case of sequence dependent, the problem is more difficult and there are few researches around it. In related researches, setup time matrix was defined as a deterministic matrix.
In this paper sequence dependent single machine scheduling with earliness - tardiness penalty considerations using fuzzy setup time is presented. It is assumed that setup times are dependent on both of, level of technology (for machine) and degree of operation's similarity (between two precedence operations in the sequence).
When setup time matrix (base on real situations) obtained, the problem can be solved using common methods available in the literature. Two- way analysis of variance clearly indicated the superiority of the proposed approach in comparison of simulated results

Key words:
Fuzzy setup time, single machine scheduling, Earliness and tardiness penalty, Simulation, ANOVA

1. Introduction

1.1. Introduction to fuzzy rule based

The pioneering work of Zadeh (1965) [1] concerning the processing of the linguistic uncertainties by the fuzzy sets has opened a wide range of applications in many diverse fields. Fuzzy application areas include estimation, prediction, control, approximate reasoning, intelligent system design, machine learning, image processing, machine vision, pattern recognition, medical computing, robotics, optimization, civil, chemical and industrial engineering, etc.
Fuzzy rule-based systems have been successfully applied to various application areas such as control and classification [2]. While the main objective in the design of fuzzy rule-based systems has been the performance maximization, their comprehensibility has also been taken into account in some studies [3, 4].
The comprehensibility of fuzzy rule-based systems is related to various factors:
(i) Comprehensibility of fuzzy partitions (e.g., linguistic interpretability of each fuzzy set, separation of neighboring fuzzy sets, the number of fuzzy sets for each variable).
(ii) Simplicity of fuzzy rule-based systems (e.g., the number of input variables, the number of fuzzy if-then rules).
(iii) Simplicity of fuzzy if-then rules (e.g., type of fuzzy if-then rules, the number of antecedent conditions in each fuzzy if then rule).
(iv) Simplicity of fuzzy reasoning (e.g., selection of a single winner rule, voting by multiple rules).

1.2. Single machine scheduling

In this case, a set of n jobs, N = {1,…, n}, with a large common due date d must be scheduled on a single machine to minimize the total earliness and tardiness. All jobs are available at time zero and preemption is not allowed. The processing time for a job j is Pj and sequence-dependent setup times Sij are considered. In general, Sij and Sji are not equal to each other, that is, the time needed to setup for job j after job i may differ from that needed to setup for

Manuscript revised November 2006.

job i after job j. Setup times are represented by a matrix [S](which in this research it is calculated using fuzzy approach) . This problem is known as the early/tardy (E/T) problem, where the earliness of job j is: Ej = Max (0, d – Cj) and its tardiness is Tj = Max (0, Cj – d). Here, Cj is the completion time of job j. The objective is then to minimize total earliness – tardiness penalty.

The objective of this problem is consistent with the principles of the just-in-time manufacturing approach, where jobs are desired to be completed as close as possible to their due dates to avoid holding cost (due to earliness) and penalties for missing the due date (due to tardiness). Due to the existence of setup times, the problem becomes similar to the Traveling Salesman Problem, which is NP-hard and obtaining an optimal solution is very time consuming as n increases.

This problem with unrestricted due date *d* and equal earliness and tardiness penalty was first introduced in Kanet (1981)[5]. The problem is considered unrestricted when d is large enough not to restrict the scheduling decision. Kanet introduced a polynomial-time algorithm to solve the problem optimally. Since then many researchers introduced optimal procedures for various extensions of the problem ([6], [7], [8], [9]). Other researchers introduced approximate algorithms for different versions of the problem to solve for larger number of jobs ([10]). Baker and Scudder (1990) [11] published a comprehensive state-of-the-art review for different versions of the problem. Rabadi (1999) [12] developed a branch-and-bound (B&B) algorithm to solve the E/T problem with sequence-dependent setup times where it solved problems with up to 25 jobs. When the problems become large, obtaining optimal solutions becomes infeasible.

1.3. Fuzzy scheduling

So far, fuzzy sets have been used in scheduling under two conditions [13]:

Modeling: Although scheduling implies determinism, the imprecision was introduced in the general scheduling model. It deals with fuzzy constraints, fuzzy due dates and fuzzy processing time.

Solving deterministic models: Approximate reasoning based on fuzzy logic was used to build simple solvers [14, 15] or expert systems [16, 17].

However, in fuzzy scheduling, there are some researches around fuzzy due dates, fuzzy constraints, fuzzy processing times and approximate reasoning [13].

Deterministic models with fuzzy due dates have been proposed in several papers. Litoiu and Tadei [18] have proposed a periodic model with linear fuzzy due dates. The same linear fuzzy due dates with a non-periodic model have been used by Ishii et al [19].

Slany [20] has extended the term constraints over performance cost functions and resources. Expressed as soft barriers, the constraints allow the application of fuzzy concepts. Each constraint becomes a criterion in multi-criteria decision making framework.

Considering fuzzy processing times yields a more complex scheduling problem since it involves fuzzy arithmetic. Several models with processing times as fuzzy numbers have been used so far.

McCahon and Lee [21] have used fuzzy trapezoidal numbers in a flow shop problem. Ishibuchi et al. [22] have used general fuzzy numbers to describe the processing times in scheduling models.

Turksen et al. [16, 17] have proposed an approximate reasoning approach to solve a deterministic job shop scheduling problem. A job shop is a particular case of the general scheduling model described above, and it consists of a set of tasks and constraints.

As a related work, Yi and Wang [23] addressed a model for scheduling grouped jobs on identical parallel machines, which the model assumes set-up time is incurred when a machine changes from processing one type of component to a different type of component, and the objective is to minimize the total earliness-tardiness penalties. They developed a fuzzy logic embedded Genetic Algorithm and tested the efficiency of the algorithm for practical applications in larger scale production systems.

However, there are few closely related researches around applying fuzzy rule based in single machine with earliness – tardiness penalty.

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2. Problem Methodology

2.1. Problem status

In sequenced dependent setup single machine scheduling, a setup time matrix is prescribed and deterministic, it means that for precedence jobs a specific setup time is defined. In this research sequence dependent single machine scheduling with earliness and tardiness penalty using fuzzy setup time is assumed. Main focus of this research is to calculate setup time in fuzzy environment according to real condition that will be occurred in a shop. Certainly the results of fuzzy system (setup time) will be used in single machine scheduling problems with E/T penalty.

2.2. System identification

System is relationship between input and output variables that outputs variables are depend on inputs variables.
In this research due to real status in a shop, setup time is considered as output, that is depends on "degree of operation's similarity (for precedence jobs in sequence) and level of machine's technology.
The main structure of the proposed system is given in figure 1.

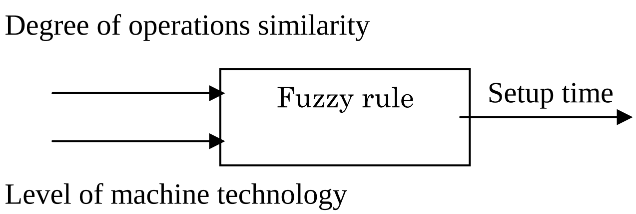

Figure1. Main structure of proposed system

3. Fuzzy Rule Base

In any diagnostic or prognostic study in meteorology for the application of fuzzy reasoning there are three interdependent steps. A successful execution of these steps leads to the solution of the problem in a fuzzy environment; i.e. the solution procedure digests any type of uncertainty in the basic evolution of the event concerned.
A fuzzy system is a mapping between linguistic terms, such as "medium complexity" and "high cost" that are attached to variables. Thus an input into a fuzzy system can be either numerical or linguistic with the same applying to the output [24]. A typical fuzzy system is made up of three major components: fuzzifier, fuzzy inference engine (fuzzy rules) and defuzzifier as described below.

3.1. Fuzzification

The fuzzifier transforms the input into linguistic terms using membership functions that represent how much a given numerical value of a particular variable fits the linguistic term being considered.
All meteorological events are considered as having ambiguous characteristics and therefore their domain of change are divided into many fuzzy subsets, which complete, normal and consistent with each other.

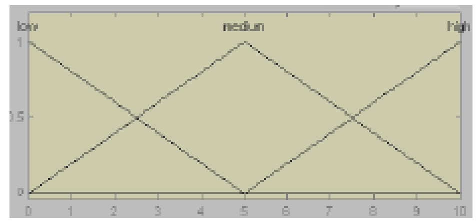

Figure2. Membership function of degree of operation's similarity

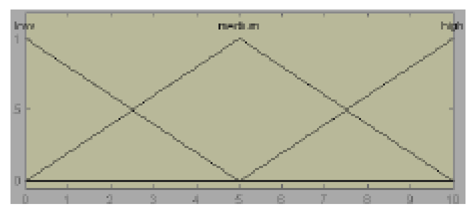

Figure3. Membership function of machine's technology

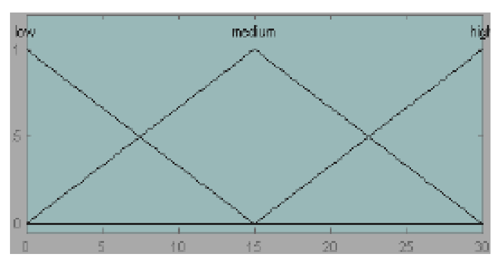

Figure 4.Membership function for setup time

3.2. Fuzzy Inference System (FIS)

This step systematically relates all the factors that take place in the solution depending on the purpose of the problem. In fact, this part includes many fuzzy conditional statements to describe a certain situation.

The fuzzy inference engine performs the mapping between the input membership functions and the output membership functions using fuzzy rules that can be obtained from expert knowledge of the relationships being modeled. The greater the input membership degree, the stronger the rule fires, thus the stronger the pull towards the output membership function.

Let us consider two events X and Y which are interactive and so dependent. This dependency can be verbally expressed as follows.

IF X is A (1) THEN Y is B(1)
ALSO
IF X is A (2) THEN Y is B (2)
ALSO
IF X is A (3) THEN Y is B(2)
ALSO
IF X is A (n) THEN Y is B (n),

where A(1), A(2),: : :, A(n) and B(1), B(2),: : :, B(n) are the linguistic description of X and Y, respectively. These fuzzy subsets cover whole domain of change of X and Y. The fuzzy conditional statements mentioned above can be expressed in the form of a fuzzy relation as R(X,Y)=ALSO (R1; R2; R3; : : :; RN), where Ri denotes the fuzzy relation between X and Y stated by the i-th fuzzy conditional statement. The term ALSO combines Ri's into the fuzzy relation R(X,Y). Having the fuzzy relationship R (X,Y) established, the compositional rule inference is applied to infer the fuzzy subset B from a given fuzzy subset A. This inference can be represented by the term B=AoR(X,Y), where o is a compositional operator.

Due to real conditions in a shop, some linguistic rules are defined.

(1) If degree of operations similarity is low, and level of machine technology is low, then setup time is high.

(2) If degree of operations similarity is medium and level of machine technology is medium, then setup time is medium.

(3) If level of machine technology is high then setup time is low.

All rules have the same weight (equal to one). Obviously number of rules and their structure is an open field to investigate, which may be different in various situations.

3.3. Defuzzification

The final result from previous step is in the form of fuzzy statement. A defuzzification method must then be applied to calculate the deterministic value of a linguistic variable.

Since several different output membership functions are included in the consequents of rules triggered, a defuzzifier carries out the defuzzification process to combine the output into a single label or numerical value as required.

There are many defuzzification methods such as centre of gravity (COG, centroid), bisector of area (BOA), mean of maxima (MOM) leftmost maximum (LM), etc [13]. In this study, we employed the centroid method, which is commonly used in several related papers [25].

This process can be done through equations (1-3) for a linguistic variable Y as follows:

$$y_j = \frac{\sum_{i=1}^{L} y_i}{L} \tag{1}$$

$$p_j(x) = \frac{\prod_{i=1}^{M} \mu_{A_{ij}}(x_i)}{\sum_{j=1}^{L} \prod_{i=1}^{M} \mu_{A_{ij}}(x_i)} \tag{2}$$

$$y = f(x) = \sum_{j=1}^{L} p_j(x) y_j \tag{3}$$

Where *p(x)* is fuzzy basis function and y is a particular value of the linguistic variable, *yj* is the support value in which the membership function reaches its maximum grade, L is the number of rules and M is the number of inputs. In this paper the center-average method is selected to defuzzify the problem. In figure 5, fuzzy logic modeling process is given:

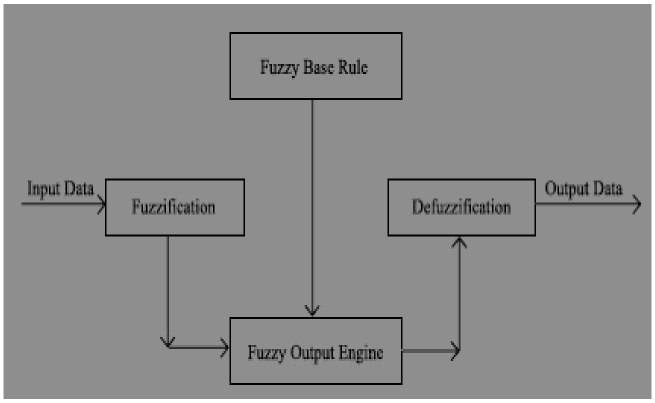

Figure5. Fuzzy logic modeling process [26]

The form of a fuzzy rule base can be illustrated through a matrix. The number of fuzzy regions in the input defines the size of matrix of combined fuzzy rule base. For a two input – one output case, a combined rule base can be formed with a two dimensional matrix. Data in table 1 illustrates setup time values.

		Degree of operation similarity		
		Low	Medium	High
Level of machine Technology	Low	22	15	7
	Medium	16	15	14
	High	12	9	5

Table1. Transition matrix (setup times)

In this case, after finding setup time using fuzzy system, the results will be considered (instead of using deterministic setup time matrix) in common methods available in the literature. Therefore, finding minimum total penalty for sequence dependent single machine scheduling problems will be done more realistic.

4. Validation

In order to evaluate validity of fuzzy approach, Monte Carlo simulation is used. After running simulation model, the results are compared with fuzzy setup times.

4.1 Monte Carlo simulation

No.	Number of Jobs	Simulation results	Fuzzy results
1	5	18.36	14.6
2	5	16.61	10.4
3	5	17.01	12
4	5	14.08	9.4
5	5	8.02	9.6
6	5	21.35	13
7	5	15.97	13.4
8	5	10.2	12.4
9	5	12.84	12.8
10	5	16.88	9.8
Average	**5**	**15.132**	**11.74**
11	10	20.12	13.1
12	10	17.27	13.1
13	10	17.54	12.1
14	10	16.52	15
15	10	20.54	15.2
16	10	16.55	13.9
17	10	12.73	12.6
18	10	11.19	10.9
19	10	8.83	13.2
20	10	12.64	12.1
Average	**10**	**15.393**	**13.12**
21	15	16.23	13.26
22	15	17.01	14
23	15	11.85	10.46
24	15	18.81	14.2
25	15	17.45	13.1
26	15	16.56	13.13
27	15	17.14	15.26
28	15	16.76	14.8
29	15	15.08	13.73
30	15	12.75	10.53
Average	**15**	**15.964**	**13.247**
31	20	19.13	12.53
32	20	15.5	13.6
33	20	16.94	10.33
34	20	14.78	14.8
35	20	14.31	14.06
36	20	11.98	11.53
37	20	16.1	14.13
38	20	12.89	12.73
39	20	14.7	12.53
40	20	12.49	10.93
Average	**20**	**14.882**	**12.717**

Table2. Comparative analysis between proposed approach and simulated results

While common methods available in the literature considered setup time as a deterministic and pre-specified matrix, the proposed fuzzy approach, emphasis on applying fuzzy rule based and estimating setup times resulting from real conditions that may occur in the shop. Also setup time matrix is considered as an input for sequence dependent single machine scheduling problem with earliness-tardiness penalty.

However, in this section, validation of the proposed approach is evaluated. For this purpose, simulated setup times are compared with setup times obtained from fuzzy rule based method. The model has tested the cases of 5, 10, 15 and 20 jobs on a single machine to demonstrate the superiority of the proposed approach. In all cases, fuzzy setup times are smaller than simulated setup times. Comparative analysis between fuzzy and simulated setup time is given in table 2.

4.2 Two way analysis of variance (ANOVA)

A two-way analysis of variance tests the equality of population's means when classification of treatments is by two variables or factors which in this research the number of jobs and the methods are considered as two important factors affecting the setup times. However, the results of analysis of variance as shown in table 3, indicates that the fuzzy approach is significantly different from the results obtained by simulation.

Source	DF	SS	MS	F	P-Value
J	3	15.767	5.256	0.85	0.473
Method	1	139.049	139.049	22.37	0.000
Interac.	3	4.659	1.553	0.25	0.861
Error	72	447.480	6.215		
Total	79	606.954			

Table3. The results of two way analysis of variance

In the other way, due to high P-value there is no significantly difference between numbers of jobs (5, 10, 15 & 20). Also there is no interaction between number of jobs and the methods. Thus obtained results, indicate proposed fuzzy approach has a lower mean in comparison of simulated results at all levels of the number of jobs.

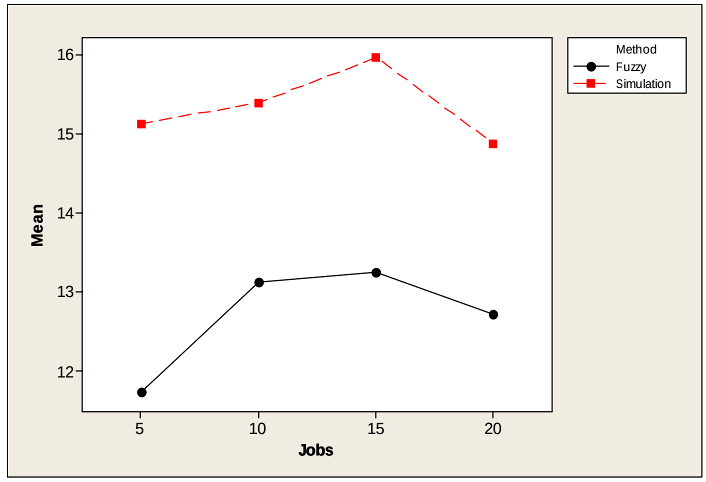

Figure6. The interaction plot for setup times

5. Conclusion & further research

In this paper sequence dependent single machine scheduling with Earliness – Tardiness penalty using fuzzy setup time is presented. The approach is to use degree of operations similarity and level of machine technology is considered as two inputs of the proposed fuzzy system.

This approach would be used to determine fuzzy setup time instead of using deterministic setup time matrix. Finally the output of fuzzy system (setup time) will be used in known methods available in the literature to find minimum total penalty in sequence dependent single machine scheduling problems with E/T penalty. The results were compared with simulated setup times by applying two- way analysis of variance and in all cases (5, 10, 15, 20 jobs), fuzzy setup time were found to be less than the simulated setup times.

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