Consideration for Chance Losses with Inventory Optimizations

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Summary

Even for the retail business, the optimization for the inventory is one of the key activities, because they trade the items that have a limit-time to eat.

Depending on the demand, there are two losses in the food inventories; the abandoned loss and the chance loss.

Both the losses are the same in the point to miss the sales, but concerning to the technique of the stock control among the grocery with the best-before date, the notion for the chance loss had not been examined enough.

This thesis proposes the inventory optimization methods for the grocery in which have the best-before date.

From the examination with the model in this thesis, it is effective to improve EOQ for the original item that intends to purchase to promote to purchase something, in case of the appearance for the chance loss.

At the same time, it is also effective to reduce the safety stock with this measurement.

Key words: *Inventory control, Optimization, Chance loss, Newsboy problem, EOQ*

1. Introduction

It is important to optimize the inventory even for the retail business. Especially, even for a certain shops which sell the items that have limit-time to eat.

If the items come to the best-before date, they have to abolish the items, in such a reason; the inventory is also the key activity for the retail business.

This thesis proposes the inventory optimization methods for the grocery in which have the best-before date.

On the other hand, It is also difficult for the store operator to forecast the following customers' activities; When, Which and How many items going to take in the store. For those reason, there are two losses in the food inventories.

One is called the "Chance loss" that defined which items have been sold out more than demand. And the other is the "Abandoned loss" which amount of sales less than forecasted and ordered.

Both the losses are the same in the point to miss the sales; the abandoned loss is to become worthless as an item for the coming of the time limit.

As for the abandoned loss, amount of control of the abandoned loss is possible by the discount sales or the control of the order amount. On the other hand, the chance loss means the condition for the stockout of the items; this accident is hard to search out precisely for the store operator.

Therefore, the method for discovering and decreasing the chance loss is requested.

The store items of the groceries are composed of three units; the year round items, the seasonal items, and the events items. This composition makes more difficult to find out the accident of the chance loss precisely.

The optimization of the stock is from on the restriction and the capital operation side of the area of the store to the important management elements.

In this thesis, the optimization methods for achieving both the reduction for chance loss and abandoned loss is proposed.

2. The inventory controlling and optimization

As for the inventory controlling, the ordering lot size has been estimated from the balance of the ordering cost and the keeping cost for the long time, to which is not comparatively deteriorated at a long term, especially the manufacturing business domain.

On the other hand, concerning to the technique of the stock control among the commodity with the best-before date, the notion for the chance loss had not been examined enough.

Those items have been controlled by the ordering lot size based on the less quantity of inventory.

Here, the Economic Order Quantity shows the amount of ordering that minimize the total of the procurement cost, the ordering cost, and the maintenance cost.

Hereafter EOQ stands for the Economic Order Quantity.

2.1 EOQ for the items that have a best-before date

As for the ordering quantity for the items with the best-before date, Ota at al [1] were formulated by attaching the correction variable with the basic formula for estimating the ordering quantity with the original price such as perishable foods and newspaper.

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They were formulated the new presumption for the minimum ordering quantity with the case of the original price and discount price, and attached the discount rate formula as a new variable for presuming the quantity of the items with easy to deterioration.

2.2 Improving points for the chance loss

In the thesis of Ota at la, an inventory is only controlled with the amount of the ordering without examination and consideration for the storage stock.
And a demand frequency is assuming the Poisson arrival even though another frequency has to examine for the arrival rate and the demand pattern.
Finally, it is formulated only the abandoned loss as the loss, therefore, the chance loss is left for formulating. Those issues are rose as the improvement points.
The inventory is the control of both the ordering method and the ordering time, and to optimize the stock is not only to reserve the amount that supplies the demand but to request three saddle points of the sales number, the profit and the loss at the same time.
In this thesis, we will propose the chance loss reduction from the two losses.

3. Concept for the inventory control

3.1 The chance loss and the abundant loss

Controlling of the losses for the items is to understand the timing those who wants the items.
It is clear to figure out that the gross number for the abandoned loss if we count the number after the business hour. On the other hand, it is difficult to understand how many losses at chance have happened throughout the business hour.
It is not reasonable to make the excessive amount of the inventories in preparation for the chance loss not only from the view point of the restriction of the inventory budget but of the best-before

3.2 The inventory controlling for the grocery

The ordering methods for the grocery foods adopt the regular ordering cycle and quantity fixed order in general.
As for the inventory for the grocery food, the amount of the safety stock, the ordering point, the theory stock and the maximum inventory of each item are calculated based on the average sales amount and standard deviation each day led from the amount of the sales number as well as the stock controlling technique of manufacturing items.

3.3 Concept for the safety stock

Let the D for the demand for a certain day and assumed to be a constant and the L for the delivery time, when the amount of the stock comes to D x L will be an ordering point.
It is also happened to increase or decrease in the demand between L days and run out of the stock.
The quantity of inventory to prevent running out of stock is called a safety stock.
Assuming the standard deviation of a certain item's sales quantity for one day in σ , a standard deviation (SD) on L days is;

$$\text{SD}=\sigma\sqrt{L} \quad (1)$$

Then, it is possible to think about the safety stock as a stock to cover random demand in a delivered period of the item.
Assuming the demand to be a normal distribution, the amount of the safety stock with a hazard ratio α is given by the following formula;

$$\text{Safety stock} = \alpha\ \sigma\sqrt{L} \quad (2)$$

Then, the ordering point is given as follows;

(The amount of the stock at the ordering point)
= (Average sales amount between the delivery time)
+ (Amount of safety stock)

$$= D \times L+\alpha\ \sigma\sqrt{L} \quad (3)$$

The safety stock keeps the prevention of running out of stock in case of the demand expansion in mind. On the other hand, the amount of stock grows immediately after the arrival of items when the demand reduces on the contrary.
The gap for the demand becomes the amount of the safety stock assuming a hazard ratio to be a constant.
Therefore, it is possible to think about the maximum inventory as follows;

(Maximum stock) = (Ordering point) + 2 (Safety stock)

$$=Q+2\alpha\ \sigma\sqrt{L} \quad (4)$$

3.4 Safety coefficient, Hazard ratio and service ratio

The safety coefficient means the error margin between the forecast and the result happens based on the normal distribution, as a result, the gap both the demand forecasting and the result happen based on the normal distribution during the distribution period.

That is, the safety coefficient means the possibility for interrupting to the amount of the safety stock.

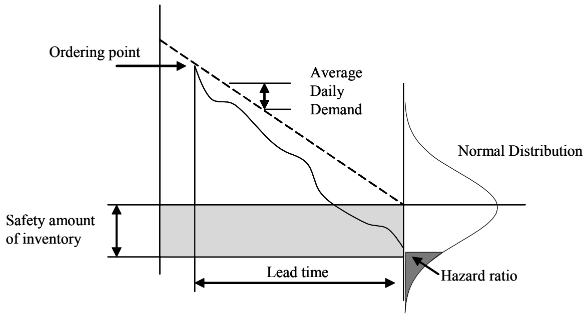

Fig.1: Ordering point and hazard ratio

The service ratio in the inventory control means the ratio for the demand fulfillment in a certain period.
Namely,

1-(Service ratio) = Hazard ratio (5)

As for the concept for the hazard ratio, it is formulated by the following the three viewpoints;
1: the amount of the customer
2: the amount of the item
3: the amount of the item price

In this thesis, the hazard ratio is defined as the item based and formulated as follows;

$$\text{The hazard ratio} = \frac{\text{the amount of out of the stock}}{\text{the amount of the demand}}$$

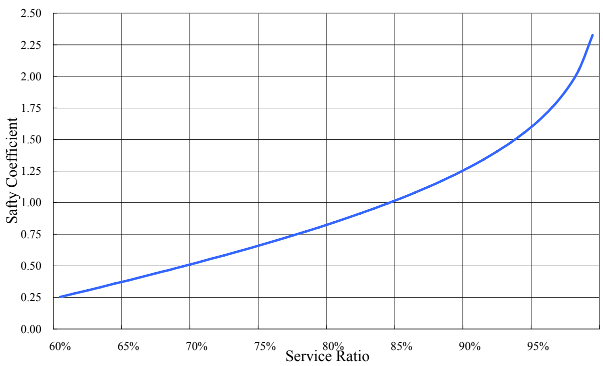

Fig.2: Safety Coefficient and Service ratio

The safety coefficient is decided depending on the ratio that can afford the stock out. In other words, it depends on the estimation for the hazard ratio.
For example, if it is set the stock out ratio for 5%, then 95% for the service ratio is set, and also the safety coefficient becomes 1.65 as a constant.

This safety coefficient shows the upper percentage point of the probability for the hazard ratio.
Therefore, the relation both the service ratio and the safety coefficient is shown as the following graph.
From the graph, when service rate rises, the safety coefficient also increases.

3.5 EOQ model

Next, the optimum ordering lot size in which frequency for both the remainders and the out of the stock as few as possible is formulated.
The optimum ordering lot size that maximize the profit and minimize the loss at the same time is formulated as follows;
Given that x is the amount of order and y is the amount of demand, then the profit $e(x, y)$ is formulated as follows;

$$e(x,y) = \begin{cases} ay - b(x-y) & (x \geq y) \\ ax & (x < y) \end{cases} \quad (6)$$

Given that a is a gross margin for one item and b is a loss for the remainders, then the expectation profit at the amount of order for x to the demand distribution for $p(y)$ is formulated as follows;

$$E(x) = \sum_{y=0}^{\infty} e(x,y)p(y) \quad (7)$$

Then from (1) and (2),

$$E(x) = \sum_{y=0}^{x}[ay - b(x-y)]p(y) + \sum_{y=x+1}^{\infty} axp(y) \quad (8)$$

Thus, EOQ x_{opt} that maximizes the formula $E(x)$ is as follows;

$$x_{opt} = \begin{cases} E(x) - E(x-1) \geq 0 \\ E(x+1) - E(x) \leq 0 \end{cases} \quad (9)$$

As a result, x_{opt} is solved as follows;

$$x_{opt} = \begin{cases} \sum_{y=0}^{x-1} p(y) \leq \frac{a}{a+b} \\ \sum_{y=0}^{x} p(y) \geq \frac{a}{a+b} \end{cases} \quad (10)$$

3.6 EOQ with cost minimized

In this model, an EOQ model to minimize the ordering cost by using the newsboy's problem theorem in consideration of only the cost of stocking and the opportunity loss is formulated.

Let *c* $(a<c)$ donate a cost for the loss at opportunity, cost $t(x, y)$ provided that x for the amount of order and y for the amount of demand, is formulated as follows;

$$t(x,y)=\begin{cases} ax & (x>y) \\ ax+c(y-x) & (x\le y) \end{cases} \quad (11)$$

Define the expectation cost as $T(x)$,

$$T(x)=\sum_{y=0}^{\infty} t(x,y)p(y) \quad (12)$$

Then from (11) and (12),

$$T(x)=\sum_{y=0}^{x-1} axp(y)+\sum_{y=x}^{\infty}[(ax+c(y-x)]p(y) \quad (13)$$

Thus, EOQ x_{opt} that minimizes the formula $T(x)$ is as follows;

$$x_{opt}=\begin{cases} T(x)-T(x-1)\le 0 \\ T(x+1)-T(x)\ge 0 \end{cases} \quad (14)$$

As a result, x_{opt} is solved as follows;

$$x_{opt}=\begin{cases} \sum_{y=0}^{x-1} p(y)\le \dfrac{c-a}{c} \\ \sum_{y=0}^{x} p(y)\ge \dfrac{c-a}{c} \end{cases} \quad (15)$$

According to $a<c$, $0<\frac{c-a}{c}<1$ is approved.

3.7 EOQ with the chance loss

This model formulates EOQ that considers the chance loss in the case of the item that runs out of the stock.

This model also includes the cases that purchase another item or unable to buy.

Profit $e(x, y)$ provided that x for the amount of order and y for the amount of demand is formulated as follows;

Let c donate a cost for the loss at opportunity, d donates the gross profit that transits to another items and μ donates the provability for the chance loss.

$$e(x,y)=\begin{cases} ay-b(x-y) & (x\ge y) \\ ax & (x\le y) \\ d\mu(y-x)-c(1-\mu)(y-x) & (x+1\le y) \end{cases} \quad (16)$$

Define the expectation profit as $E(x)$,

$$E(x)=\sum_{y=0}^{x}[ay-b(x-y)]p(y)+\sum_{y=x}^{\infty} axp(y) + \sum_{y=x;1}^{\infty}[d\mu-c(1-\mu)(y-x)]p(y) \quad (17)$$

As a result, x_{opt} that minimizes the formula $E(x)$ is solved as follows;

$$x_{opt}=\begin{cases} \sum_{y=0}^{x} p(y)\le \dfrac{a+d\mu-c(1-\mu)}{a+b+d\mu-c(1-\mu)} \\ \sum_{y=0}^{x} p(y)\ge \dfrac{a+d\mu-c(1-\mu)}{a+b+d\mu-c(1-\mu)} \end{cases} \quad (18)$$

According to $a<a+b$, If $a+d\mu>c(1-\mu)$ is approved, then $0<\frac{a+d\mu-c(1-\mu)}{a+b+d\mu-c(1-\mu)}<1$ is also approved.

4. Model evaluation

In this chapter, we will discuss the evaluation for the models that we examined the last chapter with the POS (Point Of Sales) data with transaction ID that were gathered at a certain supermarket in Shimane Pref., Japan.

Generally speaking, most of the super markets in JAPAN composed of the following series of items; the regular items, the seasonal items and the event items.

Furthermore, most of the sales are composed of the regular items; therefore, it is hereafter defined the condition for the inventory as following conditions; the inventory as the number of the regular items, the amount of the inventory both amount of the rack and the back yard.

4.1 Data attributes

The POS data were gathered and counted by the following attributes. Form the result of the POS, the store has a JPY 72,163,933 sales with 372,438 pieces consist of 1596 items during the period;

Table1: Data attribute for POS with the transaction ID

Term	July 1st, 2006～Dec 31st , 2006
Place	Supermarket in Shimane Pref., JAPAN
Research categories	Groceries
Number of category	14 categories
Name of code	3101:Japanese seasoning 3102:Western seasoning 3103:Farm products 3104:Marine products 3105:Bottled and canned products 3106:Noodle 3107:Cooking flavor 3108:Coffee 3109:Grocery foods 3110:Dirnk 3111:Cake materials 3112:Rice cake 3113:Gifts 3199:others
Number of items	1,596 items
Total amount of items	372,438 items
Total amount of sales	JPY 72,163,933

At first, we have examined both monthly and seasonal demand fluctuation with the Weighted Moving Average method (WMA), respectively.

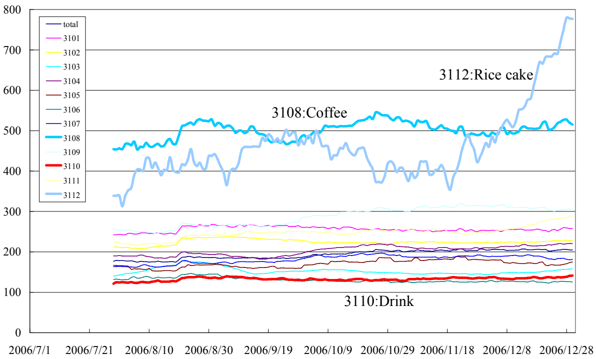

Fig.3: 30days Weighted Moving Average

4.2 Demand fluctuation form POS data

As the result from the WMA, both Coffee and Rice cake category are specified the monthly demand fluctuation.
Next, both Coffee and Rice cake category were also specified the seasonal demand fluctuation. Furthermore, grocery foods were increasing to the demand toward the year end.
This category contains green tea and black tea and so on.
On the other hand, another category seemed to steady demand throughout the year.

Here, the categories for the evaluation for the model are examined.

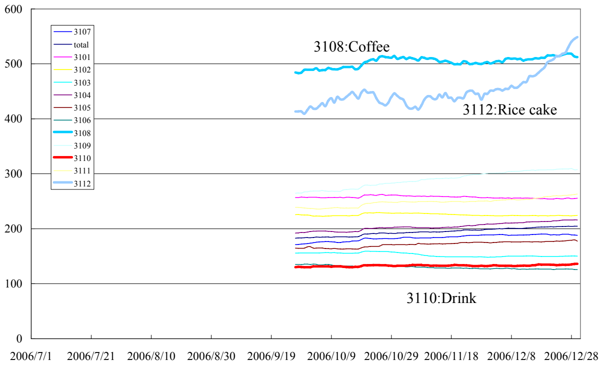

Fig.4: 90days Weighted Moving Average

The item with a high sales figure in a half year from June 2006 to December 2006 was drink. Sales figures within higher the 100th place receive drink for 32 items.
As for drink category, there are little variations both of seasonally and monthly, but on the other hand this figure shows that the category is supported by the customer through out the year.
On the other hand, as for the category of the coffee and the rice cake were supported seasonally. Especially, as for the rice cake category might be considered to be a seasonal item with a tendency to sale but from the sales policy, the category in the rice cake is handled and regarded as a regular item.

Table2: Number of the sales and distribution by category

code	Name	# of sales	%	# of items within 100th
3101	Japanese seasoning	59,836	16.1%	19
3102	Western seasoning	58,479	15.7%	15
3103	Farm products	26,652	7.2%	5
3104	Marine products	25,311	6.8%	3
3105	Bottled & canned	9,944	2.7%	1
3106	Noodle	58,095	15.6%	16
3107	Cooking flavor	19,835	5.3%	2
3108	**Coffee**	**10,117**	**2.7%**	**5**
3109	Grocery foods	8,354	2.2%	2
3110	**Drink**	**88,606**	**23.8%**	**32**
3111	Cake materials	6,704	1.8%	0
3112	Rice cake	505	0.1%	0
3113	Gifts	-	0.0%	0
3199	others	-	0.0%	0
	total	372,438	100.0%	100

As the result of examination, the POS data from both the drink category with a high support and the coffee category with a gradual seasonal variation are used for the model evaluation.

4.3 Evaluation for the models

Sales price, original cost, and gross margin with the weighted average are shown in the table below.

The items with a high support throughout the year are small profit ratio compared with seasonal high support items.

Therefore, the items with a high support throughout the year have to pay more attention to chance loss.

In the following, each category evaluates the models.

Table3: WMA for Coffee and Drink

	Weighted Moving Average			
Category	Sales price	Original cost	Gross margin	Margin %
All Groceries	193.76	150.94	42.82	22.10%
Coffee	569.25	462.07	107.18	18.83%
Drink	124.09	102.60	21.48	17.31%

EOQ model

EOQ for coffee is 57 and for drink is 333 with this model.

EOQ with cost minimized

This model considers only the cost at chance as for the cost other than stocking, therefore, as the cost for the loss goes up, EOQ also goes up, too.

It is necessary to raise the amount of the safety stock for the chance loss control, and is necessary to set a severe hazard rate as a result.

Give the safety stock from the quotient material with the seasonal variation to the quotient material without the seasonal variation.

The items without seasonal variation have to raise the number for the safety stock than with seasonal variation.

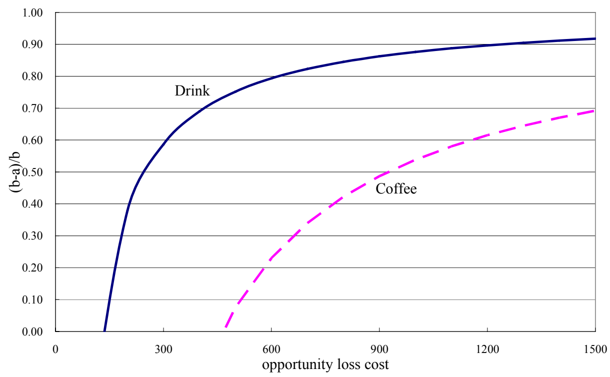

Fig.5: Relation for Chance loss and EOQ

EOQ with the chance loss

In this model, as the range for $0 < \dfrac{a + d\mu - c(1-\mu)}{a + b + d\mu - c(1-\mu)} < 1$ grows, in case the cost for the opportunity loss goes up, it becomes difficult to compensate the opportunity loss with other items.

Therefore, EOQ comes to increase as a result. EOQ also rises when the purchase transition probability α rises.

EOQ for the item of originally intended to purchase can be easily controlled more than the item that have equivalent gross margin.

It is effective to promote to sell something chap item to improve the gross margin, in consideration for the cost of the opportunity loss when the chance loss is happened.

As for the drink category without the seasonal variation is also same result.

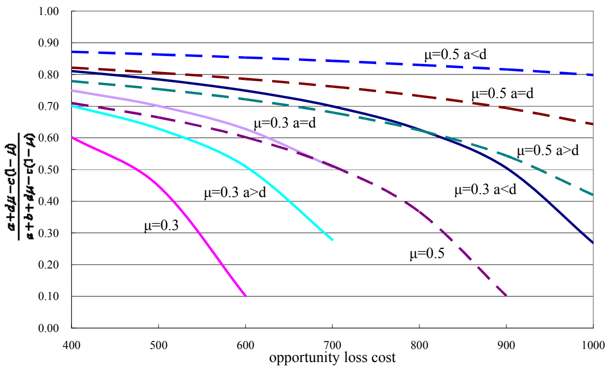

Fig.6: EOQ and purchase transition probability

Therefore, from the proposed model with the chance loss, it is effective to promote to sell something similar item to improve EOQ for those who have no target to purchase.

It is effective to improve EOQ for the original item to purchase something if the chance loss has happened.

Finally, it is important to purchase something for the measures of the chance loss.

5. Conclusion

Form the model for the chance loss with this thesis, it is effective to improve EOQ for the original item that intend to purchase to promote to purchase something, in case of the appearance for the chance loss.

At the same time, it is also effective to reduce the safety stock with this measurement. For it is released from the necessity for setting the high hazard ratio.

Therefore, it is more important to promote purchasing for something to improve the chance loss more than ever, without depending on seasonal or monthly variation. This measurement is for reducing EOQ index.

Especially, the items without seasonal variation have to control the loss with setting the hazard ratio high.

Because this measurement of the high hazard ratio means increasing the total amount of the inventory, therefore, it is more important to measure such kind of the inventory indexes as safety stock, ordering point and maximum stock to each item.

Furthermore, the purchase transition probability is defined as conjunction with both the item that intended to purchase and other in one unit; therefore, a revised model with the separation the item intended to purchase from another item is required, as a proceeding approach.

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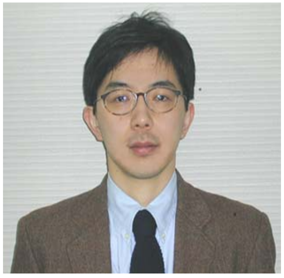

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