Geodesic-Fan Based Mesh Parameterization in Shoe CAD System

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Summary
Parameterization of a 3D mesh is a fundamental problem in a variety of applications such as shoe CAD system and geometric modeling. There are two major paradigms in mesh parameterization: energy functional minimization and the convex combination approach. In general, the convex combination approach is widely used because of simple concept and fast computation. However, the approach has some problems such as fixed boundary and high distortion near the boundary. In this paper, we present an extension of the approach based on the geodesic-fan representation and mean value coordinates, which resolves the drawbacks of the convex combination approach. Moreover, we apply our result to a 3D shoe CAD system in order to resolve the practical problems of industry.
Key words:
Parameterization, Mean Value Coordinates

1. Introduction

Parameterization of a 3D triangular mesh is a fundamental problem in various applications of computer graphics and computer animation. Mesh parameterization of 3D surface is a one-to-one mapping without any fold-over. This special property provides an elegant and robust solution to various problems in geometric modeling and computer graphics, such as texture mapping, mesh morphing, and smooth surface fitting [1-8]. In general, the mesh cannot be flattened over a plane without distortion. The main purpose of research on mesh parameterization is to minimize distortion. Especially, such distortion minimization is very important problem in the practical applications such as shoe cad system [9,10].

There exist two major approaches in mesh parameterization. One is energy functional minimization and the other is convex combination approach. For the first approach, several methods have been developed to define and minimize an energy functional that measures distortion in the embedded mesh. Maillot et al. proposed a method to minimize a norm of the Green–Lagrange deformation tensor based on elasticity theory [5]. Eck et al. minimized the metric dispersion of harmonic imbedding instead of [1]. A non-deformation criterion (i.e., Dirichlet energy per parameter area) is introduced in [11] with extrapolating capabilities. One of the most widespread methods for parameterization of 3D mesh surface with the topology of a disk is proposed by Floater [12]. In general, this approach is called as convex combination approach because it is based on the convex combination of neighborhood vertices. The convex combination approach is an extension of the barycentric mapping approach proposed by Tutte [13]. This approach obtains parameterization by fixing the boundary vertices of a 3D mesh onto a 2D convex polygon and solving a linear system to determine the 2D embedded positions of the interior vertices. The linear system is constructed by representing each interior vertex as a convex combination of its neighborhood. In this approach, the major problem concerns how to determine the coefficients of the convex combination for each interior vertex. Floater proposed shape preserving parameterization, where the coefficients are determined by using conformal mapping and barycentric coordinates [12]. The harmonic embedding [1] is also a special case of this approach, except that the coefficients may be negative.

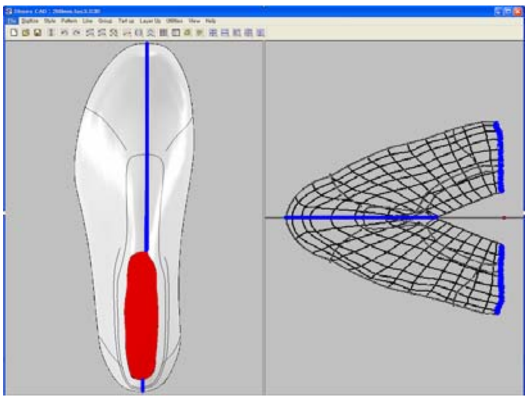


Fig.1. Shoe CAD system: (a) 3D shoe last
(b) 2D parameterization of the last

Manuscript received June 5, 2007
Manuscript revised June 20, 2007

There are several constraints in practical applications of parameterization such as 3D shoe cad systems. The famous shoe design systems are Crispin Dynamics, Shoe-master, Dimensions, and etc. Crispin is a CAD system focusing on the shoe styling for shoe-masters [9,10]. Shoe-master designed by CSM3D Ltd shows a great advantage in the field of shoe's heel and sole design. Figure 1 shows a 3D shoe last and the result of parameterization satisfying the constraints of 3D shoe cad system. The surface of the last in the left part of Figure 1 consists of two regions: exterior region and interior region divided by blue lines. The exterior and the interior regions are positioned in the left and the right parts of the 3D last, respectively. The right part of Figure 1 is the result of our method, which consists of two parameterizations. In general, there are two constraints of mesh parameterization in 3D shoe system. One is to preserve the length of special curves on the boundary between the exterior and the interior regions The lengths of toe, heel, and instep have to be preserved under parameterization. The other is that the difference between the areas of the last's surface and the result of parameterization has to be minimized as much as possible.

In this paper, we present a new method of mesh parameterization based on geodesic-fan representation and mean value coordinates so that it satisfies the shoe cad constraints. We exploit geodesic-fan representation and edge tweaking method in order to satisfy the practical constraints, and mean value coordinates and convex combination approach to speed up the process of parameterization.

The remainder of the paper is as follows. In Section 2 we review the existing convex combination approach. In Section 3, we explain our algorithm that is called as geodesic-fan based approach. First, we introduce the concepts of geodesic-fan representation and mean value coordinates. We explain the edge tweaking method so that the result resembles the shape of the boundary of the mesh and satisfies the length constraints. The experimental results in shoe cad system are shown in Section 4. Section 5 provides a summary and discusses some future work.

2. Convex Combination Approach

First of all, we introduce a pioneer method of the convex combination approach to mesh parameterization, which was developed by Floater [12]. Let $u_1,\cdots,u_N$ be the 2D embedded positions of 3D vertices, $v_1,\cdots,v_N$, where $v_1,\cdots,v_n$ and $v_{n+1},\cdots,v_N$ are the interior and the boundary vertices of a mesh, respectively. The convex combination approach determines the values of $u_{n+1},\cdots,u_N$ by mapping the 3D boundary onto a given convex polygon in a 2D parameter space. To obtain the values of $u_1,\cdots,u_n$ the approach represents u_i as a convex combination of u_j , where v_j are the one-ring neighborhood vertices of v_i . In other words,

$$u_i = \sum_{j=1}^{N} \lambda_{i,j} u_j , \; i = 1,\cdots,n, \qquad (1)$$

where

$$\begin{cases} \lambda_{i,j} > 0, & (i,j) \in E, \\ \lambda_{i,j} = 0, & (i,j) \notin E. \end{cases}$$

$$\sum_{j=1}^{N} \lambda_{i,j} = 1.$$

Here, E is the edge set of the mesh. We can compute the values of $u_1,\cdots,u_n$ by solving the linear system in Eq. (2), which is derived from Eq. (1):

$$u_i - \sum_{j=1}^{n} \lambda_{i,j} u_j = \sum_{j=n+1}^{N} \lambda_{i,j} u_j , \quad i = 1,\cdots,n . \quad (2)$$

The right hand of the equation is already known value because the position of the boundary vertices is determined by the mapping. So, Eq. (2) becomes the following matrix equation

$$\begin{bmatrix} 1-\lambda_{1,1} & -\lambda_{1,2} & \cdots & -\lambda_{1,n} \\ -\lambda_{2,1} & 1-\lambda_{2,2} & \cdots & -\lambda_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ -\lambda_{n,1} & -\lambda_{n,2} & \cdots & 1-\lambda_{n,n} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix},$$

where

$$b_i = \sum_{j=n+1}^{N} \lambda_{i,j} u_j .$$

Floater proved that a unique solution of the linear system in Eq. (2) always exists. He also proved that if mapping between the 3D boundary and the 2D convex polygon is one-to-one, mapping for the interior vertices becomes an embedding without overlaps. The shape of the embedding depends on the coefficients $\lambda_{i,j}$ of the convex combination. Floater proposed three methods to obtain the coefficients: uniform parameterization, chord length parameterization, and shape-preserving parameterization.

Among them, the last method best reflects the shape of a mesh since the method has the affine invariant property

We ascertain a drawback with the convex combination approach: high distortion occurs near the boundary. We guess that the fixed boundary causes such a high distortion. So we propose an extension of this convex combination approach with floating boundary and mean value coordinates. The floating boundary is obtained by one-layer virtual boundary vertices and edge tweaking method. The mean value coordinates are based on geodesic-fan representation. We call our method as geodesic-fan based parameterization.

3. Geodesic-Fan Based Approach

The major problem of parameterization of a 3D triangular mesh concerns how to embed a mesh onto a 2D parameter space so that the shapes of triangles can be well preserved. In the convex combination approach, fixing the boundary of a mesh onto a convex polygon highly deforms the triangles near the boundary in 2D parameter space. In contrast, if boundary vertices can move to reflect the 3D boundary shape of a mesh, we can decrease distortion near the boundary of an embedding. Hence, our basic idea starts from how to make vertices on the boundary move to reflect the 3D boundary shape of a mesh in 2D parameter space. To freely move boundary vertices, we apply edge tweaking method to the boundary. And then we adopt the geodesic-fan representation in order to efficiently represent the relationship between vertices on the mesh surface [14]. Based on this representation, we compute the barycentric coordinates $\lambda_{i,j}$ of an inner vertex u_i with mean value coordinates. Lastly, we rescale the parameterization in order to apply a 3D shoe cad system as shown in Figure 2.

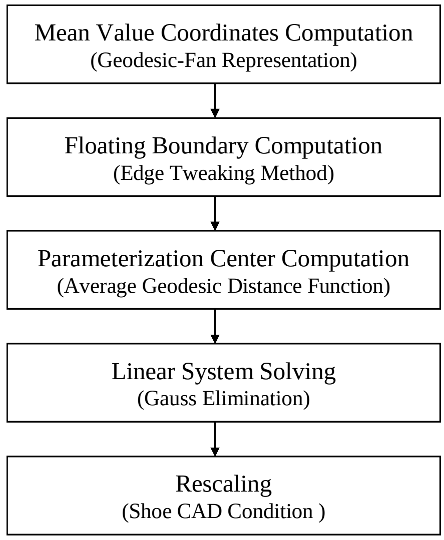


Fig. 2 Geodesic-Fan Parameterization Process

3.1 Geodesic-Fan Representation

We use the geodesic polar parameterization of a point to establish sample positions on mesh surfaces. If one arbitrary geodesic passing through p is designated a polar base, every other geodesic passing through p may be parameterized by its angle θ with respect to the polar base in the conformal plane at p . Any point q on the mesh surface may be parameterized with respect to p as (θ, r) , where θ identifies a geodesic through both p and q , and r the length on this geodesic of q Note that these coordinates may not be unique for a given point, but any particular set of coordinates identifies a unique point on the surface. The pair (θ, r) are called the geodesic polar coordinates of q with respect to p . For a given contour containing the point p , we may construct a geodesic–fan with respect to p by connecting the point p and each vertex on the contour.

The average geodesic distance function was introduced by Hilaga et al. for the purpose of shape matching [**]. This is a function $A(p)$ that takes on a scalar value at each point p on the surface S . Let $g(p, g)$ be the geodesic distance between two points p and q on S . The average geodesic distance of p is defined as follows:

$$A(p) = \frac{\int_{q \in S} g(p,q)dq}{Area(S)}.$$

$A(p)$ is a member of the following set of functions with $n = 1$.

$$A_n(p) = \sqrt[n]{\frac{\int_{q \in S} g^n(p,q)dq}{Area(S)}}.$$

When the number n increases to the infinity,

$$A_\infty(p) := \lim_{n \to \infty} A_n(p) = \max_{q \in S} g(p,q),$$

which measures the maximal distance between p and any point on S.

3.2 Mean Value Coordinates

We introduce a generalization of barycentric coordinates proposed by Floater, which allows a vertex in a planar triangulation to be expressed as a convex combination of its neighboring vertices [15]. The coordinates are motivated by the Mean Value Theorem for harmonic functions and can be used to simplify and improve methods for parameterization and morphing.

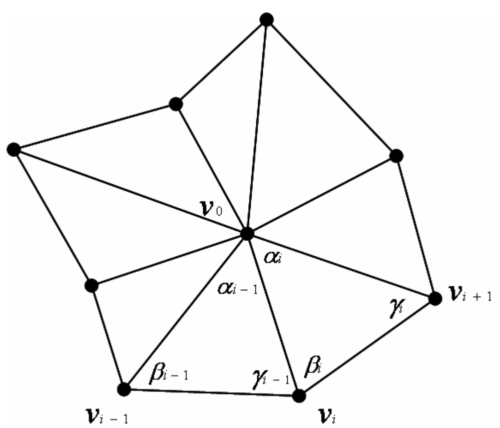


Fig. 3 Star-Shaped Polygon and its angles

Let $v_0, v_1, \cdots, v_n$ be points around v_0 in the plane with $v_1, \cdots, v_n$ arranged in an counter-clockwise order, as shown in Figure 3. The points $v_1, \cdots, v_n$ form a star-shaped polygon with v_0 in its kernel. The aim of barycentric coordinates systems is to study the sets of weights $\lambda_1, \lambda_2, \cdots, \lambda_n$ such that

$$\sum_{i=1}^{n} \lambda_i v_i = v_0, \sum_{i=1}^{n} \lambda_i = 1. \qquad (3)$$

Equation (3) expresses v_0 as a convex combination of the neighboring points $v_1, \cdots, v_n$ In the simplest case $n = 3$, the weights $\lambda_1, \lambda_2, \lambda_3$ are uniquely determined by Eq. (3). All of the weights are positive. In general, we call the weights as barycentric coordinates of v_0 with respect to v_1, v_2, v_3.

There has long been an interest in generating barycentric coordinates to k-sided polygons. For a convex polygon, Wachspress [2] found a solution, in which the coordinates can be expressed in terms of rational polynomials,

$$\lambda_i = \frac{w_i}{\sum_{j=1}^{n} w_j},$$

$$w_i = \frac{A(v_{i-1}, v_i, v_{i+1})}{A(v_{i-1}, v_i, v_0)A(v_i, v_{i+1}, v_0)} = \frac{\cot \gamma_{i-1} + \cot \beta_i}{d\ (v_i, v_0)^2},$$

where $A(a,b,c)$ is the signed area of triangle $[a,b,c]$ and γ_{i-1} and β_i are the angles shown in Figure 3. The latter formulation in terms of angles is due to Meyer et al. [3]. This coordinates do not work for a star shaped polygon.

Floater proposed a new coordinate system called as mean value coordinates, which well work for a star-shaped polygon [***]. The coordinates are defined by

$$w_i = \frac{\tan(\alpha_{i-1}/2) + \tan(\alpha_i/2)}{d(v_i, v_0)}.$$

3.3 Floating Boundary: Edge tweaking

To obtain a bounding convex polygon that resembles the 3D boundary, we first derive a 2D polygon that preserves the adjacent angles and lengths of 3D boundary edges. In general, such a 2D polygon may not exist when the 3D boundary is non-planar. Sederberg et al. considered a similar problem for 2D shape blending and proposed the edge tweaking method [16]. In the edge tweaking, the changes of edge lengths are minimized while the angles between edges are preserved. In this paper, we adopt the edge tweaking method to obtain a 2D polygon from the 3D boundary. The bounding convex polygon is determined as the convex hull of the 2D polygon. Now we explain how the edge tweaking method is used in this paper. For the edge tweaking method, we represent the original edge lengths, the amounts of length adjustment, and the adjacent angles between edges as i_i, s_i, and θ_i, respectively, for $i = n+1, n+2, \cdots, N$ Fig. 2(a) shows an illustration. The problem is to find the values of s_i that minimize the objective function

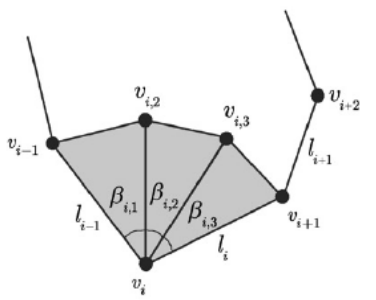
$$f(s_{n+1}, \cdots, s_N) = \sum_{i=n+1, i \notin T}^{N} \frac{s_i^2}{l_i^2},$$

subject to the equality constraints:

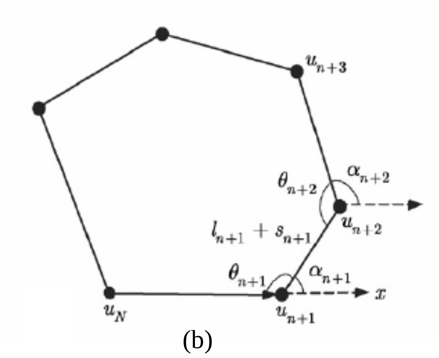
$$\varphi_1(s_{n+1}, \cdots, s_N) = \sum_{i=n+1, i \notin T}^{N} (l_i + s_i)\cos(\alpha_i) = 0,$$

$$\varphi_2(s_{n+1}, \cdots, s_N) = \sum_{i=n+1, i \notin T}^{N} (l_i + s_i)\sin(\alpha_i) = 0.$$

Here, $N - n$ is the number of the boundary edges, *T* is the set of boundary edges the length of which have to be preserved, and α_i is the angle between the x-axis and the directed edge from u_{i+1} to u_{i+2}. The two equality constraints mean that the 2D polygonal curve with the edge lengths of $l_i + s_i$ is a closed polygon. The solution for the values of s_i can be obtained by using Lagrange multipliers.



(a)



(b)

Fig.4. Edge tweaking for the bounding convex polygon: (a) 3D mesh boundary (b) 2D polygon.

Let v_i and u_i for $i = n+1, n+2, \cdots, N$, denote vertices on the 3D mesh boundary and the corresponding vertices on the 2D polygon, respectively. To obtain u_i from v_i by using the edge tweaking method, we must determine the original lengths l_i and the angles θ_i For the lengths l_i, we can simply set l_i as the length of the 3D boundary edge between v_i and v_{i+1}. In the case of the angles θ_i, we compute θ_i from the angles $\beta_{i,j}$ around v_i of the boundary triangles adjacent to vi as follows (see Fig. 4(b)).

A simple way to compute θ_i is to sum the angles $\beta_{i,j}$ However, the angles $\beta_{i,j}$ may be changed when the boundary triangles are embedded onto a 2D polygon, and the sum of $\beta_{i,j}$ in 3D may be improper for θ_i To estimate the values of angles $\beta_{i,j}$ in the final embedding of the 3D mesh, we compute the averaged conformal angles $\gamma_{i,j}$ Let V_i be the set of vertices of the 3D mesh which are adjacent to the boundary vertex v_i The set V_i is divided to two subsets; a boundary vertex set $\{v_{i,1}, v_{i,c})$ and an interior vertex set $\{v_{i,2}, \cdots, v_{i,c-1})$ where $v_{i,1} = v_{i-1}$ and $v_{i,c} = v_{i+1}$, and c is the number of vertices in V_i. See Fig. 2(b) for an illustration. When we apply conformal mapping to the one-ring neighborhood of an interior vertex $v_{i,j}$, we obtain $\beta_{i,j-1}^2$ and $\beta_{i,j}^1$ for the angles $\beta_{i,j-1}$ and $\beta_{i,j}$, respectively. We define the averaged conformal angles $\gamma_{i,j}$ by

$$\gamma_{i,1} = \beta_{i,1}^2,$$
$$\gamma_{i,c-1} = \beta_{i,c-1}^1,$$
$$\gamma_{i,j} = \frac{\beta_{i,j}^1 + \beta_{i,j}^2}{2},$$

for $j = 2, \cdots, c-2$. Finally, we determine the angle θ_i as the sum of the averaged conformal angles $\gamma_{i,j}$;

$$\theta_i = \sum_{j=1}^{c-1} \gamma_{i,j} .$$

In Fig. 5, the bounding convex polygon is the convex hull of the 2D polygon generated by the edge tweaking method. The embedding result is better than those in Fig. 6 because the bounding polygon reflects the 3D boundary of the mesh. Fig. 9 shows an example of parameterizing a mesh with a more complicated boundary.

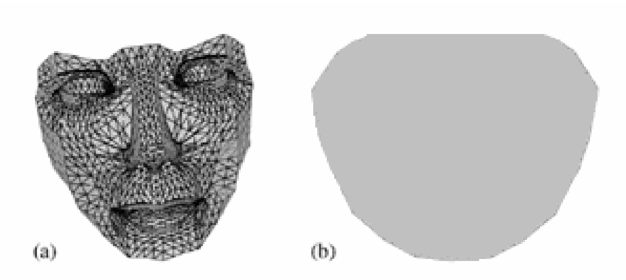


Fig.5 Embedding result with the bounding polygon that resembles the 3D boundary: (a) 3D mesh; (b) bounding polygon from edge tweaking.

3.4 Linear System with constraints

Now, we are ready to solve the linear system of mesh parameterization in 3D shoe cad systems. Our algorithm is an extension of the convex combination approach proposed by Floater [12]. First of all, we compute the center of parameterization in order to apply the geodesic-fan based representation. The problem is formally set by the following minimization:

$$A_\infty(v^* : B) = \min_{i=1}^{n} \max_{q \in B} g(v_i, q),$$

where B is the set of boundary vertices.

We apply the edge tweaking method in order to get the 2D convex boundary of the mesh parameterization. The boundary edges belonging to the front and back constraint sets preserve their length and the shape of the boundary well resembles that of 3D mesh.

The linear system in Eq. (2) needs to get the convex combination of interior vertices with respect to their one-ring neighborhoods. We adopt the mean value coordinates so that the local shape may be well preserved under the linear system. The linear system may be efficiently solved by Gauss-elimination method.

4. Experimental Results

Figure 6 shows a 3D shoe mesh and the result of a parameterization without shoe's constraints. The yellow region is the exterior part of a 3D shoe last, the white region is the interior part. The right of Figure 6 has two parts: one is the parameterization for the interior part (blue color), the other is for the exterior part (black color). In general, the most of shoe cad systems need the optimal layout of two parameterizations because of efficient productivity. Hence, the right part of Figure 6 should be changed to that of Figure 1. Moreover, the lengths of the front instep lines and the back heel lines are the same as those of the other part's parameterization.

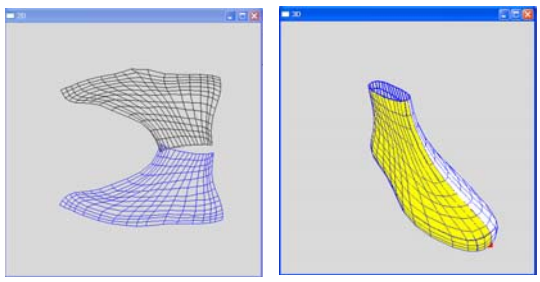


Fig.6. Shoe's parameterization with two regions

Table 1 shows the comparison of distortion measurements for the face model with three algorithms: (a) shape preserving algorithm proposed by Floater [12], (b) virtual boundary algorithm proposed by Lee et al., [17] (c) Our algorithm which is based on geodesic-fan and mean value coordinates. We use the texture stretch metric L^2 and L^∞ defined in [**] as the measures for comparison. The L^2 norm corresponds to the mean stretch over all directions, and the worst-case norm L^∞ relates to the greatest stretch.

Table 1.Comparison of distortion measurements

Algorithm	L^2 **-norm**	L^∞ **-norm**
Square (Floater)	1.246	5.236
Virtual Boundary (Lee)	1.221	4.968
Geodesic-Fan (Kim)	1.131	4.659

From Table 1, we observe that the distortion measurement changes according to the shape of the bounding polygon, and the value decreases as bounding polygon is closer to the shape of the 3D boundary. Hence, the embedding with the edge tweaking method has a better result than the others. The use of mean value coordinates is the main difference between Lee's algorithm and ours. The mean value coordinates well resemble the shape of one-ring neighborhoods so that our algorithm should have less distortion.

Table 2 shows the computation time required to parameterize the models shown in this paper with Lee's and our algorithms. In the current implementation, we use the Gauss–Seidel method to solve the linear system from the convex combinations. Our algorithm is faster than Lee's. The time complexity to obtain the convex combination of shape preserving method is $O(m^2)$, where m is the number of one-ring neighborhood vertices. On the other hand, the process of the mean value coordinates needs $O(m)$ time complexity.

Table 2.Computation time

Model	**No. of Vertices**	**No. of Boundary Vertices**	**Time(ms) (Lee)**	**Time(ms) (Kim)**
Face	1607	68	471	367
Shoe	195	52	16	9

Figure 7 shows a function of 3D shoe cad system related with mesh parameterization. Shoe designer traditionally draws patterns only on 2D parameter space without exact 3D geometric information on the last. If we have 3D shoe cad system, we may directly design the 3D pattern on the last. So this function is very useful to design a variety of patterns intuitively.

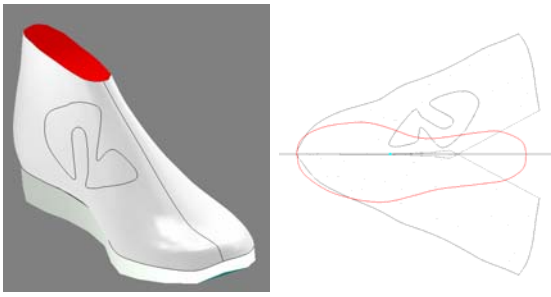


Fig.7. Pattern design on the shoe cad system

5. Concluding Remarks

The contribution of this paper is to present an efficient method to compute mesh parameterizations in shoe cad system. Our method is an extension of the convex combination approach proposed by Floater. Our algorithm uses geodesic-fan based representation and edge tweaking method in order to decrease distortion of boundary regions. Moreover, we adopt the mean value coordinates to compute the one-ring neighborhood's relationship efficiently. The process to obtain the mean value coordinates is faster than Lee's, and our algorithm well resembles the local shapes of mesh parameterization In future, we will develop a mesh parameterization with holes and seams.

Acknowledgments

This work was supported by Dong-Eui University Grant (2005AA185).

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