The Problem of Restoration of Distribution Networks: a Heuristic Method

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Summary
This paper approaches the problem of restoring a faulted area in an electric power distribution system after locating and isolating the faulted block. Since service restoration is an urgent matter in the operation distribution systems, the proposal is to determine a sequence of operations to restore power supply to as much as possible of the out-of-service area, in the shortest time. For such, it was developed a Heuristic Search Method (Restoration Heuristic Procedure, RHP) that considers the operators' experience. The effectiveness of the proposed heuristic procedure is demonstrated by simulating tests in a real distribution network of *COPEL*, the utility that serves the city of *Curitiba*, state of *Paraná*, *Brazil*. It was found that the restoration plan(s) can be reached very efficiently, so this method can be considered as a satisfactory alternative to be used in this task.
Keywords: Distribution Network Restoration; Restoration Heuristic Procedure; Real Application.

1. Introduction

Utilities have been encouraged to improve the services provided to consumers by supervising, controlling and evaluating power supply continuity indices. Thus, the essential objectives are to maintain adequate service and continued power supply to consumers, to the maximum possible extent.
When a fault takes place in a certain area of an electrical distribution system, it is essential for the system operators to locate the fault, isolate the faulted block and restore the service to the out-of-service area. This paper is concerned with the problem of service restoration based on the assumption that the fault location has already been identified and the faulted block has already been isolated.
The purpose here is to present a Restoration Heuristic Procedure (RHP) to determine some restoration plans (a list of solutions with objective functions values listed in increasing order of quality, i.e., the best solution will be the first one, with the minimum value for the objective function, as will be seen later) for a fault on distribution feeders. The idea is to present the plan list to the system operators to help them make the final decision on how to restore the area that is out of service.
The RHP must meet some practical needs of dispatch system that are presented below [4]:

- the restoration plan must find the new configuration in the shortest time that is possible in order to avoid inconveniences for the customers and without violating the constraints (block voltages and load sections). Of course, it is expected that the response time will be proportional to the magnitude and complexity of the area under analysis. Results may provide optimal and sub-optimal configurations that reflect an improvement in the operators' decision making;
- the plan has to minimize the number of operations involved in each configuration. An increase in the number of operations in large centers increases operating costs and the time to restore the faulted area, and decreases the life of switches;
- restore as much load as possible within the out-of-service area;
- the plan must not overload any equipment or system component;
- the system's radial structure must be maintained;
- the configuration of the restored area should be as close to the original configuration as possible, i.e., only those switches that are in the faulted area's vicinity may be operated.

Typically, the restoration problem is a combinatorial optimization problem, because there is a great number of possible switches to be opened or closed in the distribution system. The magnitude of this kind of problem depends essentially on the quantity of switches involved in the search for an optimal configuration; *2n* combinations might be generated, where *n* is the number of switches involved. In major urban centers there are complex networks and several normally open (NO) and normally closed (NC) switches, thus increasing the magnitude of the problem. That is why the RHP has to narrow down the potential candidates and reach the restoration plan in a short period.
This problem is also multi-objective, because it has to satisfy all the practical needs listed above and, finally, is also a non-linear problem considering the operation constraints.
This paper is divided into five sections. Section 2 shows a literature review of some papers that also approach the restoration problem. In section 3, we present a description of the real problem and the RHP developed to solve the restoration problem. In section 4, some simulations are made using an academic example and a real example and, finally, in section 5, we present the conclusions and some considerations about this work.

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2. Literature Review

In recent years, several papers about service restoration have appeared and several approaches have been proposed. Some of them use optimization methods, some use expert systems, some use heuristics and some use combinations of more than one method. Some papers, among several ones, are summarized below.

Nagata *et al.* [6] developed a method for the restoration problem using an Expert System and a Mathematical Programming approach. The optimal target system for restoration is formulated as a Mathematical Programming problem and solved by the decomposition-coordination strategy. The whole problem is divided into sub-problems according to the generic knowledge of experts. The introduction of a new concept (restorative operation cost) that mirrors the several restoration strategies has decreased the number of rules in the knowledge base of a typical Expert System. In another paper, Nagata *et al.* [7], proposed an algorithm for a quick and efficient computational solution based on a Mathematical Programming approach. The computational time was greatly improved by dividing the problem into two stages and incorporating strategies derived from the operators' experience.

Morelato & Monticelli [5] presented a general framework for heuristic searches, which can be used to solve distribution restoration problems, including, but not limited to the problem dealt with in this paper. Ciric & Popovic [1] proposed a methodology combining a heuristic approach and Mixed-Integer Programming to solve the network restoration problem. Due to its computational efficiency, this approach can be used both in planning and in operational environments.

Shirmohammadi [8] described the principle and the implementation of a heuristic methodology to restore service to the isolated portions of a distribution system via network reconfiguration, by operating over the whole network all at once. The methodology determines a minimum number of switching operations required to restore service to network branches that are isolated. Hattori *et al.* [2] developed a new algorithm for power distribution restoration - named "Dynamic Area determination Algorithm based on Incremental Expansion Approach" - based on a Modern Heuristic Method, which is an original algorithm and does not depend on any conventional approaches. The most remarkable characteristic of this new algorithm is the incremental expansion of restoration areas.

Hsu *et al.* [4] proposed a heuristic search method for service restoration, following a fault on distribution feeders, which are clearly separated in a *trunk* (the main part of the feeder) and *laterals* (branches starting at the trunk). To reach a restoration plan that satisfies all practical requirements, a set of heuristic rules was compiled through interviews with experienced operators at the Taiwan Power Company. Restoring a fault in a distribution system within the service area of the Taiwan Power Company's Taipei City District Office checked the proposed heuristic's effectiveness.

3. Problem Description

COPEL - Companhia Paranaense de Energia Elétrica (Electrical Power Company of Paraná), the utility that supplies the 399 cities and 216 small locations in the state Paraná, Brazil, has 349 substations to supply around 2,800,000 consumers (residences, industries and others). Specifically in Curitiba, the capital of Paraná, and in its metropolitan area, there are 32 substations and 250 feeders, with an average of 8 feeders per substation, supplying around 170,000 consumers.

When they are part of the city of Curitiba's GIS (Geographical Information Systems) database, distribution feeders have in each feeder, 400 to 5,000 objects (also called elements) that may be transformation stations (demand nodes), sections or switches. Switches in feeders can be normally closed (NC) or normally open (NO). Normally open switches can be those that connect two feeders, or loop switches inside one feeder. The state of all switches determines the network's state before the fault.

In order to meet the processing needs, georeferenced circuits were simplified, and the electrical network was represented as a $G(X, A)$ graph, where the set of nodes X is formed by the network's blocks (a block is a set of transformers, which is limited by switches), and the set of arcs A is formed by the network's switches (an arc interconnects two blocks and can be open or closed). For each block, we must know the demand in *kW* of the region represented by the block. It is also necessary to know the capacity (also in *kW*) of each feeder. Knowing the load demands of each block in a feeder, we can calculate the total demand for the feeder. Subtracting this value from the feeder's capacity, we obtain the feeder's *reserve* capacity. One can see that each feeder (considering only its blocks and closed switches) forms a tree (that is, all blocks are connected, without loops); feeders must always remain as a tree, in order to maintain the distribution feeder's radiality.

Figure 1, at the end of this paper, represents a simplified network (used for testing purposes) with three substations, three feeders, 35 blocks and 44 switches.

Let's suppose that a fault happened somewhere inside block 7 in this network. The first thing that has to be done is to open all switches connected to block 7, in order to isolate the fault. The network, after the fault has been isolated, is represented in figure 2 (at the end).

The substation is still feeding normally blocks 2, 3, 4 and 5, because they are located before the faulted area and thus are not affected by the fault. However, blocks 6 to 13 are now not being supplied. Block 7 will need to remain in this state until the fault is restored, but some of the other blocks can have their power restored by closing normally open switches and connecting them to adjacent feeders, such as the switch between blocks 9 and 26.

The company aims at restoring power as much as possible and as fast as possible, otherwise it may have to pay fines because of service interruptions. The problem is to find a plan (a list of switches to be closed or opened) that will restore the area that is not being served.

In this case, one can see that each of the switches connected to block 7 that has been opened produces a sub-tree that has to be restored. There are three sub-trees: one formed by block 6, one formed by block 8 and one formed by blocks 9, 10, 11, 12 and 13. Each of these sub-trees is in fact a sub-problem that must be solved separately - restoring the sub-tree formed by blocks 9 to 13 does not restore block 8, or vice versa.

In many cases, there will be only one sub-tree to be restored. This happens when there are only two switches connected to the faulted block. This is the case of most of the blocks of feeder 3, as can be seen in Figures 1 and 2.

Again, with block 7 as the faulted block, we can see that it is not possible to restore block 6 because it is not connected to any other block. Block 8 only connects to another block of the same feeder by means of a loop switch (normally open). If block 11 is restored, then we can close this switch and restore block 8, if there is enough load reserve (capacity reserve). For the biggest sub-tree there are several connection possibilities to two different adjacent feeders. If the reserve of one of them is big enough, it might be possible to close just one switch and restore the whole sub-tree. If not, it will be necessary to use both feeders, each restoring a part of the sub-tree. To keep the radiality, a switch inside this sub-tree has to be opened.

By the time the research was made, operators in *COPEL*'s Control Center accomplished this process manually. Even tough these operators are very experienced people with a broad knowledge of the network, often the solution they find is not the optimal one and sometimes areas that could be restored are left unfed. Depending on the problem's complexity it might take too long for them to reach a solution. Because of that, the algorithm that is described in this paper in sections 3.1 and 3.2, tries to find a plan to restore the faulted area, presenting the switches that have to be operated, thus helping the operators in their task.

3.1 A Heuristic Search Method: Restoration Heuristic Procedure

The literature presents many heuristic procedures to solve the distribution network restoration problem. In this paper we will present a Heuristic Search Method: the RHP already mentioned, which was developed taking into account some ideas from papers written by Hsu *et al.*, 1991 and Nagata *et al.*, 2000, with adaptations to fit the problem to *COPEL*'s needs and to the structure of its distribution network.

Our restoration algorithm works by closing and opening switches in the network and calculating the effects of these changes. It maintains a list of solutions, which is initially empty and to which solutions are added as they are found. The maximum number of solutions to be kept in the list can be changed by the operator. Whenever a solution is found, be it a solution that restores the faulted area wholly or partially, its objective value is calculated by formula (1) below.

$$Z = \alpha C + \beta \Sigma_i \; k_i \qquad (1)$$

where C is the total load that is not being served; k_i is the weight of switch i, where i ranges from 1 to the number of operated switches. Switches with a priority have lower weights, and switches without a priority have greater weights, so as to prevent their excessive use. Parameters α and β prioritize either the minimization of the non-served area or the minimization of the number of switching operations. The smaller the value of Z, the better the solution is.

If the list of solutions has not reached the maximum number of solutions, a newly found solution is added to the list. Additionally, if the list is full, but the new solution is better than a solution in the list, this new solution is added, and the worst solution in the list is removed. The list of solutions is always sorted decreasingly from the best to the worst one.

When the algorithm ends, a list of solutions is presented to the operator, so he/she can choose a solution. The best solution the algorithm found (the first one in the list) may not be the best to be executed, because of subjective criteria, such as switches that are operated too frequently, switches with difficult access, the operational team's present position and so on.

3.2 The steps in the Algorithm

The steps in the RHP algorithm are the following:

1. Identify the block in which the fault has happened.
2. Open all switches connected to that block in order to isolated it. For each switch that is opened, a sub-tree is generated, which will have to be restored, except for the switch that connects the faulted block to a block the substation is still feeding normally (a block that is "before" the fault).
3. Create a list of the blocks still being fed and containing at this moment only the blocks the substation is still feeding, if any.
4. Check if there is a loop switch between a fed block and an unfed one.
   (a) If there isn't, go to step 5 below.
   (b) If there is, then close this switch and add the blocks to the sub-tree that is now being fed to the list of fed blocks. Remove this sub-tree from the list of sub-trees that still have to be restored. If there still are sub-trees to be restored, go to step 4. Otherwise, the algorithm ends with full restoration, and no choices have to be made.
5. For each sub-tree that has to be restored, do the following:

(a) Start with an empty list of solutions and room for a certain number of possible solutions, which is determined by the operator.
(b) Create a list of switches connecting a block in this sub-tree to an adjacent feeder. Call this list *S*.
(c) Calculate the total load demand for this sub-tree, as well as the reserves for the adjacent feeders.
(d) For each switch in *S*, close that switch and determine that maximum extent to which the sub-tree can be fed from the adjacent feeder when that is switch closed. It may, or may not, be possible to restore the whole sub-tree. Either way, a solution is found for each switch, its objective value is calculated and if the solution is considered good it is added to the list (possibly replacing another solution).
(e) If a solution restoring the whole sub-tree is found, go to step *5i*; otherwise, continue.
(f) Now, for each pair of switches that connect to different adjacent feeders in S, do the following:
    i. Close both switches.
    ii. Since this makes the network non-radial, at least one switch must be opened. This switch must be in the path between the closed switches, so as to establish this path.
    iii. For each switch in the path, determine the maximum area that can be restored when this switch is open and the other two closed. Each switch in the path produces a solution that is analyzed and added to the list, if good.
(g) If a solution restoring the whole sub-tree is found, go to step *5i*; otherwise, continue.
(h) Now, a similar procedure to the one described in *5f* is executed, but with three switches to adjacent feeders. In this case, at least two switches must be opened. This generates several other solutions, some of which may be added to the list.
(i) Present the solutions in the list to the operator, and let him/her choose one.
(j) The chosen solution is adopted, the state of the network is updated according to the solution and the load used for the restoration is subtracted from the reserves of the feeders that were used.
(k) The process restarts at *5a* with another sub-tree, if there still are unsolved sub-trees.

6. After all sub-trees have been restored, the algorithm ends.

4. Simulations and Results

In this section we present an academic example for the distribution network restoration problem in order to clarify how the RHP works. After that, we show some results obtained in a part (consisting of three substations and eight feeders) of *Curitiba*'s electrical network distribution system.

4.1 Academic Example

The RHP was implemented in the MATLAB language in order to test it and verify its efficiency and efficacy. Let us first consider the network $G(X, A)$ represented in figure 1, where X is the set of blocks (35 blocks) and A is the set of switches (44 arcs, where 12 are normally open (NO) and 32 are normally closed (NC)) subdivided into the S^1, S^2 and S^3 feeders, where each feeder is supplied by one substation at blocks 1, 14 and 25. NO switches interconnect the feeders.
This network's arcs have maximum capacities (I_{max}) to be complied with, as shown in table 1 for feeder 1, which has 19 arcs. Table 1 also shows the terminal blocks in each arc (switch), their codes and their types (NC or NO).

Switch	Code	Status	Imax
[1, 2]	1083	NC	400
[2, 3]	1084	NC	150
[2, 4]	1085	NC	400
[2, 7]	1086	NC	400
[4, 5]	1087	NC	400
[7, 6]	1088	NC	150
[7, 8]	1089	NC	150
[7, 10]	1090	NC	400
[8, 11]	1091	NO	150
[11, 12]	1092	NC	150
[10, 9]	1093	NC	150
[10, 11]	1094	NC	150
[10, 13]	1095	NC	400
[9, 26]	1096	NO	150
[13, 26]	1097	NO	400
[13, 29]	1098	NO	400
[12, 17]	1099	NO	150
[12, 33]	1100	NO	150
[5, 24]	1101	NO	400

Table 1. Feeder 1 Data.

If, for instance, a fault is generated in block 2, the first thing to do is to isolate this block from the rest of the system.
Assuming that the weights to minimize time and loads are the same (α = 1 and β = 1) and running the RHP, the sequence of operations used in the attempt to restore the system is obtained by the computational program and is the following:

- number of total operations: 5;
- switches used to isolate the fault: (these operations are not considered in the total number of operations, because in any case there is always the need to isolate the faulted block): (1,2), (3,2), (4,2), (7,2);
- switches that must be opened: (7,8), (10,11);

- switches that must be closed: (5,24), (12,17), (13,26);
- there is a load cut in block 8. The system cannot supply block 8. Additionally, block 3 is also not fed, because there are no connections to it.

Figure 3 (at the end) represents the state of the network after these operations have been executed.

4.2 Example with a Real Network

Typically, small networks end up by being of use only for academic conclusions on the use of algorithms. Now, a part of a real network distribution system with two substations and eight feeders is considered. This network is located in the urban area of Curitiba, with an average consumer density and comprising approximately 200 blocks. It is represented in figure 4.

For the network represented in figure 4, several faults were simulated in the feeders in order to check the RHP's effectiveness. Parameters α and β were changed so as to analyze the impact on results. Table 2 shows some of the simulated cases. In this table, N_x and N_y are the number of switches that were opened or closed, respectively, after applying the RHP.

Feeder	**Weights**	**Block**	**Switches used**	**Time** (sec)	**Not served** (%)
1	$\alpha = 1$; $\beta = 1$	2	$N_x = 3$; $N_y = 4$	< 1	0
1	$\alpha = 1$; $\beta = 0$	19	$N_x = 4$; $N_y = 1$	1	27
5	$\alpha = 1$; $\beta = 0$	2	$N_x = 2$; $N_y = 3$	< 1	0

Table 2. Results of some simulations.
(Pentium IV, 512 MB RAM, 2.0 GHz)

Next, the algorithm is executed once, showing the input (the faulted block) and the output, and the list of solutions obtained by the computational program (in this case, four solutions were presented). In italics there are some comments.

In which block the fault occurred? 3 (*the operator supplies this number*)

* Open switch 3-2 (*these two procedures are carried out to isolate*

* Open switch 3-5 *the faulted block*)

*** results for the sub-tree that starts at block 5

*** total demand: 1,235 kW

Solution 1: full restoration with 3 switches (Z = 5) *(the sequence of operations must be:)*

open switch 11-5; close switch 11-12; close switch 7-10.

Solution 2: full restoration with 3 switches (Z = 6) *(the sequence of operations must be:)*

open switch 5-6; close switch 11-12; close switch 7-10.

Solution 3: partial restoration with 2 switches (Z = 256) *(the sequence of operations must be:)*

restored load: 982 kW (253 kW not restored, i.e., 20.4858%)

open switch 6-7; close switch 11-12.

Solution 4: partial restoration with 2 switches (Z = 481) *(the sequence of operations must be:)*

restored load: 757 kW (478 kW not restored, i.e., 38.7045%)

open switch 5-11; close switch 7-10.

Please, select the number of the solution to be implemented: 3 *(the operator chooses the solution's number).*

The solutions (1, 2, 3 and 4) are listed in order of priority. In this example four solutions were listed, as follows: after opening the two indicated switches (to isolate the fault) it is necessary, for solution 1: three operations (one switch has to be opened and two have to be closed); for solution 2: three operations (one switch has to be opened and two have to be closed); for solution 3: two operations, but with partial service restoration and, finally, for solution 4: two operations, also with partial service restoration.

After choosing the solution, we have to run the simplified load flow in order to get the voltages in the blocks of the considered network (voltages have to be greater or equal to 95% of the maximum voltage) and the loads in the sections of the considered network (loads have to be smaller or equal to 80% of the maximum allowed load). In the given example (real network), the chosen solution was number three, even though this solution offers partial service restoration.

5. Conclusions

In this paper we presented a Heuristic Search Method - the Restoration Heuristic Procedure (RHP) - to solve the problem of restoration in a distribution network.

We show that the RHP is adequate to solve restoration problems and may be used in real-size networks. The heuristic procedure that was presented is able to considerably reduce the magnitude of the problem by using the local network concept (considering only adjacent feeders), in a very objective approach to the problem. This way, the heuristic procedure offers substantially fast solutions, usually with times that are smaller than one second.

We have to emphasize that although the example considered in this paper was a partial one - a real network in the city of Curitiba, with only two substations from a total of 32, and just eight feeders from a total of 250 - the computational time to run the RHP (about one second) will not be affected, because the procedure will always work locally, considering only the adjacent feeders.

Besides providing solutions in a very short time, the RHP presents a list of possible solutions that can be implemented in a prioritized order. The reader may think that the first solution should always be implemented (because it is the best solution the procedure found), but this is not necessarily true, because sometimes there are subjective criteria to be considered, such as those cited in the text, at the end of Section 3.1.

If there is more than one faulted block, in the same or in different feeders, the procedure presented in this paper can also be used. For this situation, the operator has to indicate first the faulted block in which the affected area has the greatest urgency for restoration (hospitals, industries and so on). After that, once the adjacent reserve feeder capacity is updated, the operator will indicate the second faulted block in priority and so on.

The RHP presented in this paper is very simple, interesting, efficient and effective. The restoration plan(s) presented to the operator will help him/her reach a final decision on how to restore the area(s) that are out of service. It reaches its practical objectives in terms of use in *COPEL*'s Operation and Distribution Centers' (ODCs) environment, for cities of all sizes.

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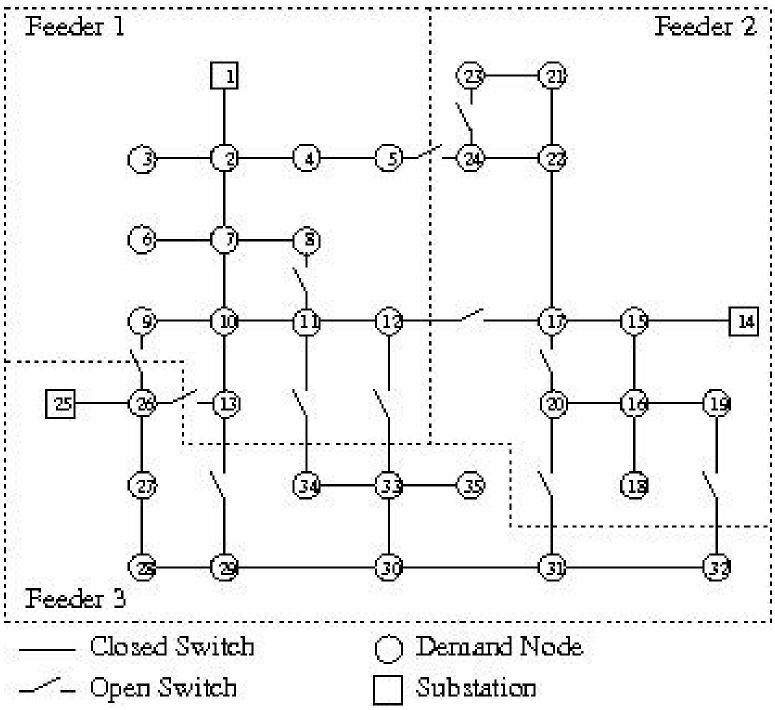

Figure 1. An electrical network represented as a $G(X, A)$ graph.

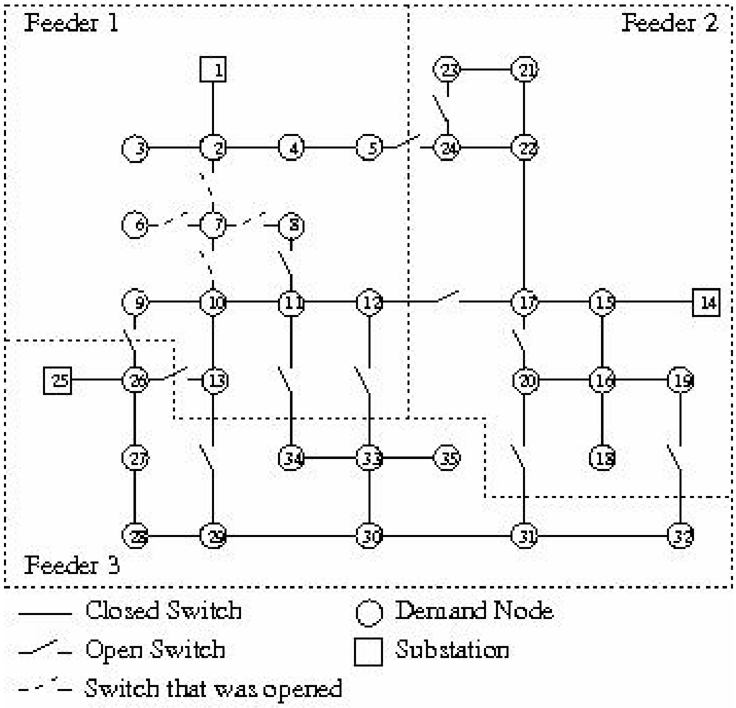

Figure 2. The network after the fault (block 7) has been isolated.

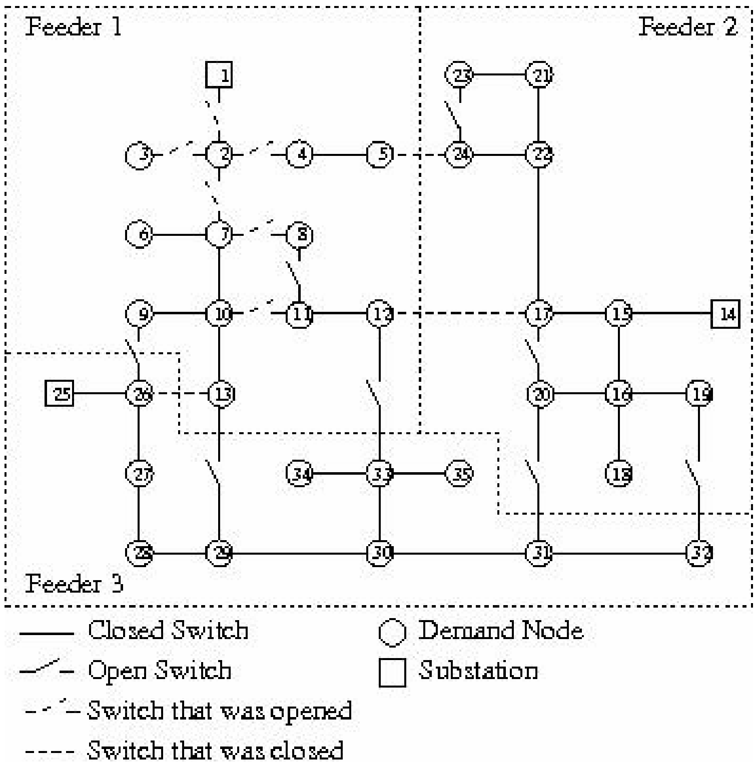

Figure 3. Result after running the RHP.

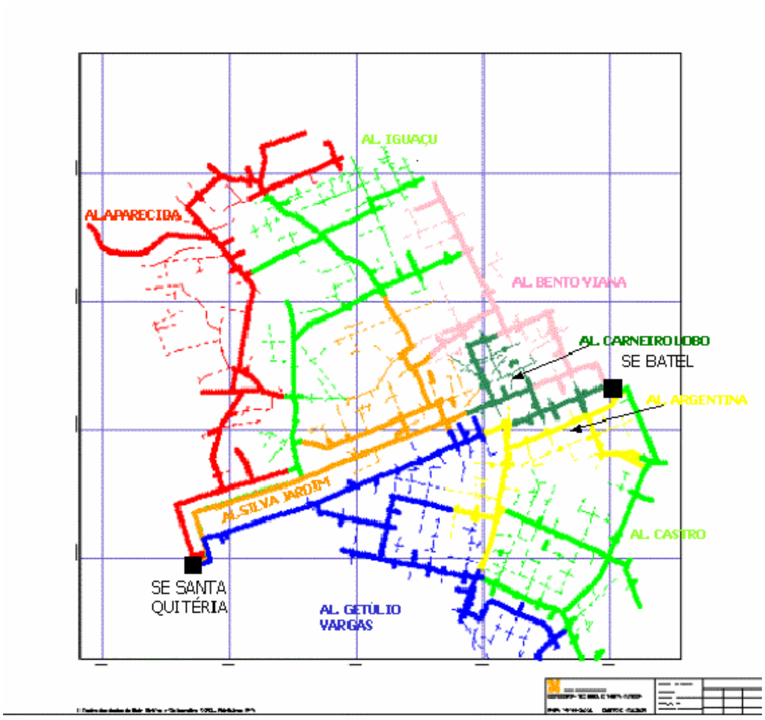

Figure 4. Illustration of a part of Curitiba's real network distribution system, showing two substations (SE SANTA QUITÉRIA and SE BATEL), eight feeders and some important lanes in the city.

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