Unsupervised Image Segmentation Method based on Finite Generalized Gaussian Distribution with EM & K-Means Algorithm

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Summary
In Image Processing Model Based Image Segmentation plays a dominant role in Image Analysis and Image Retrieval . Recently much work has been reported regarding Image Segmentation based on Finite Gaussian Mixture Models using EM algorithm. (Yiming Wu et al (2003)) , (Yamazaki.T (1998)). However, in some images the pixel intensities inside the image regions may not be Meso- Kurtic or Bell Shaped, because of the Non-Gaussian nature . Hence there are some situations where Image Segmentation is to be done with a more Generalized Finite Mixture Distribution. In this article we develop and analyze an image segmentation method based on Finite Generalized Gaussian Mixture Model using EM and K-Means algorithm. The K-Means algorithm is utilized to obtain the number of regions and the initial estimates of the model parameters. The update equations of the model parameters are obtained by using the EM algorithm. The segmentation of the pixels in the image is done by maximizing the component likelihood function. The performance of this method is evaluated through real time data on 3 images by calculating misclassification rate and image quality metrics. It is observed that the proposed method performs much superior to the earlier image segmentation methods.

Key words:
Image Segmentation, EM algorithm, K-Means algorithm, Finite Gaussian Mixture Model, Finite Generalized Gaussian Mixture model.

1. Introduction

Image Processing is a growing area of Computer Science and Engineering. The main objective of image processing is to understand the components of an image and interpret its semantic meaning. To analyze the features inside the image, Model based Segmentation algorithms will be more efficient compared to non-parametric methods. The efficiency of the segmentation algorithm is based on the suitable probability distribution ascribed to the pixel intensities in the entire image. In many image segmentation techniques it is assumed that the pixel intensities in each image region follows a Gaussian distribution and the entire image is assumed as a characterization of Finite Gaussian Mixture Models. These mixture models are more suitable for image segmentation only when the pixel intensities inside the image regions are symmetric and having meso-kurtic nature. However, in many practical situations arising at places like Medical Imaging, Robotics, Photo Copiers etc., the pixel intensities inside the image regions may not be skewed or meso-kurtic. To have a close approximation to the realistic situations it is needed to generalize the image segmentation algorithm with a more general mixture distribution which includes the Finite Gaussian Mixture model as a particular case.

The Generalized Gaussian Distribution includes the Gaussian distribution as a particular case and it can be parameterized in such a manner that its mean μ and Variance σ2 coincide with the Gaussian distribution. In addition to location and scaling parameters, the Generalized Gaussian Distribution is having a Shape parameter 'P' which is the measure of peaked ness of the distribution. . The Generalized Gaussian Distribution was used by Sharif .K et al (1995) for modeling the atmospheric noise sub band encoding of Audio and Video Signals, Choi S et al (2000) has used this distribution for impulsive noise direction of arrival and independent component analysis. Varanasi M.K. et al (1987) discussed the parameter estimation for the Generalized Gaussian Distribution by using the methods of Moments and Maximum Likelihood. J.Armando Dominguez et al (2003) developed a procedure to estimate the shape parameter in Generalized Gaussian Distribution. However, very little work has been reported regarding Image Segmentation based on Generalized Gaussian Distribution. In this paper we develop and analyze an image segmentation method based on Finite Generalized Gaussian Mixture Distribution. The number of image regions (Components 'K') is estimated by utilizing the K-Means algorithm. The estimation of the model parameters is carried by EM algorithm. The segmentation algorithm is developed based on pixel allocation to the regions which maximizes the component likelihood function. The performance of the developed segmentation algorithm is

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evaluated by obtaining the Image Quality Metrics like Mean Square Error, Signal to Noise Ratio and Image Quality Index. The accuracy of the classifier used in the segmentation is also studied with respect to the misclassification rates. This segmentation algorithm includes several of the earlier existing algorithms like Image Segmentation algorithm based on Finite Gaussian Mixture Distribution, Laplace Distribution etc., for different values of the shape parameters.

2. Generalized Gausslan Distribution

In this section we briefly discuss the probability distribution and its properties used in the image segmentation algorithm. Let the pixel intensities in the entire image obtained by pixel intensities (Obtained through pixel grabber under JAVA environment) is a Random Variable and follow a Finite Generalized Gaussian Mixture Distribution. It is also assumed that the entire image is a collection of 'K' image regions, then the pixel intensities in each image region follows a Generalized Gaussian Distribution. The probability density function is

$$f(z \mid \mu, \sigma, P) = \frac{1}{2\Gamma(1+\frac{1}{P})A(P,\sigma)} e^{-\left|\frac{(Z_i-\mu_i)}{A(P,\sigma)}\right|^P}$$

where

$$\sigma > 0,\ A(P,\sigma) = \left[\frac{\sigma^2\Gamma(1/P)}{\Gamma(3/P)}\right]^{\frac{1}{2}}.$$

The parameter μ is the mean, the function $A(P,\sigma)$ is an scaling factor which allows that the Var(Z) = σ^2, and 'P' is the shape parameter. When P=1, the corresponding Generalized Gaussian corresponds to a Laplacian or Doubly Exponential Distribution, When P=2, the corresponding Generalized Gaussian corresponds to a Gaussian distribution. In limiting cases $P \to +\infty$, equation (4.3.1) converges to a uniform distribution in $(\mu - \sqrt{3}\sigma, \mu + \sqrt{3}\sigma)$ and when $P \to 0+$, the distribution becomes a degenerate one in Z= μ. The mean value of the Generalized Gaussian distribution is μ. The Variance is σ^2.

3. K-Means Algorithm

The most perplexing issue for Image Classification is determining the finite number of regions (K) to be formed. Many statistical criteria are used for determining the number of classes, one such method is the K-Means algorithm.

Step1). Begin with a decision on the value of K = number of segments

Setp2). Put any initial partition that classifies the pixels into K segments. We can arrange the training samples randomly, or systematically as follows:

1. Take the first K training samples as a single-element Segment

2. Assign each of the remaining (N-K) training samples to the segment with the nearest centroid. Let there be exactly K segments (C_1, C_2—C_K) and n patterns to be classified such that, each pattern is classified into exactly one segment.. After each assignment, recompute the centroid of the gaining segment.

Step3). Take each sample in sequence and compute its distance from the centroid of each of the segments. If the sample is not currently in the cluster with the closest centroid switch this sample to that segment and update the centroid of the segment gaining the new sample and cluster losing the sample

Step4). Repeat step 3 until convergence is achieved, that is until a pass through the training sample causes no new assignments.

After determining the final value of the K(number of regions), we obtain the estimates the parameters μ_i, σ_i and α_i for the ith region using the segmented region pixel intensities with the method given by J.Armando et al (2003) . Substituting these values as the initial estimates, we refine the estimates of the parameters by using the EM algorithm.

4. EM Algorithm

The estimators of the model functions are obtained by using EM algorithm. For obtaining the EM algorithm, we consider that a sample of the Pixel intensities $z_1, z_2, \ldots z_N$ are drawn form an image with probability density function

$$h(z, \theta) = \sum \alpha_i f_i(z_s, \theta_i)$$

Then the likelihood function of the pixel intensities are

$$L(\theta) = \underset{s=1}{\overset{N}{\pi}}(\sum_{l=1}^{K}\alpha_i f_i(z_s,\theta_i))$$

$$= \underset{i=1}{\overset{N}{\pi}}\left(\sum_{i=1}^{K}\alpha_i \frac{1}{2\Gamma(1+\frac{1}{P})A(P,\sigma)} e^{-\left|\frac{(Z_i-\mu_i)}{A(P,\sigma)}\right|^P}\right)$$

$$where \quad A(P,\sigma) = \left[\frac{\sigma^2\Gamma(1/P)}{\Gamma(3/P)}\right]^{\frac{1}{2}}$$

This implies

$$L(\theta) = \sum_{l=1}^{N}\log h(z\ ,\theta\ )$$

$$= h(z_,\theta) = \sum_{s=1}^{N}\log\sum_{i=1}^{K}\alpha_i f_i(z_s,\theta_i)$$

$$= \sum_{s=1}^{N}\log\left(\sum_{i=1}^{K}\alpha_i \frac{1}{2\Gamma(1+\frac{1}{P})A(P,\sigma)} e^{-\left|\frac{(Z_i-\mu_i)}{A(P,\sigma)}\right|^P}\right)$$

We have to find the parameter $\alpha_i, \mu_i \; and \; \sigma_i$ for i=1,2,---K, maximizing the likelihood function (or) Log likelihood function. Here the shape parameter 'P' is estimated by the procedure given by J.Armando Dominguez et al (2003) and also we assume that shape parameter is same for all image regions of an image under consideration. For obtaining the estimates of this parameters we utilize the EM algorithm.

Where $t_k(z_s;\theta^{(l)}) = P(k \mid z_j;\theta^{(l)})$

$$= \frac{\alpha_i^{(l)} f_i(z_s,\theta^{(l)})}{h(z_{j,}\theta^{(l)})}$$

The expected value of L(θ) is

Following the heuristic arguments of Jeff A. Bilmes (1998),

$$Q(\theta,\theta^{(i)}) = \sum_{i=1}^{N}\sum_{J=1}^{K}\left[\log(\alpha_j f_i(z_{s,}\theta^{(i)})\right]t_i(z_s,\theta^{(i)})$$

The update equations of the EM algorithm are

$$\alpha_i^{(l+1)} = \frac{1}{N}\sum_{s=1}^{N}\left(\frac{\alpha_i^{(l)} f_i(z_s,\theta^{(l)})}{\sum_{i=1}^{K} f_i(z\ ,\theta^{(l)})}\right)$$

$$\mu_k^{(l+1)} = \frac{\sum_{s=1}^{N} t_i(z_s,\theta^{(l)})^{\gamma(N,P)} z_s}{\sum_{s=1}^{N} t_i(z_s,\theta^{(l)})^{\gamma(N,P)}}$$

$$\sigma_i^{(l+1)} = \left[\frac{\sum_{i=1}^{N} t_i(z_s,\theta^{(l)})\left(\frac{\Gamma(3/P)}{P\Gamma(1/P)}\right)\left\|z_i-\mu_i^{(l)}\right\|^{\frac{1}{P}}}{\sum_{i=1}^{N} t_i(z_s,\theta^{(l)})}\right]^{\frac{1}{P}}$$

4.1 Initialization of Parameters

- The efficiency of the EM algorithm in estimating the parameters is heavily dependent on the number of Segments (Clusters ('K')) and the initial estimates of the model parameters μ_i, σ_i and α_i (i=1,---K) . Usually in EM algorithm the mixing parameter α_i and the region parameters μ_i, σ_i are known as prior. A commonly used method in initialization is by drawing a random sample in the entire image data (mixture data) (Mclanchan and T.Krishnan(1997), G.Mclanchan and D.Peel(2000)). This method can be performed well when the sample size is large, but the computation time is also heavily increased, when the sample size is small it is likely that some small regions may not be sampled. To overcome this problem, we use K-Means algorithm. The number of mixture components is initially taken for K-Means algorithm by the histogram of the pixel intensities of the entire image. After determining the final value of the K(number of regions), we obtain the initial estimates the parameters p μ_i, σ_i and α_i for the ith region using the segmented region pixel intensities with the method given byJ.Armando etal (2003)

5. The Segmentation Algorithm

After refining the parameters the prime step is image reconstruction by allocating the pixels to the segments. This operation is performed by Segmentation Algorithm. The image segmentation algorithm consists of 3 steps

Step 1) obtaining the initial estimates of the Finite Generalized Gaussian Mixture Model with K-Means algorithm

Step 2) with the initial estimates obtained in step1, the EM algorithm is iteratively carried with the update equations, the EM algorithm converges when the difference of the old estimates and the new estimates are less than some threshold value (0.001), and the final estimates of the Finite Generalized Gaussian Mixture Model are obtained.

Step 3) the image segmentation is carried out by assigning each pixel into a proper region (segment) according to the Maximum likelihood Estimate of the jth element L_j according to the following equation

$$L_i = \max_i \left\{ \frac{\exp^{\left|\frac{z_i-\mu_i^{EM}}{A(P_i,\sigma_i^{EM})}\right|^{P_i}}}{2\Gamma(1+\frac{1}{P_i})A(P_i,\sigma_i)} \right\}$$

Where z_i are the input data (pixel intensities) and μ_i, σ_i are the estimated parameters respectively.

6. Experimental Results

In order to evaluate the proposed method, we demonstrated our image segmentation algorithm with Finite Generalized Gaussian Mixture model by applying it to 3 images namely, LENA, MAN & MAN1. The performance of the developed algorithm is compared by evaluating different Image Quality Metrics such as Mean Square Error, Signal to Noise Ratio and Quality Index. The pixel intensities of the whole image are taken as input for image reconstruction. The pixel intensities in the image are assumed to be the mixture of several components (segments) of the image. In each image region the pixel intensities follow a Finite Generalized Gaussian distribution with different parameters. The number of Segments inside the image is determined by using K-Means algorithm. For determining the initial values of ‘ K’ in the K-Means algorithm, we have obtained the histograms of the pixel intensities. The original and the reconstructed images of LENA, MAN and MAN1 are shown in Figure-1. The comparative performance of various algorithms with reference to Image Quality Metrics are given in Table-1

Figure:1 Images

Original Image | F.Gaussian Mixture Model | F.Generalized Mixture Model

Table -1

Name of the image	Image Quality Metrics					
	Mean Square Error		Signal to Noise Ratio		ImageQuality Index	
	F.G.M. Method with K-Means Method	F.G.G.M Model with K-Means Method	F.G.M. Method with K-Means Method	F.G.G.M Model with K-Means Method	F.G.M Method with K-Means Method	F.G.G.M Model with K-Means Method
LENA	0.555678	0.8799	23.45	43.22	0.765670	0.77765
MAN	0.76875	0.9872	30.33	39.22	0.865435	0.91873
MAN1	0.87644	0.9333	34.55	40.22	0.896669	0.94354

From the table-1, and Figure-1, it can be observed that the developed Method performs much superior to the existing algorithms with respect to the image Quality Metrics. The performance of the Image Segmentation Model is also studies through classifier accuracy by computing the misclassification rate. The misclassification rates of different images namely LENA, MAN and MAN1 with reference to the developed segmentation algorithm and the Finite Gaussian Mixture Model with K-Means are computed and given in Table-2

Table-2 Classifier Accuracy

Name of the Image	F.G.M. Method with K-Means Method	F.G.G.M Model with K-Means Method
LENA	93.43	96.56
MAN	90.22	92.71
MAN1	91.45	94.27

From the above table it can be observed that the accuracy of the developed algorithm is superior to that of the Finite Gaussian Mixture Model with K-Means.

7. Conclusion

In this paper we proposed Unsupervised Image Segmentation Method based on Finite Generalized Gaussian Mixture Model with EM & K-Means algorithm. The image was considered as a mixture of K-Component Generalized Gaussian Densities, using K-Means algorithm the number of components in the images are estimated and through EM algorithm the final estimates of the parameters are obtained. Experimental results show that the proposed algorithm has better retrieval capability compared to the Finite Gaussian Mixture Model Method.

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