

Mobile Location Estimation in NLoS Environment Base on Interior Point Method

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Summary

Increasing applications of mobile communication services makes the challenges in location estimation as a fundamental problem in many applications. There are error sources such as multipath fading, Non Line of Sight (NLoS) propagation and multi-user interferences that deteriorate the accuracy of location estimation. To mitigate this errors two approach is feasible, statistical methods and geometric optimization algorithms. In this paper we focus on NLoS error and employ the time of arrival (ToA) for range measurement. A new location estimation is proposed based on geometric optimization approach that adopts the interior point method (IPM) for better location estimation, which is a non linear constrained optimization method. This method has a salient feature that can solve optimization problems including more inequality constraints than equality constraints. The estimation can be network or terminal based and doesn't discriminate between LoS and NLoS base stations. The proposed algorithm requires ToA measurements only from three BSs and doesn't require any priori probabilistic information. Simulations were conducted to evaluate the performance of the algorithm for different NLoS error distributions, and the results show significant improvement than previous works such as range scaling (RSA) and density clustering (DCA) algorithms and also satisfies the location accuracy demand of FCC E-911 specification.

Key words:

Location Estimation, None Line of Sight (NLoS), Time of Arrival (ToA), Interior Point Method (IPM)

Introduction

In recent years the widespread use of mobile communications has stimulated the research on it's related subjects. The mobile location estimation is a critical parameter that has many applications such as location-sensitive billing, fleet tracking, package and personnel tracking, mobile yellow pages, location-based messaging, route guidance, hand over, topology inference and providing traffic information. The frequency with which location requests are made and the desired accuracy varies with the application. The first specification that defines the accuracy needs for mobile location estimation was published from Federal Communication Commission

(FCC) as Enhanced-911 (E-911) services [1]. The accuracy requirement of E-911 for phase II is 100m for 67% of the time and 300m for 95% of the time in network-based location system.

There are several methods for location estimation that can be categorized as received signal strength (RSS), Angle of arrival (AoA), time of arrival (ToA) or time difference of arrival (TDoA). For an overview of various wireless location techniques and technologies, see [2] and [3]. The accuracy of radio location schemes depends on the propagation conditions of the wireless channels. The main error sources in location estimation are measurement noises, multipath artifacts, non line of sight (NLOS) propagation and multiple access interference [2]. Usually measurement noise could be neglected in versus of other errors [5],[6].

If LoS propagation exists between the MS and all BSs, a high location accuracy can be achieved. Traditional algorithms, such as those in [5]–[9], are designed to provide accurate location in LoS environments. However, in wireless communication systems in which the direct path from the MS to BS is blocked by buildings and other obstacles, the signal measurements include an error due to the excess path length traveled because of reflection or diffraction, which is termed the NLoS error [4]. Unfortunately, NLoS error is relatively large, of the order of hundreds of meters [3], and except for rural areas is quite common in all other environments [10]. Therefore this type of error causes considerable degradation of location estimation accuracy. This has led to the development of algorithms that focus on identifying and mitigating the NLoS error.

Hence, In this paper we focus on network based ToA method with NLOS error and propose a new and effective method for mitigating it's inaccuracy.

In ToA methods, the propagation time of signal between MS and BS is estimated and the range between them is calculated by multiplying the ToA with velocity of propagation. The NLOS affects the ToA and makes it greater than the correct value and therefore the estimation would be biased.

A variety of approaches for NLOS mitigation is available in literatures. The techniques for NLoS mitigation considered in [4] and [11] require a time series of range measurements from a mobile and are based on the assumption that the standard deviation of NLoS range measurements is greater than that for LoS measurements. These techniques may prove to be useful to track the MS when a mixed measurement set including both LoS and NLoS measurements is observed over a time span of a few seconds, but they will not give good results for only NLoS paths. Several approaches have been proposed to mitigate NLoS effects with only single measurements at a set of participating BSs. The algorithm in [12] attempts to selectively remove or weight NLoS corrupted measurements by examining the range residuals of the measurements. If the difference between a BS's measured range and the range for the computed position is large, then that range measurement is weighted proportionately to minimize its effect.

Similar algorithms have been proposed to deal with NLoS in TDoA [13] and AoA [14]. Although increasing the number of BSs, improves performance metrics but this may not be realizable in practical systems. Some statistical algorithms have been designed to work in NLoS environments that are described by channel-scattering models, such as the ring/disk of scatterers and the Gaussian scattering models [15], [16]. These algorithms utilize the distribution function of the ToAs that depend on the scattering model to estimate the true range measurements. However, it becomes important to characterize the scattering environment in a particular area before the algorithm can be applied.

Recently, a constrained optimization procedure has been proposed that estimates the bias by means of a sequence of decreasing bound constraints and corrects the location estimate formed from the biased range measurements [17]. This method has been shown to produce accurate location estimates (40 meter in 67% of times) in field trials. However, the authors do not describe how the bounds on the bias are formed, which is critical to the success of the algorithm. Another algorithm that attempts to mitigate NLoS in ToA-based location systems utilizes the information that NLoS propagation causes the measured ranges to be greater than the true ranges and employs a quadratic programming approach to solve for an maximum likelihood (ML) estimate of the source position [18]. A linear line of position method (LLoP) is presented in [9] which make it easier estimating the unknown MS location than traditional geometrical approach calculating the intersection of the circular lines of position (LoP) [7]. LLoP algorithm can mitigate the NLoS error as well as the measurement noise, but it needs at least four BSs to

achieve better location accuracy, and its performance highly depends on the relative position of MS and BSs. Venkatraman et al. [24], presents a constrained optimization approach named range scaling algorithm (RSA) that utilizes bounds on the NLoS range error inferred from the geometry of the cell layout and range circles for three BSs. RSA indeed improved the location accuracy in the NLoS environment, but cannot satisfy FCC E-911 requirements. A modified version of RSA algorithm named density-based clustering algorithm (DCA), is developed by Lin et al. [25]. DCA estimates the mobile location by solving the optimal solution of the objective function of RSA, modified by the high density cluster.

Our approach in this paper is based on another non-linear constrained optimization method that is known as interior point method (IPM). This optimization method is more general and flexible than previous methods and simulation results show better accuracy and also are in agree with FCC E-911 requirements. This method also has the advantage of requiring no modifications to the subscriber equipment. The location estimation can be performed at either the MS if it has the functionality or at special location measurement units in the network. Knowledge of the statistics of the measurement noise and NLoS error is not required. it is assumed that at any time instant, the BSs can be LoS or NLoS and the error in the range measurements is positive due to NLoS. it only needs three range measurements from three BSs, and does not discriminate between NLoS and LoS BSs.

We would like to emphasize that the proposed algorithm applies to any system based on ToA estimation and that we do not address the estimation of ToAs, but rather their use for location determination in a NLoS environment. Methods for estimating ToAs can be found in [19]–[23].

The remainder of this paper is organized as follows. The problem statement and derivation of geometrical constraints is outlined in Section 1. In section 2 we summarize IPM method and then adopt the location estimation problem and its constraints to it in section 3. The simulation setup and results are discussed in Section 4, followed by some concluding remarks in Section 5.

1. Problem Statement

Our method is based on TOA measurement from three BSs, denoted as BS_i , $i=1,2,3$. Assume that the measured range from i th BS is l_i and true range is R_i . the corresponding relations of these parameters are as:

$$l_i = TOA_i \times c$$

$$R_i = \sqrt{(x-x_i)^2 + (y-y_i)^2}; \quad i=1,2,3 \quad (1)$$

Where c is the light velocity, (x_i, y_i) is the i th BS location and (x, y) is the true location of MS.

Then according to NLOS error, we can write:

$$R_i = \delta_i l_i \quad (2)$$

That δ_i is a correction factor. Because the NLOS error is positive, the measured range is greater than the true range. Hence, the value of δ_i is bounded by $0 < \delta_i < 1$. If and only if there is no NLOS error, δ_i equals 1. It is assumed that the

measurement noise is a zero-mean Gaussian random distribution with relatively small standard deviation and is negligible as compared to the NLOS error.

Each range l_i specifies the MS location as a point on a circle with radius l_i centered at BS_i . But l_i includes NLOS error and if we apply the correction factor δ_i to each l_i and consider the above mentioned bound for δ_i s, then the true MS location lies in the region of overlap of the range circles (region enclosed by U,V,W) shown in Fig. 1.

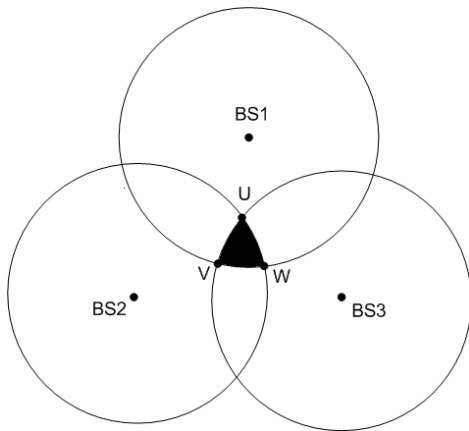


Fig. 1 the possible region of MS

Therefore the MS location problem reduced to a constrained estimation problem and the optimal estimated location should be finding in specified region. For this purpose we select an objective function as in [24], and apply a constrained non linear optimization method to find it's optimal solution. We select the interior point method (IPM) because of its generality and flexibility. This method can accept the desired constraints easily and use them automatically in its search process. In order to declaring IPM constraints, at first step we should determine the minimum values of δ_i s in order to maintaining in valid region of UVW.

Let the NLoS range error of the BS_i be η_i . Assuming the measured range of BS_2 is LoS, it can be seen from Fig. that if the true range from BS_1 , namely, R_1 is less than $l_1 - \overline{AB}$, then the true range circles of BS_1 and BS_2 will not overlap or intersect. But the true range circles should intersect at the MS location, which is impossible, and η_1 , the NLoS error of BS_1 , cannot be larger than \overline{AB} .

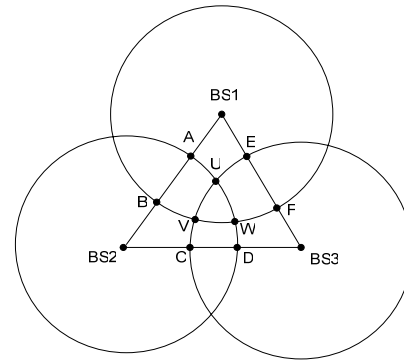


Fig. 2 Geometry of ToA-Based location.

Applying the same argument to the ranges from BS_2 and BS_3 , the value of η_1 cannot be larger than \overline{EF} . thus the proper bound of η_1 is:

$$\max \eta_1 = \min\{\overline{AB}, \overline{EF}\}$$

Similarly, the upper bound of η_2 and η_3 are:

$$\max \eta_2 = \min\{\overline{AB}, \overline{CD}\}$$

And

$$\max \eta_3 = \min\{\overline{CD}, \overline{EF}\}$$

We know that $\eta_1 = l_1 - R_1 = (1 - \delta_1)l_1$ so that $\delta_1 = 1 - \eta_1 / l_1$.

Thus the minimum value that δ_1 can take is given by

$$\min \delta_1 = 1 - \frac{\max \eta_1}{l_1} = 1 - \frac{\min\{\overline{AB}, \overline{EF}\}}{l_1}$$

Define the distance between BS_i and BS_j as L_{ij} . From figures 2 and 3 we can see that:

$$\overline{AB} = l_1 + l_2 - L_{12}$$

$$\overline{CD} = l_2 + l_3 - L_{23}$$

$$\overline{EF} = l_1 + l_3 - L_{13}$$

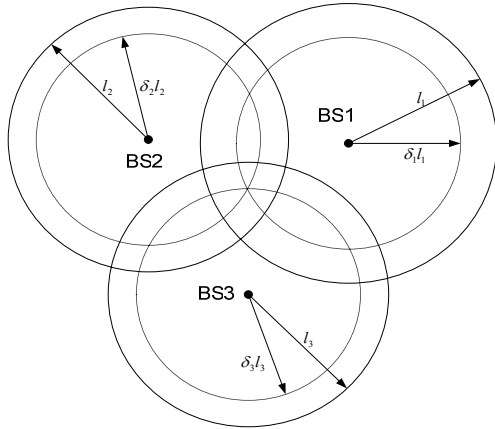


Fig. 3 The geometry of the three circular equations formed by BS_i and $\delta_i l_i$

Since \overline{AB} , \overline{CD} and \overline{EF} are positive, $\min \delta_i$ can be written as

$$\begin{aligned} \min \delta_1 &= 1 - \frac{\min\{\overline{AB}, \overline{EF}\}}{l_1} \\ &= 1 - \frac{\min\{l_1 + l_2 - L_{12}, l_1 + l_3 - L_{13}\}}{l_1} \\ &= \max\left\{\frac{L_{12} - l_2}{l_1}, \frac{L_{13} - l_3}{l_1}\right\} \end{aligned}$$

Similarly, the lower bounds of δ_2 and δ_3 also can be written as

$$\begin{aligned} \min \delta_2 &= \max\left\{\frac{L_{12} - l_1}{l_2}, \frac{L_{23} - l_3}{l_2}\right\} \\ \min \delta_3 &= \max\left\{\frac{L_{13} - l_1}{l_3}, \frac{L_{23} - l_2}{l_3}\right\} \end{aligned}$$

When the range of the MS from a serving BS is small, it is possible that the NLoS error is large enough so that the range circle of the serving BS lies fully within the range circle of the other BS, as illustrated in Fig. 4. In this scenario, δ_i calculated by the previous equations may be negative. So the equations are modified as

$$\begin{aligned} \min \delta_1 &= \max\left\{\frac{L_{12} - l_2}{l_1}, \frac{L_{13} - l_3}{l_1}, \rho\right\} \\ \min \delta_2 &= \max\left\{\frac{L_{12} - l_1}{l_2}, \frac{L_{23} - l_3}{l_2}, \rho\right\} \\ \min \delta_3 &= \max\left\{\frac{L_{13} - l_1}{l_3}, \frac{L_{23} - l_2}{l_3}, \rho\right\} \end{aligned} \quad (3)$$

Where $0 < \rho < 0.1$, and $0 < \delta_i \leq 1$ [24].

In section 3 we apply above constraints in IPM optimization bounds.

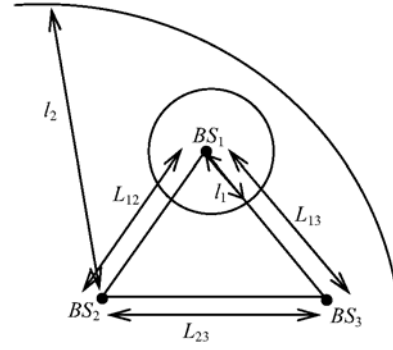


Fig. 4 Range circles of BS₁ and BS₂ that do not intersect

2. Interior Point Method

In general, the interior-point method [26] is a solution for convex optimization problem which include both equality and inequality constraints,

$$\begin{aligned} &\text{minimize } f_0(x) \\ &\text{subject to } f_i(x) \leq 0, i = 1, \dots, m \\ &Ax = b \end{aligned} \quad (4)$$

Where $f_0, \dots, f_m : R^n \rightarrow R$ are convex and twice continuously differentiable, and $A \in R^{p \times n}$ with rank $A = p < n$. We assume that the problem is solvable, i.e., an optimal x^* exists. We denote the optimal value for $f_0(x^*)$ as p^* .

We also assume that the problem is strictly feasible, i.e., there exists $x \in D$ that satisfies $Ax = b$ and $f_i(x) < 0$ for $i = 1, \dots, m$. This means that Slater's constraint qualification holds, so there exist dual optimal $\lambda^* \in R^m$, $v^* \in R^m$, which together with x^* satisfy the KKT [26] conditions:

$$\begin{aligned} Ax^* &= b, f_i(x) \leq 0, i = 1, \dots, m \\ \lambda^* &\geq 0 \\ \nabla f_0(x^*) + \sum_{i=1}^m \lambda_i^* \nabla f_i(x^*) + A^T v^* &= 0 \\ \lambda_i^* f_i(x^*) &= 0, i = 1, \dots, m \end{aligned} \quad (5)$$

Interior-point methods solve the problem (1) (or KKT conditions (2)) by applying Newton's method to a sequence of equality constrained problems, or to a sequence of modified versions of the KKT conditions. We will concentrate on a particular interior-point algorithm, the barrier method, for which we give a proof of convergence and a complexity analysis.

We can view interior-point methods as another level in the hierarchy of convex optimization algorithms. Linear equality constrained quadratic problems are the simplest. For these problems the KKT conditions are a set of linear equations, which can be solved analytically. Newton's method is the next level in the hierarchy. We can think of Newton's method as a technique for solving a linear equality constrained optimization problem, with twice differentiable objective, by reducing it to a sequence of linear equality constrained quadratic problems.

2.1 Logarithmic barrier function

Our goal is to approximately formulate the inequality constrained problem (1) as equality constrained problem to which Newton's method can be applied [26]. Our first step is to rewrite the problem (1), making the inequality constraints implicit in the objective:

$$\begin{aligned} &\text{minimize} && f_0(x) + \sum_{i=1}^m I_-(f_i(x)) \\ &\text{subject to} && Ax = b \end{aligned} \tag{6}$$

Where $I_:\mathbb{R}\rightarrow\mathbb{R}$ is the indicator function for the non-positive reals,

$$I_-(u) = \begin{cases} 0 & u \leq 0 \\ \infty & u > 0 \end{cases} \tag{7}$$

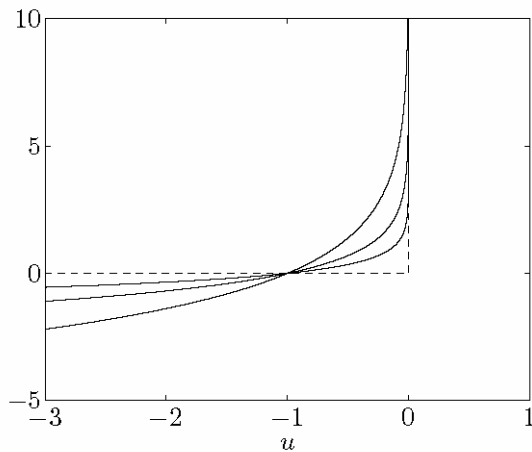


Fig. 5 the dashed lines show the function $I_-(u)$, and the solid curves show $\hat{I}_-(u) = -(1/t)\log(-u)$ for different values of t .

Since above function is not differentiable, equation (7) can not be solve by Newton method. The basic idea of the barrier method is to approximate the indicator function $I_$ by the function

$$\hat{I}_-(u) = -(1/t)\log(-u), \quad \text{dom}\hat{I}_- = -R_{++} \tag{8}$$

Where $t > 0$ is a parameter that sets the accuracy of the approximation. Substituting this new function in (3) gives approximation:

$$\begin{aligned} &\text{minimize} && f_0(x) + \sum_{i=1}^m (-1/t)\log(-f_i(x)) \equiv \\ &&& tf_0(x) + \phi(x) \\ &\text{subject to} && Ax = b \end{aligned} \tag{9}$$

The objective here is convex, since $-(1/t)\log(-u)$ is convex and increasing in u , and differentiable.

The function

$$\phi(x) = -\sum_{i=1}^m \log(-f_i(x)) \tag{10}$$

is called the logarithmic barrier or log barrier for the problem (1).

For future reference, we note that the gradient and Hessian of the logarithmic barrier function are given by

$$\begin{aligned} \nabla\phi(x) &= -\sum_{i=1}^m \frac{1}{-f_i(x)} \nabla f_i(x) \\ \nabla^2\phi(x) &= \sum_{i=1}^m \frac{1}{f_i^2(x)} \nabla f_i(x) \nabla f_i(x)^T + \sum_{i=1}^m \frac{1}{-f_i(x)} \nabla^2 f_i(x) \end{aligned} \tag{11}$$

The barrier method can be summarized as

given strictly feasible x , $t := t^{(0)} > 0$, $\mu > 1$, tolerance $\varepsilon > 0$.
repeat:

1. Centering step (Newton Optimization Method).
 Compute $x^*(t)$ by minimizing $tf(x) + \Phi(x)$, subject to $Ax = b$, starting at x .
2. Update. $x := x^*(t)$.
3. Stopping criterion: quit if $m/t < \varepsilon$.
4. Increase t . $t := \mu t$.

Choose of Barrier step size (μ):

The choice of the parameter μ involves a trade-of in the number of inner and outer iterations required. If μ is small (i.e., near 1) then at each outer iteration t increases by a small factor. As a result the initial point for the Newton process, i.e., the previous iterate x , is a very good starting point, and the number of Newton steps needed to compute the next iterate is small. Thus for small μ we expect a small number of Newton steps per outer iteration, but of course a large number of outer iterations since each outer iteration reduces the gap by only a small amount. On the other hand if μ is large we have the opposite situation. In practice for values of μ in a fairly large range, from around 3 to 100 or so, the two effects nearly cancel, so the total number of Newton steps remains approximately constant. This means that the choice of μ is not particularly critical;

values from around 10 to 20 or so seem to work well. When the parameter μ is chosen to give the best worst-case bound on the total number of Newton steps required, values of μ near one are used.

Choose of Starting value ($t^{(0)}$):

Another important issue is the choice of starting value of t . Here the trade-off is simple: If $t^{(0)}$ is chosen too large, the first outer iteration will require too many iterations. If $t^{(0)}$ is chosen too small, the algorithm will require extra outer iterations, and possibly too many inner iterations in the first centering step. Since $m/t^{(0)}$ is the duality gap that will result from the first centering step, one reasonable choice is to choose $t(0)$ so that $m/t^{(0)}$ is approximately of the same order as $f_0(x^{(0)})-p^*$, or μ times this amount. Then we can take $t^{(0)}=m/\eta$, where η is the duality gap.

Now we describe the Centering Step which follows the Newton Method.

2.2 Newton’s optimization method

The first step in Newton method [26] is calculation of Newton step and Newton decrement at x . For $x \in \text{dom} f$,

Newton step: $\Delta x_n = -\nabla^2 f(x)^{-1} \nabla f(x) = -H^{-1} g$

Newton decrement: $\lambda(x) = (\nabla f(x)^T \nabla^2 f(x)^{-1} \nabla f(x))^{1/2}$ (12)

Where H and g are Hessian and gradient matrices respectively.

Now by above mentioned, we can state the Newton’s algorithm in this way:

given a starting point $x \in \text{dom} f$, tolerance $\varepsilon > 0$.
repeat:
 1. Compute the Newton step and decrement.
 2. Stopping criterion: quit if $\lambda^2/2 \leq \varepsilon$.
 3. Line search. Choose step size t by **backtracking line search**.
 4. Update. $x := x + t\Delta x_n$.

2.3 Backtracking line search

Most line searches used in practice are inexact: the step length is chosen to approximately minimize f along the ray $\{x+t\Delta x \mid t \geq 0\}$, or even to just reduce f ‘enough’. Many inexact line search methods have been proposed. One inexact line search method that is very simple and quite effective is called backtracking line search. It depends on two constants, with α, β with $0 < \alpha < 0.5, 0 < \beta < 1$.

given a descent direction Δx for f at $x \in \text{dom} f$, $\alpha \in (0, 0.5)$, $\beta \in (0, 1)$.

$t := 1$.
 while $f(x + \Delta x) > f(x) + \alpha \nabla f(x)^T \Delta x$, $t := \beta t$

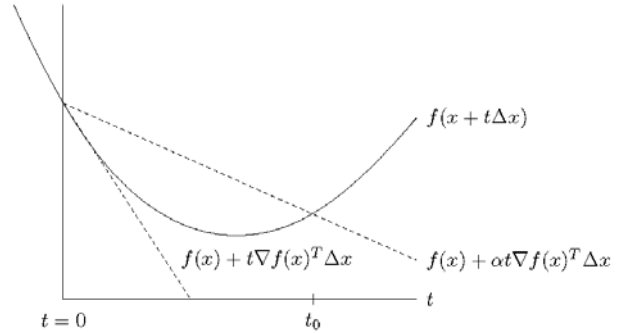


Fig. 6 backtracking line search. The curve shows f , restricted to the line over which we search. The lower dashed line shows the linear extrapolation of f , and the upper dashed line has a slope a factor of smaller. The backtracking condition is that f lies below the upper dashed line, i.e., $0 \leq t \leq t_0$.

The line search is called backtracking because it starts with unit step size and then reduces it by the factor until the stopping condition $f(x + \Delta x) \leq f(x) + \alpha \nabla f(x)^T \Delta x$ holds. Since Δx is a descent direction, we have $\nabla f(x)^T \Delta x < 0$, so for small enough t we have

$f(x + \Delta x) \approx f(x) + t \nabla f(x)^T \Delta x < f(x) + \alpha \nabla f(x)^T \Delta x$ (13)

This shows that the backtracking line search eventually terminates. The constant α can be interpreted as the fraction of the decrease in f predicted by linear extrapolation that we will accept.

Choose α, β :

The parameter α is typically chosen between 0.01 and 0.3, meaning that we accept a decrease in f between 1% and 30% of the prediction based on the linear extrapolation. The parameter β is often chosen to be between 0.1 (which corresponds to a very crude search) and 0.8 (which corresponds to a less crude search).

3. Mobile Location Estimation Problem

The location estimation problem can be formulated as a nonlinear optimization problem. The cost function should be minimized is taken to be the sum of the square of the distances from the MS location to the points of intersections of the range circles closest to it (i.e., points U, V, and W in Fig. 3). The coordinates of U, V, and W are (U_x, U_y) , (V_x, V_y) , and (W_x, W_y) , respectively. The objective

function to be minimized for the nonlinear optimization problem is, therefore [24]

$$f_0(x, y) = (x - U_x)^2 + (y - U_y)^2 + (x - V_x)^2 + (y - V_y)^2 + (x - W_x)^2 + (y - W_y)^2 \quad (14)$$

Now we can formulate the optimization problem as

$$\text{minimize } f_0(x, y) = (x - U_x)^2 + (y - U_y)^2 + (x - V_x)^2 + (y - V_y)^2 + (x - W_x)^2 + (y - W_y)^2$$

$$\text{subject to } \delta_{\min} \leq \delta_i(x, y) \leq 1, \quad i = 1, 2, 3 \quad (15)$$

As we define in equation (1), we should calculate the inequality constraint $f_i(x) \leq 0$.

$$\delta_i(x, y) = \frac{(x - x_i)^2 + (y - y_i)^2}{l_i^2}, \quad i = 1, 2, 3 \quad (16)$$

Where x, y are the mobile position and x_i, y_i are the position of i_{th} base station and l_i is the TOA base measurement of mobile's distance from base station i .

The inequalities will be formulated as,

$$f_i(x, y) = \begin{cases} \delta_{\lfloor \frac{i}{2} \rfloor + 1}(x, y) - 1 \leq 0, & i = 1, 3, 5 \\ (\delta_i)_{\min} - \delta_i(x, y) \leq 0, & i = 2, 4, 6 \end{cases} \quad (17)$$

$\lfloor i/2 \rfloor$ means the biggest integer next to $i/2$ value.

Now we should calculate the logarithmic barrier function for these constraints. From (7) and (8) we need to Gradient and Hessian of constraints. Then,

$$\nabla f_i(x, y)|_{\text{odd}} = \begin{bmatrix} \frac{\partial f_i(x, y)}{\partial x} \\ \frac{\partial f_i(x, y)}{\partial y} \end{bmatrix} = \begin{bmatrix} \frac{\partial \delta_{\lfloor \frac{i}{2} \rfloor + 1}(x, y)}{\partial x} \\ \frac{\partial \delta_{\lfloor \frac{i}{2} \rfloor + 1}(x, y)}{\partial y} \end{bmatrix} = \nabla \delta_{\lfloor \frac{i}{2} \rfloor + 1}(x, y) \quad (18)$$

$$\nabla f_i(x, y)|_{\text{even}} = \begin{bmatrix} \frac{\partial f_i(x, y)}{\partial x} \\ \frac{\partial f_i(x, y)}{\partial y} \end{bmatrix} = - \begin{bmatrix} \frac{\partial \delta_{\frac{i}{2}}(x, y)}{\partial x} \\ \frac{\partial \delta_{\frac{i}{2}}(x, y)}{\partial y} \end{bmatrix} = -\nabla \delta_{\frac{i}{2}}(x, y) \quad (19)$$

$$\nabla \delta_k(x, y) = \frac{2}{l_k^2} \begin{bmatrix} x - x_k \\ y - y_k \end{bmatrix}; \quad k = 1, 2, 3 \quad (20)$$

The same as above, we can find the Hessian matrix of the constraints as,

$$\nabla^2 f_i(x, y) = \nabla^2 \delta_{\lfloor \frac{i}{2} \rfloor + 1}(x, y); \quad i = 1, 3, 5 \quad (21)$$

$$\nabla f_i(x, y)|_{\text{even}} = -\nabla^2 \delta_{\frac{i}{2}}(x, y); \quad i = 2, 4, 6 \quad (22)$$

$$\nabla^2 \delta_k(x, y) = \begin{bmatrix} \frac{\partial^2 \delta_k(x, y)}{\partial x^2} & \frac{\partial^2 \delta_k(x, y)}{\partial x \partial y} \\ \frac{\partial^2 \delta_k(x, y)}{\partial y \partial x} & \frac{\partial^2 \delta_k(x, y)}{\partial y^2} \end{bmatrix} = \frac{2}{l_k^2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \quad k = 1, 2, 3 \quad (23)$$

Now the Gradient and Hessian of log barrier function will be as follows:

$$\nabla \phi(x, y) = \sum_{i=1}^3 \frac{-2}{l_i} \left(\frac{2\delta_i(x, y) - (1 + (\delta_i)_{\min})}{\delta_i^2(x, y) + (1 + (\delta_i)_{\min})\delta_i(x, y) + (\delta_i)_{\min}} \right) \begin{bmatrix} x - x_i \\ y - y_i \end{bmatrix} \quad (24)$$

$$\nabla^2 \phi(x, y) = \sum_{i=1}^3 \frac{2}{l_i} \left(\frac{1}{(\delta_i(x, y) - 1)((\delta_i)_{\min} - \delta_i(x, y))} \right) \left(\frac{2 \cdot 2\delta_i^2(x, y) - 2((\delta_i)_{\min} + 1)\delta_i(x, y) + 1 + (\delta_i)_{\min}^2}{(\delta_i(x, y) - 1)((\delta_i)_{\min} - \delta_i(x, y))} \begin{bmatrix} (x - x_i)^2 & (x - x_i)(y - y_i) \\ (x - x_i)(y - y_i) & (y - y_i)^2 \end{bmatrix} + (2\delta_i(x, y) - ((\delta_i)_{\min} + 1)) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \quad (25)$$

Base on (14) we can define the Gradient and Hessian of cost function as,

$$\nabla f_0(x, y) = 2 \begin{bmatrix} 3x - (U_x + V_x + W_x) \\ 3y - (U_y + V_y + W_y) \end{bmatrix} \quad (26)$$

$$\nabla^2 f_0(x, y) = 6 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (27)$$

If we define the problem as (9) then $f(x, y)$ (the function that should be minimize by Newton method) is equal:

$$\text{minimize } f(x, y) = t f_0(x, y) + \phi(x, y) \quad (28)$$

Then the Gradient and Hessian of this function which should be optimum by Newton method (12) is:

$$g = \nabla f(x, y) = t \nabla f_0(x, y) + \nabla \phi(x, y) \quad (29)$$

$$H = \nabla^2 f(x, y) = t \nabla^2 f_0(x, y) + \nabla^2 \phi(x, y) \quad (30)$$

By substituting the (24), (25), (26) and (27) in (29) and (30), the algorithm could be run.

The flow-chart of Fig. 7 shows the algorithm overview.

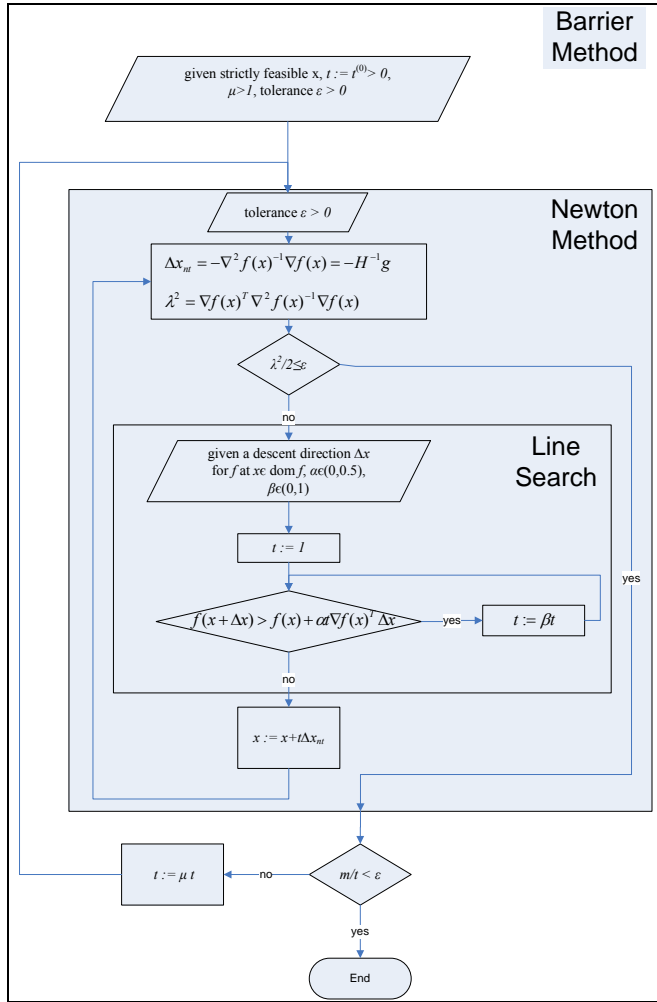


Fig. 7 Flow Chart of IPM Algorithm

4. Simulation results

In simulation of problem, we consider three BSs, measure the TOA from MS in NLoS environment. The location of BSs are (1000,1000), (4500,1000) and (2750,4500). The support of NLoS error range is assumed in [0,400] interval. four different distribution for NLoS error is considered as shown in figure 8.

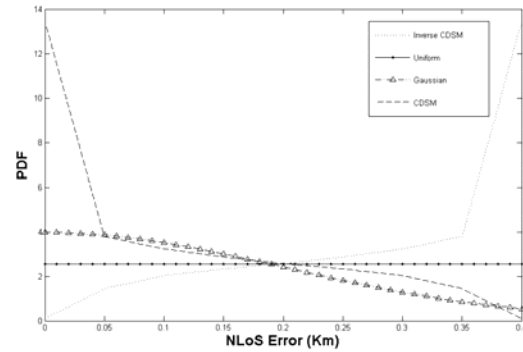


Fig. 8 different types of NLoS error distribution

Circular Disk of Scatterers Model (CDSM) is a relatively standard distribution for studying scattering effects [28]. CDSM is used for situations that both LoS and NLoS exist, but the LoS is more considerable than NLoS. In contrast, for opposite situations, we consider the reverse CDSM distribution that emphasize on NLoS compared to LoS. The variance of Gaussian error distribution assumed to be 0.04.

In simulation at first we find the intersection points of range circles by solving the system of equation of each two circles and find two points of intersection. As in Fig.1 U is the nearest intersection point of BSs 2, 3 from BS 1, V is the nearest intersection point of BSs 1, 3 from BS 2 and W is the nearest intersection point of BSs 1, 2 from BS 3.

In the next step, we find the constraints from equation (3) and applying them to the IPM algorithm. The simulation results are shown in Fig. 9 for different distributions.

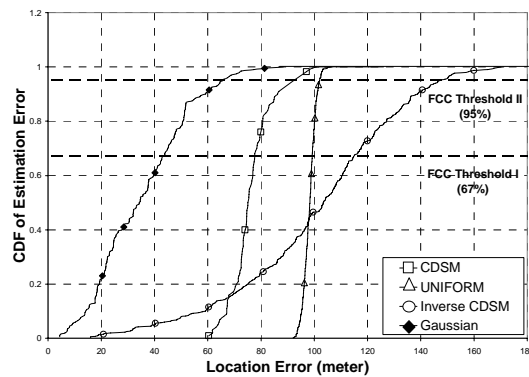


Fig. 9 the CDF of location error of the IPM on CDSM, Uniform, Reverse CDSM and Gaussian NLoS models

As Fig. 9 shows, all of the curves are started with a non-zero bias. This effect is caused by averaging from input

measurements. In this algorithm we average from 1000 sequential range measurements and this average is assumed as input to IPM algorithm. Now it helps us to prevent from variation of error but add a bias to measurements (error' bias). This means that we omit the samples which have been contained zero NLoS error for obtaining smaller range of variation (error's variance).

Also it can be seen from Fig. 9 that while the NLoS conditions is increased then the estimation error will grow. For the reverse CDSM that has strongest NLoS features, the estimation error is greater than other situations.

Finally, for a performance comparison with previous works, we show the results of proposed algorithm and other methods for CDSM case in Fig. 10.

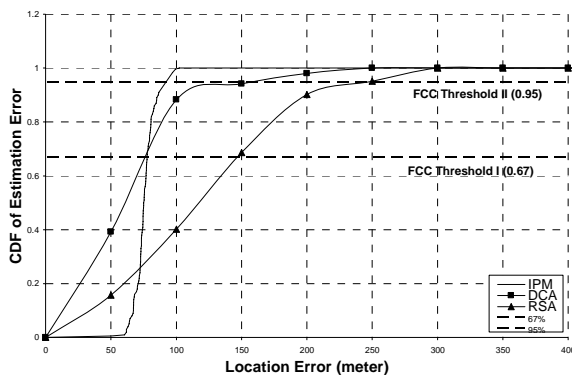


Fig. 10 the CDF of Location Error of the IPM, RSA and DCA algorithms for CDSM NLoS model

The FCC E-911 requirement thresholds are also depicted in Figures 9 and 10. It is shown for CDSM in 67% of times the error is less than 75 meter and for 95% of times, error is less than 95 meter. This will be approved the FCC E-911 specification.

5. Conclusion

A new location estimation algorithm based on interior point method for mitigating NLoS error was developed and evaluated. In practice no priori probabilistic distribution of NLoS is required and only three ranges obtained from ToA measurements is adequate. We simulated the algorithm subject to four different probability distributions and for all of them the results showed superior performance over the previous works, such as RSA and DCA. For pervasive analysis of problem, we considered both LoS and NLoS conditions by applying CDSM and Reverse CDSM probability distributions. It can be seen from the error CDFs that the location error of

IPM is less than 75 meter for 67% of the time, and less than 95 meter for 95% of the time and hence the obtained accuracy is beyond the FCC E-911 requirements. Combining the statistical estimation and IPM geometrical optimizations is still an open problem that in future may be studied.

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