Sensitivity of Uplink OFDMA, MC-CDMA and MC-DS-CDMA to Different Phase Jitter Models

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Summary

In this contribution, we consider the effect of carrier phase jitter on the performance of uplink OFDMA, uplink multicarrier CDMA (MC-CDMA) and uplink multicarrier direct sequence CDMA (MC-DS-CDMA) systems, assuming orthogonal spreading sequences. For this aim, it is proposed, first to consider a small random jitter, then, to model phase jitter by Gaussian and Rayleigh laws. Theoretical expressions are derived for the performance degradation caused by phase jitter. Assuming full load and ideal channel, it is shown that performance degradation depends only on jitter variance and on the law followed by this jitter. Further, we point out that, first, performance degradations are similar for all uplink systems OFDMA, MC-CDMA and MC-DS-CDMA, and then, Rayleigh model performs better than Gaussian model for the waveforms studied. As a conclusion, we have to know the law followed by the phase jitter in order to have more control on system performances.

***Keywords** : multicarrier systems, OFDMA, MC-CDMA, MC-DS-CDMA, phase jitter, modelling.*

1. Introduction

New applications are emerging, not just in wired environment, but also in mobile one. Furthermore, we have witnessed a widespread deployment of digital communication services requiring an exchange of digital information at constantly increasing data rates. When solving this problem, the first question is how to put this large bit stream on air with sufficient quality of service guaranties, i.e. which modulation can compromise all contradicting requirements in the best manner. The radio environment is harsh due to the many reflected waves and other effects. Single carrier techniques are vulnerable to fading and multipath propagation, especially in the case of very high bit rates.

Recently, research and development of the MultiCarrier (MC) systems have received considerable attention and have made a great deal of progress in the world. In fact, multicarrier systems offer high data rates and an immunity to channel dispersion. Orthogonal Frequency Division Multiple Access (OFDMA)systems are wide band modulation scheme that are specially able to cope with problems of multipath reception. In OFDMA system, the data streams, transmitted on different carriers, belong to different users [5].

However, when different users want to make use of the same transmission medium, their signals must be generated following a specific multiple access scheme, such that the signals of the different users can be separated at the receiver. A possible solution to separate the users is to use spreading sequences, i.e. to use Coded Division Multiple Access (CDMA) as a multiple access technique. The combination of a multicarrier technique and CDMA is proposed in [1], [2] and [4]. Among the investigated combinations we encounter the multicarrier CDMA (MC-CDMA) and the multicarrier direct sequence CDMA (MC-DS-CDMA). In MC-CDMA technique, the spreading is done in frequency domain, whereas it is done in time domain in MC-DS-CDMA technique.

The transmitter of a digital communication system contains a carrier oscillator necessary for the upconversion of the data carrying baseband signal to the bandpass signal. At the receiver, the received bandpass signal is downconverted using a local carrier oscillator. The receiver contains a synchroniser structure to estimate the ideal frequencies and phases of carrier oscillator, based on the received signal. Because the presence of interference, noise and other disturbances, these estimations are not perfect, causing phase error synchronisation.

In this work, we study the effect of one of synchronisation errors, which is carrier phase jitter, on the performance of uplink multicarrier systems OFDMA, MC-CDMA and MC-DS-CDMA. Many papers treat about the influence of such impairments [3, 6, 7]. Carrier phase jitter was first modelled by a random process without any specification on the law followed by the jitter [2, 4]. Simulation results for OFDMA, MC-CDMA and MC-DS-CDMA systems shows that performance degradation is the same for all these systems and depends only on jitter variance [4]. Our paper will complete the previous study on phase jitter and will propose some models for the jitter based on probability laws.

Assuming small random jitter, we first, consider this jitter to be modelled by a zero mean random process without any specification on the law followed by this jitter. Then, we propose Gaussian and Rayleigh models for the phase jitter. The comparison of the effect of these models on different waveforms shows that these systems have, essentially, same sensitivities to phase jitter and perform better with a Rayleigh model.

2. Systems Description

2.1. Uplink OFDMA

The conceptual block diagram of uplink OFDMA is shown in Figure 1. For each user, the data symbols to be transmitted to the basestation are organized into $(N_F+\nu)$ blocks where ν is the length of the cyclic prefix. Let $a_{i,n}$ denotes the n^{th} data symbol of the i^{th} block. Feeding these symbols to an Inverse Fast Fourier Transform (IFFT), we obtain the samples $s_{i,n,l}$, transmitted by user l, given by:

$$s_{i,n,l}=\frac{1}{\sqrt{N_F+\nu}}a_{i,n}e^{j2\pi\frac{nl}{N_F}} \qquad (1)$$

The transmitted sequence $s_{i,n,l}$ applied to the transmit filter $p(t)$ yielding the signal $s_l(t)$ given by:

$$s_l(t)=\sum_{i=-\infty}^{+\infty}\sum_{n=-\nu}^{N_F-1}s_{i,n,l}\,p(t-(i(N_F+\nu)+n)T-\tau_{c,l}) \quad (2)$$

Where 1/T is the symbol rate per carrier and $\tau_{c,l}$ is the time delay representing the transmit clock phase of user l.

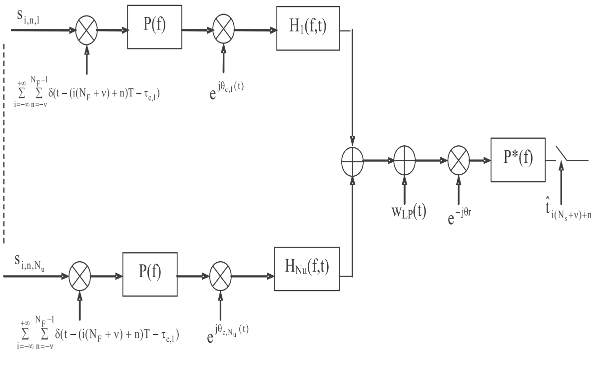

Fig. 1. The conceptual block diagram of an uplink OFDMA

The output signals of the dispersive channel are summed and disturbed by additive white Gaussian noise $w_{LP}(t)$. The resulting signal is affected by the carrier phase error $\phi_l(t)$ given by:

$$\phi_l(t)=\theta_{c,l}(t)-\theta_r \qquad (3)$$

Where θ_r is the carrier phase of the basestation and $\theta_{c,l}(t)$ is the transmitter carrier phase.

This carrier phase jitter, $\phi_l(t)$, was first modelled by a zero mean random process with jitter variance σ_ϕ^2 [2,7]. This random jitter has an unknown behaviour. We propose, here, to model carrier phase jitter by Gaussian and Rayleigh models and to see the effect of such jitter on the studied waveforms. Then, we suppose that $\phi_l(t)$ is stationary and very small that it can be considered the same for all users ($\phi_l(t)=\phi$). At the basestation, the signal is applied to the receiver filter and sampled to give $v_{i,k}$ samples. Keeping the N_F samples outside the cyclic prefix, the output samples are fed to the FFT and then to an equalizer with coefficients $g_{i,n}$. As a result, the samples $z_{i,n}$ are given by:

$$z_{i,n}=\sqrt{\frac{N_F}{N_F+\nu}}\sum_{n'=0}^{N_u-1}\sum_{i'=-\infty}^{+\infty}a_{i',n'}I_{i,i',n,n'}+w_{i,n} \qquad (4)$$

where $w_{i,n}$ is the term referred to the Additional White Gaussian Noise (AWGN) and $I_{i,i',n,n'}$ is given by :

$$I_{i,i',n,n'}=\frac{1}{N_F}g_{i,n}\delta_{i,i'}\sum_{k=0}^{N_F-1}e^{-j2\pi\frac{k(n-n')}{N_F}}e^{j\phi} \qquad (5)$$

The quantity $I_{i,i',n,n'}$ denotes the contribution of the symbol $a_{i',n'}$ on the n^{th} FFT output during the i^{th} block. The samples $z_{i,n}$ can be decomposed into four contributions: an average useful component $I_{i,i,n,n}$, a zero mean useful component $I_{i,i,n,n}-E(I_{i,i,n,n})$ or self interference, an intersymbol interference $(i'\neq i$ and $n'=n)$ and an intercarrier or interuser interference $(n'\neq n)$. So we can write:

$$z_{i,n}=\sqrt{\frac{N_F}{N_F+\nu}}a_{i,n}I_{i,i,n,n}+\sqrt{\frac{N_F}{N_F+\nu}}\sum_{i'=-\infty et i'\neq i}^{+\infty}a_{i',n}I_{i,i',n,n}$$
$$+\sqrt{\frac{N_F}{N_F+\nu}}\sum_{i'=-\infty}^{+\infty}\sum_{n'=0 et n'\neq n}^{N_F-1}a_{i',n'}I_{i,i',n,n'}+w_{i,n} \qquad (6)$$

2.2. Uplink MC-CDMA

In the multicarrier CDMA technique, the original data stream are first multiplied with the spreading sequence and then modulated on the different carriers. The conceptual block diagram of an uplink MC-CDMA system is shown in Figure 2. The data symbol $a_{i,l}$ denotes the i^{th} symbol transmitted by user l. Each data symbol is multiplied with the n^{th} chip during the i^{th} symbol interval of the sequence belonging to the user l, $c_{iN_c+n,l}$. In this work, we consider orthogonal sequences consisting of user-dependent Walsh-Hadamard (WH) sequences of length N_c. The maximum number of users, N_u, equals N_c. These orthogonal sequences make easy the separation of different users at the basestation. In fact in a multiuser scenario, each of N_u users is assigned a unique spreading sequence.

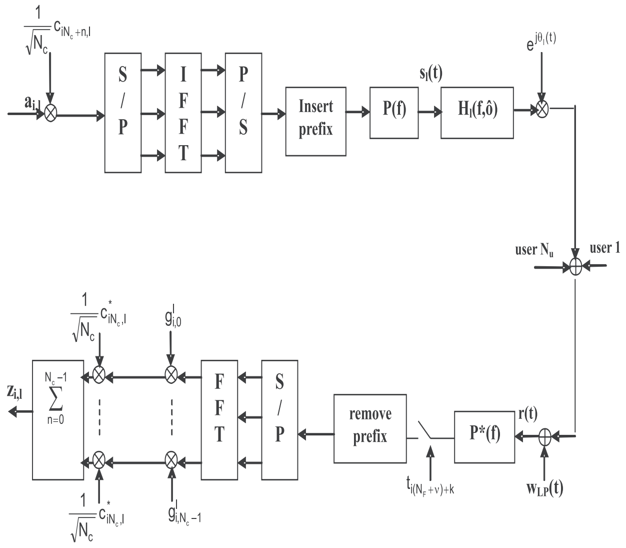

Fig. 2. Conceptual block diagram of an uplink MC-CDMA

Each of N_c components will be modulated on a different carrier using an IFFT of length N_F. In order to avoid interference between successive MC-CDMA blocks, a cyclic prefix of ν samples is used. The resulting transmitted samples $s_{i,l}$ are given by:

$$s_{i,l} = \frac{1}{\sqrt{N_c N_F}} \sum_{k=-\nu}^{N_F-1} \sum_{n=0}^{N_c-1} a_{i,l} c_{iN_c+n,l} e^{j2\pi\frac{kn}{N_F}} \qquad (7)$$

In uplink MC-CDMA, the transmitter of each user derives a clock signal and a carrier oscillator signal from a network synchronization reference signal [7,8]. Hence, the transmitter of user l has a user dependent carrier phase $\theta_{c,l}(t)$ and clock phase $\tau_{i(N_F+\nu)+k,l}$. The signal at the output of the transmit filter $P(f)$ can be written as:

$$s_l(t) = \sum_{i=-\infty}^{+\infty} \sum_{k=-\nu}^{N_F-1} s_{i(N_F+\nu)+k,l} \times p(t-(k+i(N_F+\nu))T-\tau_{i(N_F+\nu)+k,l}) \qquad (8)$$

where T is the transmit clock signal period.

The output of the dispersive channel is affected by the carrier phase difference $\phi_l(t)$ as described in section 2.1 ($\phi_l(t)=\phi$).

The received signal disturbed by the Additive White Gaussian Noise is applied to the receiver filter and sampled. After removing the cyclic prefix, the receiver demodulates the samples using Fast Fourier Transform (FFT) procedure. The output of the FFT is multiplied with the $g^l_{i,n}$ coefficients of the equalizer and the spreading sequence of the considered user l, yield the samples $z_{i,l}$ given by:

$$z_{i,l} = \sqrt{\frac{N_F}{N_F+\nu}} a_{i,l} I_{i,i,l,l} + \sqrt{\frac{N_F}{N_F+\nu}} \sum_{i'=-\infty et i'\neq i}^{+\infty} a_{i',l} I_{i,i',l,l} + \sqrt{\frac{N_F}{N_F+\nu}} \sum_{i=-\infty}^{+\infty} \sum_{l'=0 et l'\neq l}^{N_u-1} a_{i,i,l,l'} I_{i,i,l,l'} + w_{i,l} \qquad (9)$$

Where

$$I_{i,i',l,l'} = \frac{1}{N_c N_F} \delta_{i,i'} \sum_{n,n'=0}^{N_c-1} c^*_{iN_c+n,l} c_{i'N_c+n',l'} g^l_{i,n} \times \sum_{k=0}^{N_F-1} e^{-j2\pi\frac{k(n-n')}{N_F}} e^{j\phi} \qquad (10)$$

The quantity $I_{i,i',l,l'}$ represents the contribution of the symbol $a_{i',l'}$ to the output of the receiver of the l^{th} user during the i^{th} MC-CDMA block.

In equation (9), the first contribution is the useful component. This contribution can be further decomposed into an average useful component $E(I_{i,i,l,l})$ and a zero mean fluctuation $I_{i,i,l,l}-E(I_{i,i,l,l})$ or self interference (SI). The second contribution $(i'\neq i$ and $l'=l)$ is the intersymbol interference (ISI), caused by other symbols from the same user. The third contribution $(l'\neq l)$ denotes the multiuser interference (MUI). The last contribution is the additive noise term.

2.3. Uplink MC-DS-CDMA

Another form of combination of the multicarrier modulation technique and the code division multiple access (CDMA) scheme, is the multicarrier direst sequence CDMA (MC-DS-CDMA) technique which has been considered for mobile radio communication [2,4]. In MC-DS-CDMA technique, the serial to parallel converted data stream is multiplied with a spreading sequence and then chips belonging to the same symbol modulate the same carrier so that spreading is done in the time domain. The conceptual block diagram of an uplink multicarrier direct sequence CDMA (MC-DS-CDMA) system is shown in Figure 3.

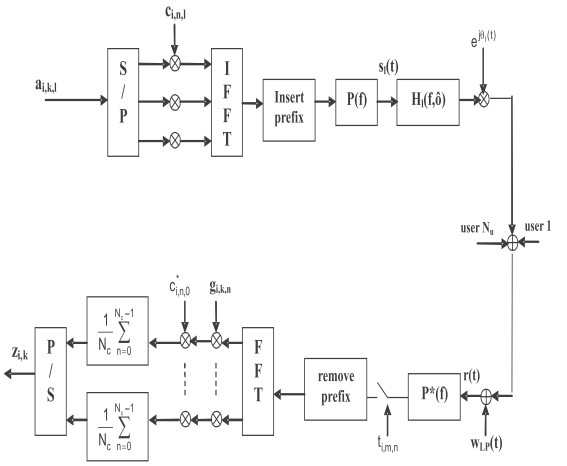

Fig. 3. Conceptual block diagram of an uplink MC-DS-CDMA

Let $a_{i,k,l}$ be the i^{th} symbol transmitted by user l on carrier k. This data symbol is multiplied with the chip $c_{i,n,l}$ where $c_{i,n,l}$ is the n^{th} chip of the sequence that spreads data symbols transmitted by user l during the i^{th} symbol interval. The spreading factor is N_c. The resulting samples are transmitted serially on the k^{th} carrier to be modulated using an inverse fast Fourier transform (IFFT). At the output, each IFFT block is extended with a cyclic prefix of ν samples. Feeding the result to the transmit filter, yielding the signal $s_l(t)$ given by:

$$s_l(t) = \sum_{i=-\infty}^{+\infty} \sum_{m=-\nu}^{N_F-1} \sum_{n=0}^{N_c-1} s_{i,m,n,l}\, p(t - t_{i,m,n}) \qquad (11)$$

where

$$s_{i,m,n,l} = \frac{1}{\sqrt{N_F+\nu}} \sum_{k=0}^{N_F-1} a_{i,k,l} c_{i,n,l} e^{j2\pi \frac{km}{N_F}} \qquad (12)$$

At the output of user channel, the signal is disturbed by carrier phase jitter $\phi_l(t)$ which is described in section 2.1. The additive white Gaussian noise disturbs the sum of signals transmitted by different users to the basestation.

At the receiver, we concentrate on the detection of data symbols transmitted by user $l = 0$. After, removing the cyclic prefix and demodulating using FFT scheme, we can multiply each sample with the corresponding chip of the reference user spreading sequence $c^*_{i,n,0}$. Summing these samples, we obtain $z_{i,k}$ given by:

$$z_{i,k} = \sqrt{\frac{N_F}{N_F+\nu}} \sum_{l=0}^{N_u-1} \sum_{k'=0}^{N_F-1} a_{i,k',l} I_{i,k,k',l} + w_{i,k} \qquad (13)$$

where

$$I_{i,k,k',l} = \frac{1}{N_c N_F} \sum_{n=0}^{N_c-1} \sum_{m=0}^{N_F-1} c^*_{i,n,0} c_{i,n,l} g_{i,k,n} e^{-j2\pi \frac{m(k-k')}{N_F}} e^{j\phi} \qquad (14)$$

Similarly to the composition done in the last paragraph, $z_{i,k}$ contains four contributions. The useful contribution $I_{i,k,k,0}$ can be decomposed into an average useful component $E(I_{i,k,k,0})$ and a zero mean self interference (SI), $I_{i,k,k,0} - E(I_{i,k,k,0})$. The second contribution for $(k' \neq k)$ corresponds to the intercarrier interference (ICI) from other symbols transmitted by user 0. The third contribution $I_{i,k,k',l}$, $(l \neq 0)$, comes from other users (IUI). The last contribution $w_{i,k}$ is referring to the additive white Gaussian noise.

3. Performance Degradation

After describing the multicarrier systems: uplink OFDMA, uplink MC-CDMA and uplink MC-DS-CDMA, we investigate here the SNR degradation for these systems when carrier phase jitter is present. To clearly isolate the effect of synchronization phase error, we consider the case of an ideal channel.

The SNR is defined as the ratio of the power of the average useful component P_U to the sum of the powers of the self interference P_{SI}, the intersymbol interference P_{ISI}, the interuser (or intercarrier) interference P_{IUI} and the additive noise AWGN. This yiels :

$$SNR(\phi) = \frac{\frac{N_F}{N_F+\nu} E_{s,n} P_u}{E(|w_{i,n}|^2) + \frac{N_F}{N_F+\nu} E_{s,n}(P_{SI} + P_{ISI} + P_{IUI})} \qquad (15)$$

where

$E_{s,n}$ is the energy per symbol on the n^{th} carrier and $E(|w_{i,n}|^2)$ is the variance of the AWGN given by :

$$E(|w_{i,n}|^2) = N_0 |g_{i,n}|^2 \qquad (16)$$

with $g_{i,n}$ the equalizer term.

In the absence of the phase jitter, the SNR resulting is :

$$SNR(0) = \frac{N_F}{N_F+\nu} \frac{E_{s,n}}{N_0} \qquad (17)$$

When phase jitter is present, the SNR is reduced, as compared to the case of the absence of synchronization error phase. The degradation of SNR due to the phase jitter is given by:

$$deg = -10 \log \frac{SNR(\phi)}{SNR(0)} \qquad (18)$$

By replacing SNR terms, the equation (18) can be simplified as follows:

$$deg = -10 \log \frac{P_u}{|g_{i,n}|^2 + SNR(0)(P_{SI} + P_{ISI} + P_{IUI})} \qquad (19)$$

This equation is the same for all multicarrier systems described above. Without loss of generality, we give the degradation value for each system and with different models of phase jitter.

3.1. Uplink OFDMA

In the expression (19), the different powers are given by:

$$P_u = \left|E(I_{i,i,n,n})\right|^2$$

$$P_{SI} = E(\left|I_{i,i,n,n} - E(I_{i,i,n,n})\right|^2)$$

$$P_{ISI} = \sum_{i'=-\infty}^{+\infty} E(\left|I_{i,i',n,n}\right|^2)$$

$$P_{ICI} = \sum_{l'=0}^{N_u-1} \frac{E_{s,n'}}{E_{s,n}} E(\left|I_{i,i,n,n'}\right|^2)$$

In order to calculate these terms, we consider, first, the case of small random process jitter with the assumption that $e^{j\phi(t)} = 1 + j\phi(t)$. Then, we introduce new models for this jitter.

3.1.1. Small Random Process

If we simplify equation (5) for $i' = i$ et $n' = n$, we have directly :

$$I_{i,i,n,n} = g_{i,n} e^{j\phi} \qquad (20)$$

The useful power P_u can be written as:

$$P_u = E(\left|g_{i,n} e^{j\phi(t)}\right|^2) = E(\left|g_{i,n}\right|^2 . \left|e^{j\phi(t)}\right|^2) = \left|g_{i,n}\right|^2 \qquad (21)$$

Using the assumption above, the power of self-interference is reduced to:

$$P_{SI} = E(\left|g_{i,n} j\phi\right|^2) = \left|g_{i,n}\right|^2 \sigma_\phi^2 \qquad (22)$$

Whereas, to calculate the intersymbol interference power, we could remark that $P_{ISI} = 0$ since $\delta_{i,i'} = 0$ for $i' \neq i$.

After some simplifications, the power intercarrier interference is given by:

$$P_{ICI} = \mid g_{i,n} \mid^2 \qquad (23)$$

Finally, we can write SNR degradation as:

$$deg = 10 \log(1 + SNR(0)(\sigma_\phi^2 + 1)) \qquad (24)$$

where σ_ϕ^2 is the jitter variance and $SNR(0)$ is the signal to noise ratio in the absence of phase error.

3.1.2. Gaussian Process

Mathematically, each average term can be computed using the quantity $E(e^{j\phi})$. At this point we have a relationship:

$$E(e^{j\phi}) = \int_0^{2\pi} p(\phi) e^{j\phi} d\phi \qquad (25)$$

where $p(\phi)$ is the jitter probability law. With the assumption that ϕ has a Gaussian probability law with variance σ_ϕ^2, then $p(\phi)$ is [10]:

$$p(\phi) = \frac{1}{\sigma_\phi \sqrt{2\pi}} \exp(-\frac{\phi^2}{2\sigma_\phi^2}) \qquad (26)$$

If we, now, simplify equation (25), then this quantity becomes:

$$E(e^{j\phi}) = \frac{1}{\sigma_\phi \sqrt{2\pi}} \int_0^{2\pi} e^{j\phi - \frac{\phi^2}{2\sigma_\phi^2}} d\phi = A \qquad (27)$$

There is no classical method to calculate the expression above. Hence we can hardly found the exact value of the integral in equation (27). An approximate method of calculation is been used, based on trapezoids surface approximation [11]. Let A be the approximate value of the integral.

By taken into consideration equation (20), then we have:

$$P_u = \mid g_{i,n} \mid^2 \mid A \mid^2 \qquad (28)$$

To give the power of self-interference, we have to do some developments. After simplification, we can write:

$$P_{SI} = \mid g_{i,n} \mid^2 (1 - \mid A \mid^2) \qquad (29)$$

Similarly to the precedent case, the power of intersymbol interference is zero.

Whereas, intercarrier interference power is:

$$P_{ICI} = \mid g_{i,n} \mid^2 \qquad (30)$$

The SNR degradation is, then:

$$deg = -10 \log \frac{\mid A \mid^2}{1 + SNR(0)(2 - \mid A \mid^2)} \qquad (31)$$

3.1.3. Rayleigh Process

Another approach can be presented to model the phase jitter. By choosing Rayleigh law instead of Gaussian law, identical theory results can be found. The probability of the Rayleigh law is given by [10]:

$$p(\phi) = \frac{\phi}{\sigma_\phi^2} \exp(-\frac{\phi^2}{2\sigma_\phi^2}) \qquad (32)$$

As a consequence the quantity $E(e^{j\phi})$, denoted B, can be written :

$$E(e^{j\phi}) = B = \int_0^{2\pi} \frac{\phi}{\sigma_\phi^2} e^{-\frac{\phi^2}{2\sigma_\phi^2} + j\phi} d\phi \qquad (33)$$

Similarly, using an approximate method to calculate the integral quantity, we found the same degradation expression given in (31) function of B instead of A:

$$deg = -10 \log \frac{\mid B \mid^2}{1 + SNR(0)(2 - \mid B \mid^2)} \qquad (34)$$

This is means that performance degradation depends, not only, on jitter variance, but also, on jitter probability law.

3.2. Uplink MC-CDMA

In the case of uplink MC-CDMA system the powers of useful component, self interference, intersymbol interference and interuser interference, are given by:

$$P_u = \left|E(I_{i,i,l,l})\right|^2$$

$$P_{SI} = E(\left|I_{i,i,l,l} - E(I_{i,i,l,l})\right|^2)$$

$$P_{IUI} = \sum_{l'=0}^{N_u-1} \frac{E_{s,l'}}{E_{s,l}} E(\left| I_{i,i,l,l'} \right|^2)$$

We easily note that $P_{ISI} = 0$ since the intersymbol interference $I_{i,i',l,l} = 0$ for $(i' \neq i)$. We formalize, now, the problem with different cases of modelling: small random jitter, Gaussian model and Rayleigh model.

3.2.1. Small Random Process

For $i' = i$ et $l' = l$, we give the general term of interference by :

$$I_{i,i,l,l} = \delta_{n,n'} g_{i,n}^{l} e^{j\phi} \qquad (35)$$

Assuming that all users have the same jitter variance σ_ϕ^2 and the same energy per symbol E_s, and with highest load, i.e. $(N_u = N_c)$, we can simplify the different power expressions, as done in section 3.1.1, we obtain :

$$P_u = \left| g_{i,n}^{l} \right|^2$$

$$P_{SI} = \left| g_{i,n}^{l} \right|^2 \sigma_\phi^2$$

$$P_{ICI} = | g_{i,n}^{l} |^2$$

If we replace each term by its expression, we can write SNR degradation as:

$$deg = 10\log(1 + SNR(0)(\sigma_\phi^2 + 1)) \qquad (36)$$

3.2.2. Gaussian and Rayleigh Process

Since we use the same method to develop different powers and for sake of simplicity, we will give directly the desired SNR degradation for jitter Gaussian model by:

$$deg = -10\log \frac{|A|^2}{1 + SNR(0)(2 - |A|^2)} \qquad (37)$$

Whereas, we just modify A by B to have SNR degradation for jitter Rayleigh model as follows:

$$deg = -10\log \frac{|B|^2}{1 + SNR(0)(2 - |B|^2)} \qquad (38)$$

where A is the approximated value of $E(e^{j\phi})$ given by expression (27) in the case of Gaussian phase jitter model, and B is the quantity introduced in equation (33).

3.3. Uplink MC-DS-CDMA

In order to give performance degradation of uplink MC-DS-CDMA system, we first describe each power term in the expression (19), than we simplify this expression. In fact, the useful component power, the self interference power, the intercarrier interference power and the interuser interference power are given by:

$$P_u = \left| E(I_{i,k,k,l}) \right|^2$$

$$P_{SI} = E(\left| I_{i,k,k,l} - E(I_{i,k,k,l}) \right|^2)$$

$$P_{ISI,l} = \sum_{k'=0}^{N_c-1} E(\left| I_{i,k,k',0} \right|^2)$$

$$P_{IUI} = \sum_{l=1}^{N_u-1} \frac{E_{s,l}}{E_{s,0}} E(\left| I_{i,k,k,l} \right|^2)$$

In order to clearly isolate the effect of the carrier phase jitter, we assume that the jitter variance is equal for all users, we are with maximum load $(N_u = N_c)$ and all users have the same energy per symbol $(E_{s,l} = E_s)$. In fact, under same assumptions and with same approximations used in MC-CDMA system to estimate the value of $E(e^{j\phi})$, SNR degradation of an uplink MC-DS-CDMA system is the same as for the uplink MC-CDMA system.
Thanks to these simplifications, we easily found same SNR degradation for small random jitter ($deg = 10\log(1 + SNR(0)(\sigma_\phi^2 + 1))$), jitter Gaussian model ($deg = -10\log\frac{|A|^2}{1+SNR(0)(2-|A|^2)}$) and jitter Rayleigh model ($deg = -10\log\frac{|B|^2}{1+SNR(0)(2-|B|^2)}$).

4. Simulations and Interpretations

The expression results indicate that different waveforms: OFDMA, MC-CDMA and MC-DS-CDMA have same sensitivities to phase jitter with any model chosen for this jitter. Simulation results confirm this fact. Figure 4 shows simulation result for small random process model. In this figure, we found that performance degradation is not only closely related to jitter variance, but also increases with increasing $SNR(0)$. Additional result is given in Figure 5. In fact, from this figure we deduce that performance of all waveforms is better with a jitter known behaviour, i.e., with Gaussian model and with low $SNR(0)$ (Figure 6). Whereas, best performances are shown in Figure 7. In fact, the Rayleigh model offers minimum degradation.

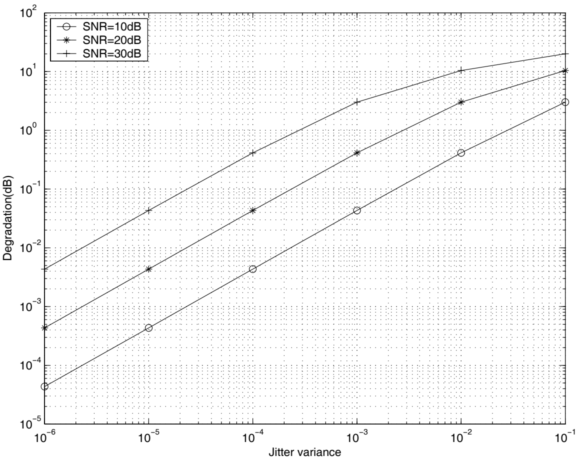

Fig. 4. Performance degradation for uplink OFDMA,

uplink MC-CDMA and uplink MC-DS-CDMA with different SNR(0) values in the case of small random process.

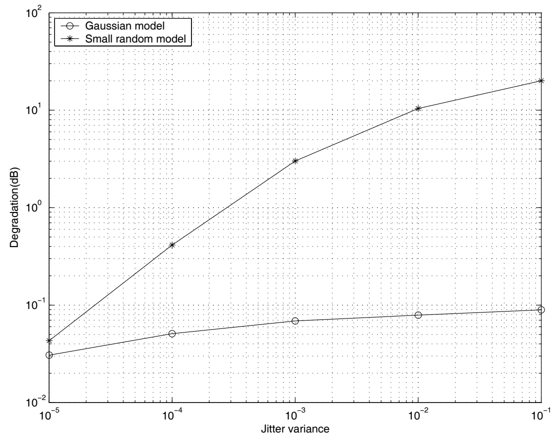

Fig. 5. Comparison of the effect of Gaussian and small random models on different waveforms performance for SNR(0)=30dB

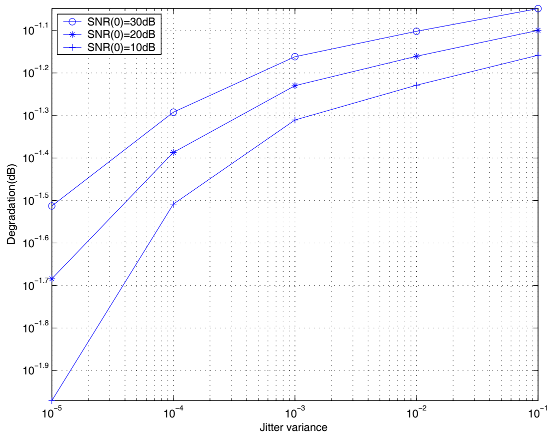

Fig. 6. Performance degradation for uplink OFDMA, uplink MC-CDMA and uplink MC-DS-CDMA with different SNR(0) values in the case of Gaussian process.

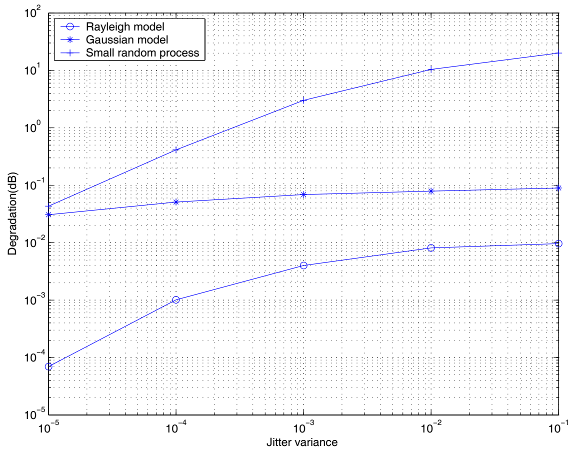

Fig. 7. Comparison of the effect of different jitter models on uplink OFDMA, uplink MC-CDMA and uplink MC-DS-CDMA.

5. Conclusion

In this paper, we investigate the sensitivity of different waveforms: uplink OFDMA, uplink MC-CDMA and MC-DS-CDMA, to different phase jitter models. As expected by theorical results, performances of these systems are similar and better with jitter Rayleigh process. As a conclusion, performance degradation of different waveforms depends, not only, on jitter variance, but also, on the law followed by this jitter. No one can deny that it would be of interest to model carrier phase jitter by Wiener process (no stationary phase jitter) and to study performance degradation of multicarrier systems under this suggestion.

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