## New One-Time Proxy Signature Scheme based on DLP using the Warrant

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#### Summary

The one-time proxy signature scheme is a kind of digital signature scheme. Using the signature scheme, the original signer can delegate his/her signing capability to the proxy signer and the proxy signer can sign the message only once. In this paper, we propose a new one-time proxy signature scheme based on Discrete Logarithm Problem(DLP). The proposed scheme uses Wang's basic proxy signature and the warrant. Because our one-time proxy signature scheme satisfies all security properties, the scheme is secure. Furthermore, we extend our scheme for multiple signing.

#### Key words:

Digital signature, Proxy signature, One-Time Signature, Public key cryptography

## **1. Introduction**

In 1996, Mambo, Usuda, and Okamoto proposed a new concept, called proxy signature [4], [5]. In a proxy signature scheme, one user Alice, called the original signer, delegates her signing capability to another user Bob, called the proxy signer, and Bob can sign messages in behalf of Alice. A verifier can validate its correctness and can distinguish between a normal signature and a proxy signature. As a result, the verifier can be convinced of the original signer's agreement on the signed message. Proxy signature schemes have been suggested for use in a number of applications, including electronic commerce, mobile agents, and distributed shared object systems, etc. In an example, the president of a company delegates a signing right to his/her secretary before a vacation. The secretary can make a signature in behalf of the president and a verifier is confident that the signature has been made by an authorized secretary, and the verifier can be convinced of the president's agreement on the signed message. Typically, a proxy signature scheme proceeds as follows. The original signer Alice sends the proxy signer Bob a signature that is associated with a specific message. Bob makes a proxy secret key using this information. Bob can then sign on a message with the proxy secret key using a normal signature scheme. After this message and signature is sent to the verifier, he/she recovers a proxy public key using public information and verifies the proxy signature using a normal signature scheme.

One-time signature schemes were proposed by Rabin [12] and Lamport [13]. They are based on the idea of committing public keys to private keys using one-way hash function. A user can sign a message only once by a public, private key pair. If the user wants to sign another message, the user has to have another public, private key pair.

One-time proxy signatures are one-time signatures for which the original signer can delegate his/her signing capability to the proxy signer. In 2003, H. Wang and J. Pieprzyk proposed a one-time proxy signature scheme [14] and another one-time proxy signature scheme was proposed by M. Mehta and L. Harn in 2005 [15]. These two schemes are secure, but they are complicated and not efficient. In this paper, we propose a new one-time proxy signature. It is a simple one-time proxy signature scheme that the proxy signer can sign the message once or for a restrictive number of times. The proposed scheme is very simple and efficient. First of all, we propose a basic scheme for signing once. If the proxy signer signs a multiple messages using the same proxy secret key, the proxy secret is known to everyone. It is illegal. Therefore, the proxy signer has to sign only a message using a proxy secret key. Our scheme uses Wang's basic proxy signature scheme [1] and satisfies all security properties. Therefore, the proposed scheme is secure. Our scheme is useful in several applications such as e-payment and e-voting. Furthermore, because our proxy scheme is a one-time signature, it is more useful.

The rest of this paper is organized as follows. Section 2 introduces the computational assumptions, definitions, and notations for a one-time proxy signature scheme. In Section 3, we briefly recall Wang's basic proxy signature. Section 4 presents the new one-time proxy signature scheme based on DLP and Section 5 discusses its security. Finally, the conclusion is presented in Section 6.

## 2. Preliminaries

### 2.1 Assumption

Our scheme uses the following known difficult problem for security.

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## Assumption 1: Discrete Logarithm (DL) assumption. Let $G_q = \langle g \rangle$ be a cyclic multiplicative group generated by g of order q. Then on inputs $(g, g^x)$ where $x \in_R \mathbb{Z}_q$ is a random number, there is no probabilistic polynomial-time algorithm that outputs the value of x with a non-negligible probability.

This computational assumption is widely believed to be true for many cyclic groups, such as the multiplicative subgroup  $G_q = \langle g \rangle$  of the finite field  $\mathbb{Z}_p$ , where p is a large prime and q is a prime factor of p-1. In practice, |p|=1024 and |q|=160 are considered to be suitable for most current security applications.

#### 2.2 Definitions

**Definition 1.** A **one-time proxy signature scheme** is usually comprised of the following phases:

-Setup: The original signer Alice and the proxy signer Bob generate their private, public key pairs, respectively. These key pairs are used in a normal signature scheme.

**-Proxy delegation:** The original signer Alice and the proxy signer Bob perform an interactive protocol to generate a proxy private, public key pair  $(x_p, y_p)$ .  $x_p$  is

known only to the proxy signer Bob and  $y_p$  is public or revocable publicly.

**-Proxy signature generation:** The proxy signer Bob signs on a message using proxy private key  $x_p$  and sends the

signature to Cindy.

**-Verification:** Cindy verifies *m* and its signature using verification equation.

The security requirements for proxy signature are specified in K. Zhang [10], [11].

**Definition 2.** A secure one-time proxy signature scheme should satisfy the following requirements:

**-Unforgeability:** Only the designated proxy signer can create the one-time proxy signature(even the original signer cannot do it).

**-Undeniability:** Neither the original signer nor the proxy signer can sign the message instead of the other party. Both the original signer and the proxy signer cannot deny their signatures against anyone.

-Identifiability: An original signer can determine the proxy signer's identity from a proxy signature.

**-Distinguishability:** The one-time proxy signature must be distinguishable from the normal signature.

-Verifiability: The one-time proxy signature can be verified by everyone.

2.3 Notations

For the convenience of describing our work, we define the parameters as follows:

- p,q: two large prime numbers,  $q \mid p-1$ .

- g : is an element of  $Z_p^*$ , its order is q.

 $\textbf{-} x_U \textbf{,} \textbf{} y_U \textbf{:}$  a participant U's private key and public key,

respectively,  $y_U = g^{x_U} \mod p$ .

- H() : a public cryptographically strong hash function

- || : denotes the concatenation of strings

 $-m_w$ : the warrant which specifies the delegation period for the kind of message *m* is delegated, the identities of the signer, etc.

-  $Sig_U[m]$ : a participant U 's signing behavior on the message m.

## 3. Related Work

In 2005, Wang proposed a basic proxy signature scheme [1] based on a provably secure two-party Schnorr signature proposed in [7]. In this section, we briefly review Wang's basic proxy signature scheme.

## 3.1 Wang's Basic Proxy Signature Scheme

It is assumed that the original signer Alice and the proxy signer Bob have agreed on a warrant  $m_w$  before generating a proxy key pair for Bob. Wang's proxy signature scheme [1] is as follows:

**Proxy Key Generation.** To generate a proxy key pair  $(x_p, y_p)$  for the proxy signer Bob, Alice and Bob execute the following interactive protocol jointly.

(1) Alice picks a random number  $k_A \in_R \mathbb{Z}_q^*$ , computes

 $r_A = g^{k_A} \mod p$  and  $c = H(r_A)$ , and then sends c to Bob.

(2) Similarly, Bob chooses a random number  $k_B \in_R \mathbb{Z}_q^*$ , computes  $r_B = g^{k_B} \mod p$ , and replies to Alice with  $(c, r_B)$ .

(3) When  $(c, r_B)$  is received, Alice checks whether  $r_B^{q} = 1 \mod p$ . If the validation holds, she computes

 $r_P = r_A r_B \mod p$ ,  $s_A = k_A + x_A H(m_w, r_P) \mod q$ , and sends the pair  $(r_A, s_A)$  to Bob.

(4) Upon receiving  $(r_A, s_A)$ , Bob first computes  $r_P = r_A r_B \mod p$ , and then checks whether  $r_A^q = 1 \mod p$ ,  $c = H(r_A)$ , and  $g^{s_A} = y_A^{H(m_w, r_P)} r_A \mod p$ . If all validations pass, he calculates  $s_B = k_B + x_B H(m_w, r_P) \mod q$ , and finally sets his proxy key pair  $(x_P, y_P)$  by

$$x_P = s_A + s_B \mod q$$
$$y_P = g^{x_P} \mod p$$

**Proxy Signature Generation.** To generate a proxy signature on a message m that conforms to the warrant  $m_w$ , the proxy signer Bob performs the same operations as in the standard Schnorr signature scheme [2]. That is, he first selects a random number  $k \in_R Z_q^*$ , and computes  $r = g^k \mod p$ ,  $s = k + x_p H(m, m_w, r) \mod q$ . The resulting proxy signature on message *m* is  $\sigma = (m_w, r_p, r, s)$ .

**Proxy Signature Verification.** To verify the validity of  $\sigma$ , the verifier Cindy operates as follows:

(1) Check whether the message m conforms to the warrant  $m_w$ . If not, stop. Otherwise, continue.

(2) Check whether Alice and Bob are specified as the original signer and the proxy signer in the warrant  $m_w$ , respectively.

(3) Recover the proxy public key  $y_P$  by computing:

$$y_P = (y_A y_B)^{H(m_w, r_P)} r_P \mod p$$

(4) Accept the proxy signature  $\sigma = (m_w, r_P, r, s)$  if and only if the following equality holds:

$$g^s = y_P^{H(m,m_w,r)} r \mod p$$

## 4. Description of the One-Time Proxy Signature Scheme based on DLP using the Warrant

We now present new one-time proxy signature scheme based on DLP. Our new scheme is constructed based on Wang's basic proxy signature scheme [1] and uses the warrant. The original signer Alice and the proxy signer Bob jointly generate a proxy key pair  $(x_p, y_p)$  for Bob, and the verifier Cindy can recover the proxy public key  $y_p$  for verification. We skip the setup phase.

### 4.1 Proxy Delegation Phase

The original signer Alice and the proxy signer Bob execute the following interactive protocol jointly.

(1) Alice picks random numbers  $k_A, k_1 \in_R \mathbb{Z}_q^*$ , computes  $r_A = g^{k_A} \mod p$ ,  $r_1 = g^{k_1} \mod p$  and  $c = H(r_A)$ , and then sends *c* to Bob.

(2) Similarly, Bob chooses a random number  $k_B \in_R \mathbb{Z}_q^*$ , computes  $r_B = g^{k_B} \mod p$ , and replies to Alice with  $(c, r_B)$ .

(3) When  $(c, r_B)$  is received, Alice checks whether  $r_B^q = 1 \mod p$ , If the validation holds, she computes  $r_P = r_A r_B \mod p$ ,  $s_A = k_A + x_A H(m_w, r_P) \mod q$ . The warrant  $m_w$  is generated by

$$m_o = [delegation lifetime,$$
  
 $Alice's and Bob's identities,$   
 $the description of message, r_1],$   
 $m_w = (m_o, Sig_{Alice}[m_o]).$ 

And then, she sends the pair  $(r_A, s_A, k_1, m_w)$  to Bob by secure manner.

Original signer(Alice)

Proxy signer(Bob)



Fig. 1 Proxy delegation phase.

(4) Upon receiving  $(r_A, s_A, k_1, m_w)$ , Bob first computes  $r_P = r_A r_B \mod p$ , and then checks whether  $r_A^q = 1 \mod p$ ,  $c = H(r_A)$ , and  $g^{s_A} = y_A^{H(m_w, r_P)} r_A \mod p$ . If all validations hold, he calculates  $s_B = k_B + x_B H(m_w, r_P) \mod q$ , and finally sets his proxy key pair  $(x_P, y_P)$  by

$$x_P = s_A + s_B \mod q$$
$$y_P = g^{x_P} \mod p$$

### 4.2 Signing Phase

To generate a proxy signature for the verifier Cindy, the proxy signer Bob executes the following operations. Because Bob has to use only  $r_1$ , he can sign a message only once. The proxy signer Bob computes  $r_1 = g^{k_1} \mod p$ ,  $s = k_1 + x_P H(m, m_w, r_1) \mod q$  and sends  $(m, r_P, m_w, r_1, s)$  to the verifier Cindy.



Fig. 2 Signing phase.

#### 4.3 Verification Phase

The verifier Cindy checks Alice's and Bob's identities and delegation lifetime of the warrant  $m_w$ . If all validations hold, Cindy follows the next operations. Cindy recovers  $y_P$  by  $y_P = (y_A y_B)^{H(m_w, r_P)} r_P \mod p$ . Next, Cindy can verify the one-time proxy signature by checking whether

$$g^{s} = y_{p}^{H(m,m_{w},r_{1})}r_{1} \mod p$$
 (1)

holds. This is because:

$$y_p^{H(m,m_w,r_1)}r_1 = g^{x_pH(m,m_w,r_1)}g^{k_1} \mod p$$
$$= g^{x_pH(m,m_w,r_1)+k_1} \mod p$$
$$= g^s \mod p$$

#### Verifier(Cindy)

$$y_p = (y_A y_B)^{H(m_g, r_p)} r_p \mod p$$
  
check  $g^s = y_p^{H(m, m_g, r_l)} r_l \mod p$ 

Fig. 3 Verification phase.

#### 4.4 Extension of the Signing Number of Times

Now, we extend our scheme for multiple signing. If the original signer Alice wants to allow the proxy signer to sign multiple messages, the original signer operates as follows. First of all, the original signer generates the warrant like this in the proxy delegation phase.

$$\begin{split} m_o = [delegation \ lifetime, \\ Alice's \ and \ Bob's \ identities, \\ the \ description \ of \ message, \\ r_1, r_2, r_3, \ldots], \\ m_w = (m_o, Sig_{Alice}[m_o]). \end{split}$$

In other words, if the original signer wants to allow the proxy signer to sign *n* messages, the original signer issues  $r_1, r_2, r_3, ..., r_n$  to the proxy signer by the warrant and sends  $k_1, k_2, k_3, ..., k_n$  to the proxy signer by secure manner. Next, in the signing phase, the proxy signer signs *n* messages using a pair  $(k_i, r_i)$  i = 1, 2, ..., n. At this time, the proxy signer has to use a pair  $(k_i, r_i)$  i = 1, 2, ..., n for a message. Lastly, in the verification phase, the verifier checks if  $r_i$ , i = 1, 2, ..., n is contained in the warrant and verifies the message and its signature

#### 5. Analysis of the Proposed Scheme

# Theorem 1. The proposed scheme satisfies the unforgeability property.

**Proof.** Firstly, we show that the original signer cannot forge the proxy signature without the proxy signer's help. Assume that the original signer Alice can forge proxy signature  $(m, r_P, m_w, r_1, s)$ . Then, we set c by:

$$c = H(m, m_w, r_1) \mod q.$$

But, we can get c', another result, using the same input  $(m, m_w, r_1)$  and the other hash function H'. In other words,

$$c' = H'(m, m_w, r_1) \mod q.$$

Therefore, the original signer Alice can forge another signature  $(m, r_P, m_w, r_1, s')$  for message m. And, because  $g^s = y_P^{H(m, m_w, r_1)} r_1 \mod p$ , then

$$g^{s} y_{p}^{-H(m,m_{w},r_{1})} = r_{1} = g^{s'} y_{p}^{-H'(m,m_{w},r_{1})} \mod p.$$

And,

$$g^{s} y_{p}^{-c} = r_{1} = g^{s'} y_{p}^{-c'} \mod p_{1}$$

Therefore,

$$g^{(s-s')} = y_P^{(c-c')} \mod p$$
$$= g^{x_P(c-c')} \mod p$$

Consequently,

$$g^{x_p} = g^{\frac{(s-s')}{(c-c')}} \mod p,$$
$$x_p = \frac{(s-s')}{(c-c')} \mod q.$$

That is to say, the original signer Alice can compute  $x_p$ , the discrete log of  $y_p$ . However, we already know that this is infeasible as previously stated in Assumption 1. Therefore, the original signer Alice cannot forge the proxy signature without the proxy signer's help. Also, in case Alice tries to generate  $(m, r_p', m_w', r_1', s')$  by replacing proxy private key,  $x_p'$ , this is also infeasible. When the proxy signature is verified, not only the proxy signature that also the original signer's public keys are used in the verification equation. Thus the forged proxy signature that is generated without valid proxy private key  $x_p$  cannot pass the verification. Also, in this case, the original signer Alice cannot forge the proxy signature.

Secondly, the third party cannot forge the valid proxy signatures as the third party knows less than the original signer Alice. Since the original signer Alice is unable to generate valid proxy signatures, the third party cannot generate valid proxy signatures either. Therefore, the proposed scheme satisfies the unforgeability property.

## Theorem 2. The proposed scheme satisfies the undeniability property.

**Proof.** As to the property of undeniability, it implies that the original and the proxy signers cannot deny having signed the message on behalf of the original signer to any person. In the proposed scheme, when the one-time proxy signature  $(m, r_P, m_w, r_1, s)$  is verified, the warrant  $m_w$  is

checked and the public keys  $y_A$  and  $y_B$  of the original signer and the proxy signer are used in the verification equation. So the original signer and proxy signer cannot deny having signed the message m on behalf of the original signer to any person. Therefore, the proposed scheme satisfies the undeniability property

# Theorem 3. The proposed scheme satisfies the identifiability property.

**Proof.** If the original signer wants to know the proxy signer's identity, the original signer can check the warrant  $m_w$ . It is easy to determine the identity. Therefore, the proposed scheme satisfies the identifiability property.

## Theorem 4. The proposed scheme satisfies the distinguishability property.

**Proof.** In the proposed scheme, when the one-time proxy signature  $(m, r_P, m_w, r_1, s)$  is verified, not only the proxy signer's but also the original signer's public keys and identities are used in the verification equation, so we can consider it as a proxy signature and not a normal signature. Thus, anyone can distinguish the one-time proxy signature from normal signatures and the proposed scheme satisfies the distinguishability property.

## Theorem 5. The proposed scheme satisfies verifiability property.

**Proof.** The security property implies that from the proxy signature, a verifier can be convinced of the original signer's agreement to the signed message. In the proposed scheme, on the one hand, from the warrant  $m_w$ , the verifier can know who the original signer and the proxy signer are. On the other hand, when the proxy signature is verified, the original signer's and the proxy signature is verified, the original signer cannot deny having delegated his signing capability to the designated proxy signer. That is to say, any verifier can be convinced of the original signer's agreement to the signed message and verify the message and its signature. Therefore, the proposed scheme satisfies the verifiability property.

## 6. Conclusion

In this paper, we propose a new one-time proxy signature scheme. Our new scheme is simple. We use Wang's basic proxy signature scheme for constructing the scheme and use the warrant for one-time signature. Our new scheme is secure because the scheme satisfies all security properties: Unforgeability, Undeniability, Identifiability, Distinguishability, and Verifiability.

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