Existence of Nash Equilibria in Wireless Networks: An Adaptive Power Control Scheme for Radio Resource Management

Nilimesh Halder and Ju Bin Song

Telecomm. Lab, Dept. of Electronic & Radio Eng.,
Kyung Hee University, Korea

Abstract

Radio resource management (RRM) is one of the most challenging and one of the most important aspects of modern wireless communication networks. Many interactive decision making processes are required to design an intelligent radio resource management scheme that can significantly improve the system performance. These interactive decision making processes of RRM can be well analyzed by the application of game theory. The purpose of this paper is to analyze the power management in cognitive radio Direct Sequence Spread Spectrum (DS-SS) wireless networks from the viewpoint of game theory. The main focus is to model and analyze an adaptive power control scheme in wireless networks using non cooperative game. Our approach is based on a model for the utility that a wireless user or node derives from using the system. Using this model, we show a distributed adaptive power control scheme that maximizes utility of each user of the networks. Formulating this as a game we show the existence and uniqueness of the Nash equilibrium that achieved by the application of game theory.

Key words: *power control, radio resource management, wireless networks, game theory, Nash equilibrium.*

1. Introduction

Now-a-days wireless networks are becoming increasingly less structured with smart and power efficient devices that can dynamically reconfigure themselves to handle any air-interface and data format, controllable QoS, global roaming and integrated services. This has the potential to radically alter communication networks so services and performance can be reconfigured to best meet the needs of the system (based on traffic and congestion) and the user.

In order to perform these activities, a framework needs to be developed such that a radio can evaluate its capabilities, the requirements of its services, its potential waveforms, and the environment to then decide and act in a way that satisfies the needs of the situation. Cognitive radio is an enhancement on traditional software radio that attempts to establish such a framework. According to Mitola, Cognitive Radios are the radios that have the capabilities to adapt with their surrounding environment [1]. Thus resource management of these radios is one of the most challenging and one of the most important aspects of modern wireless communication network. An intelligent radio resource management scheme can significantly improve the system performance of these radios. In order to achieve this goal currently game theory can be viewed as an important modeling and analyzing tools for the cognitive radio wireless networks [2][3].

In this paper we investigate the distributed RRM in terms of power control of a cognitive radio Direct Sequence Spread Spectrum wireless networks using game theory and by applying the analytical model we also showed the steady state of the network called Nash Equilibria point.

2. Application of Game Theory in Radio Resource Management

Radio resource management can be best defined as a particular infrastructure deployment, allocate resources in a manner that ideally maximize or minimize some operational parameter(s) in wireless networks. Thus resource management can be viewed as a constrained probabilistic optimization problem. It is important to note that the probabilistic aspect of RRM causes it to differ from most common mathematical optimization problems (linear and nonlinear programming problems). However like many other optimization problems, RRM has some inversely related interactive decision making processes. In an intelligent radio resource management scheme the interactive decision making processes are maximization of user resources, maximization coverage/capacity, maximization of mobility support, minimization of cost etc [4][5][6]. Thus due to the interactive nature of RRM it is anticipated that game theory can serve as a good tool for analyzing RRM algorithms in cognitive radio wireless networks.

Distributed adaptive behavior in a wireless network will generally lead to recursive behavior wherein the decisions of one radio will subsequently influence the decisions of other radios in the network. In order to successfully deploy these networks, it will be necessary to determine if the network will eventually reach a steady state. If the adaptive behavior does reach a network steady state, resources can be appropriately allocated and performance

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Manuscript received September 5, 2007
Manuscript revised September 20, 2007

anticipated; otherwise these tasks are virtually impossible. With a game theoretic analysis, these network steady states can be identified from the Nash Equilibriums (NE) of its associated game. It is important to note that not every game, and thus not every adaptive behavior, will have a steady state. Also not every steady state is desirable as in some situations the radios may be jamming each other or the network might achieve a significantly less than optimal performance. The exact steady states that a network reaches are a function of the specifically implemented adaptive behaviors and the convergence mechanisms used by the network [7][8][9].

In [10], it is stated that the following are the key considerations for the successful implementation of distributed adaptive algorithm in a wireless network:

- Steady-state existence
- Optimality of steady state
- Convergence
- Stability
- Scalability

There are a number of different convergence mechanisms that a network might employ. These mechanisms range from fully centralized to fully distributed to mixes in between. A fully distributed network will generally be easily scalable and of low complexity to implement for each node whereas a centralized network will induce high complexity on at least one node and may have scalability limitations. Thus as long as the distributed network yields the appropriate steady-state, it will generally be desirable. However, a fully centralized network will "converge" to whatever behavior is dictated by the network authority while the convergence of distributed algorithms is dependent on the improvement paths in the associated game's action space [10] [11].

In [12], we showed the game theoretical analysis of power control of wireless ad-hoc network. We designed a utility function that the user of the network tried to be maximized. The game approach of the network was as Fig. 1,

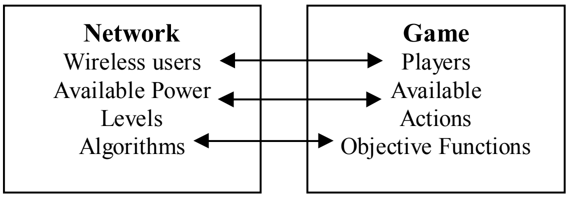

Fig. 1 Network model as a game

Thus the game can be expressed as,

$$G = \langle N, P, \{u_i\} \rangle \qquad \ldots\ldots\ldots (1)$$

In this work we have applied the concept of our previously developed game model to a cognitive radio Direct Sequence Spread Spectrum (DS-SS) wireless network and investigated the adeptness of the network to reach the steady state.

3. Game Model of DS-SS Wireless Networks

The DS-SS wireless network can be model as a game as follows:

- *Players*: A set of all decision making nodes in the participating networks. For an example here the set is $N = \{1, 2, \ldots\ \ldots\ \ldots, n\}$ nodes in the networks.
- *Actions*: The set of available power for each node $i \in N$, i.e. $P_i = \{p : p \in [P_{i,\min}, P_{i,\max}]\}$.
- *Utility Functions*: In this work, we consider a cognitive radio based single cell Direct Sequence Spread Spectrum DS-SS system with spreading factor K where mobile nodes (decision making radios) are randomly distributed within the cell and communicating with the base station to get their required services. Since DS-SS system uses single channel, power control is a serious issue for this kind of network to achieve certain QoS. The QoS for power control can be expressed in terms of signal to interference noise radio, *SINR*. Thus the *SINR* for each node i is given by, $SINR_i = \dfrac{h_i p_i}{\frac{1}{K}\sum_{j \neq i}^{N} h_j p_j + N_0}$, where N_0 designates external noise power. Each transmitting node then adapts its transmission parameters as a function of *SINR* at its node of interest constrained by a cost function that models the internal costs for a particular energy / waveform pair (battery life, complexity, distortion) and / or a cost function imposed by a network for a particular energy / waveform pair. Thus the objective function u_i can be described in terms of *SINR* as follows,

$$u_i(p) = SINR_i - c_i(p_i) \qquad \ldots\ldots\ldots (2)$$

$$u_i(p) = \frac{h_i p_i}{\frac{1}{K}\sum_{j \neq i}^{N} h_j p_j + N_0} - c_i(p_i) \ldots\ldots\ldots (3)$$

Here $c_i(p_i)$ is the cost function of each node i which can be described in unit price of power p', i.e. $c_i\left(p_i\right)=p' * p_i$. In this power control game each node will try to increase its utility by choosing a power from available power vector rationally and finally reach a steady state condition i.e. *Nash Equilibria.*

3.1 Existence of Nash Equilibria

We have applied the following Nash Existence Theorem to show the existence of Nash Equilibrium (NE) for our modeled game.

Theorem:
A strategic game $G=\langle N,A,R\rangle$ has at least one NE if $\forall i \in N$ the following condition holds

- The set A_i of actions is non empty, compact and convex subset of a Euclidean space.

The terms from set theory used in this theorem are concisely defined in [13][14].
The power game described in previous section has at least one Nash Equilibrium (NE). In order to prove this we apply Nash Existence Theorem to power game.

Proof:
The action sets P_i are non empty and convex, by definition. Each P_i is closed since it includes the boundary levels $P_{i,\min}$ and $P_{i,\max}$. All power levels in P_i lie within the boundary, thus it is bounded. Therefore the P_i 's are compact. Thus the power game must have Nash Equilibrium point.
The objective of this power game for each node can be stated as: for each node $i \in N$, given the action tuples of the remaining players, i.e. $\left(P_j\right)_{j\in N\setminus i}$ find an action P_i that maximize the utility function u_i. This motivates a distributed solution approach which proceeds as an iterative optimization problem of a scalar objective function. The iterative step is defined as for each node $i \in N$, given $\left(P_j\right)_{j\in N\setminus i}$; find the maximum utility from equation (4).

$$P_{i,eqm}=\max_{P_i}\left[u_i\left(P_i,P_{-i}\right)\right], \forall i \in N \qquad \ldots\ldots\ldots (4)$$

This iterative procedure continues until all nodes in the network find that their utilities do not change between iterations and the change in their power levels is less than a pre-defined bound or an upper limit on the number of iterations is reached.

4. Simulation and Results

We consider a cognitive radio based DS-SS network with K=63 as follows where N = 12 mobile nodes (decision making radios) are randomly distributed around the base station. In Fig. 2 the center point shows the base station of the network and the 12 mobile nodes are marked by '+' which are randomly distributed around the base station. The area in Fig. 2 is measured in kilometers.

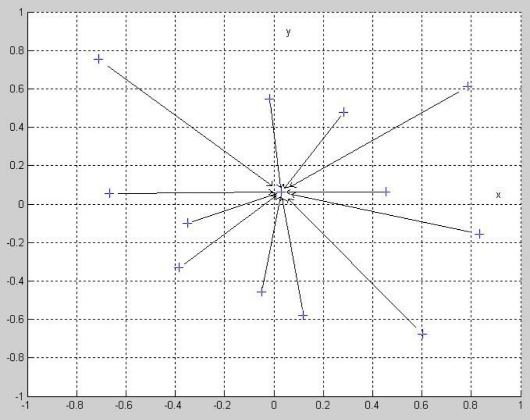

Fig. 2 Network model for simulation

Each mobile implements the power update algorithm shown in Fig. 3. In this algorithm, each node evaluates the value of its current utility function and also predicts how the value will change if the power is increased by 0.1 dB and if the power is decreased by 0.1 dB ignoring how the other mobiles might adjust their power. The power level is then adjusted to the level that produces the largest value of the utility function. This is of course constrained by the available power levels of the mobile.

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UTIL:  Calculate Utility Function $u_i$
UTIL$^+$: Predict Utility Function $u_i$
for Cur_Power_Level +0.1 dB
UTIL$^-$: Predict Utility Function $u_i$
for Cur_Power_Level - 0.1 dB

If UTIL$^+$ is largest
    Cur_Power_Level = Cur_Power_Level + 0.1 dB
If UTIL$^-$ is largest
    Cur_Power_Level = Cur_Power_Level - 0.1 dB
If UTIL is largest
    Cur_Power_Level = Cur_Power_Level
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Fig. 3 Relative iterative power update algorithm

In Fig. 4 we show that the utility of each radio of each mobile node meets the Nash Equilibria by applying the relative iterative algorithm.

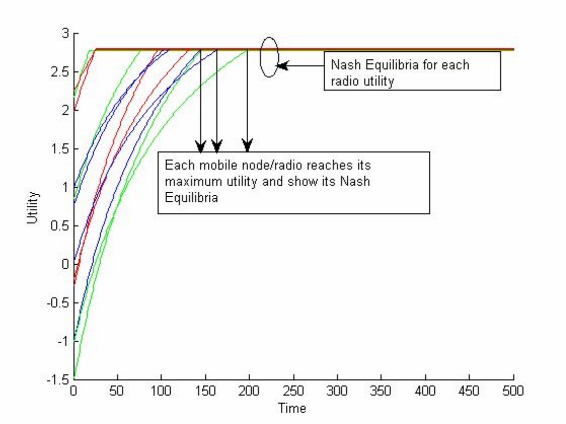

Fig. 4 Nash Equilibria for each radio utility

The power game for each mobile node is in steady state condition due to the Nash Equilibria of each node's utility which is shown in Fig. 5.

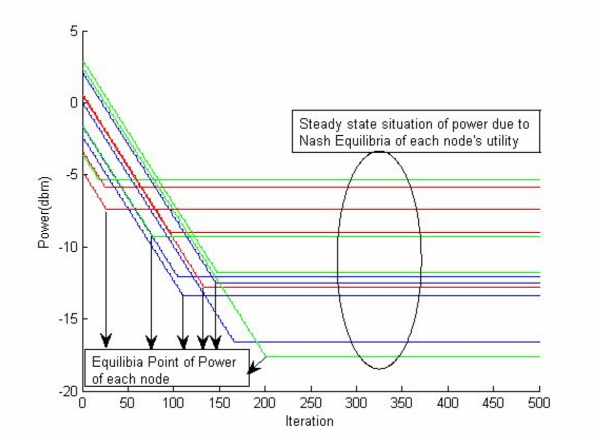

Fig. 5 Power adjustment to reach Nash Equilibria

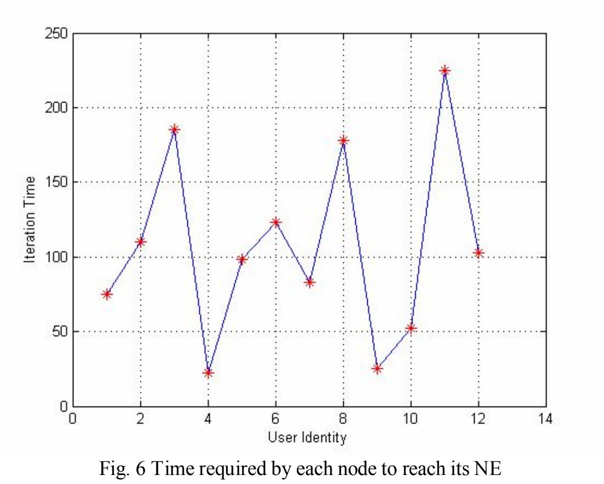

Fig. 6 Time required by each node to reach its NE

Fig. 6 shows the required iteration time of each node to reach the stead state situation i.e. NE point.

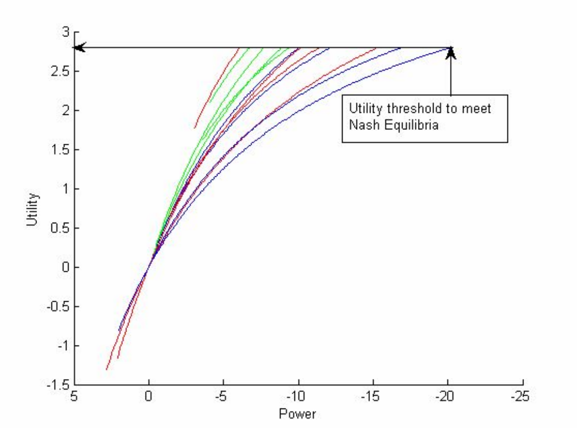

Fig. 7 Power adjustment vs utility to meet Nash Equilibria

Fig. 7 shows the trade off between power and utility of each node to meet Nash Equilibria. The power control scheme as a game is shown in Fig. 8.

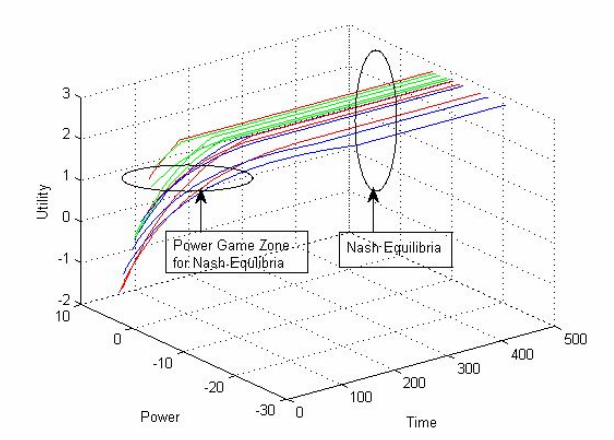

Fig. 8 Power control as a game

5. Conclusion

Distributed approach of radio resource management is a challenging task in a wireless network because all mobile nodes are communicating in a distributed manner with a central base station. In this paper we have introduced game theoretical techniques for adaptive power management in single cell DS-SS networks. We developed a utility function for the mobile nodes based on *SINR* and by applying non cooperative game theoretical techniques with relative iterative power update algorithm we got Nash Equilibria for the network. Our future approach will be to extend this work to adaptive modulation control (AMC) as well as throughput maximization of the networks.

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Nilimesh Halder is currently a PhD student and a member of Telecom. Lab in the Dept. of Electronic and Radio Eng. at Kyung Hee University. He received the B.S. degree in Computer Science and Engineering from Khulna University, Bangladesh in 2005. His research interests include wireless communications, ad-hoc mobile networks, radio resource management, power control & management, self-organizing networks, cognitive radio and game theory.

Ju Bin Song is currently a professor in the Dept. of Electronic and Radio Eng., Kyung Hee University, from 2003.He received BS and MS degree in 1987 and 1989, respectively and PhD degree in the Department of Electronic and Electrical Eng., University College of London, UK in 2001. He was senior researcher in ETRI from 1992 to 1997 and a research fellow in UCL, 2001. He was a professor in Hanbat National University from 2002 to 2003. His research interests include telecommunications, mobile multi-hop networks, radio resource management and next generation communications. He is a member of IEEE, KICS and KIEE.