Modified Stability Criterion for BAM Neural Network with both Delays and Reaction Diffusion Terms

Jianqin Wang 1 and Qiankun Song 2

1. School of Basic Medical, North Sichuan Medical College, Sichuan Nanchong 637007, China
2. Department of Mathematics, Chongqing Jiaotong University, Chongqing, 400074, P.R. China

Summary
In this paper, the bidirectional associative memory (BAM) neural network with both delays and reaction diffusion terms is considered. By employing analytic techniques, a simple criterion is presented for checking the existence and uniqueness of the equilibrium and its global exponential stability for the neural network. The criterion improves and extends some recent results.

Key words:
BAM neural network, delay, reaction-diffusion terms, exponential stability.

1. Introduction

The bidirectional associative memory (BAM) neural network model was first introduced by Kosko [1]. The classes of neural networks have been successfully applied to pattern recognition, signal and image process, artificial intelligence due to its generalization of the single-layer auto-associative Hebbian correlation to a two-layer pattern-matched heteroassociative circuits. Some of these applications require that the designed network has a unique stable equilibrium point.

In hardware implementation, time delays occur due to finite switching speed of the amplifiers and communication time [2]. Time delays will affect the stability of designed neural networks and may lead to some complex dynamic behaviors such as periodic oscillation, bifurcation, or chaos [3]. Therefore, study of neural dynamics with consideration of the delayed problem becomes extremely important to manufacture high quality neural networks. Some results concerning the dynamical behavior of BAM neural networks with delays have been reported, for example, see [2-19] and references therein. The circuits diagram and connection pattern implementing for the delayed BAM neural networks can be found in [8].

It is well-known that diffusion effect cannot be avoided in the neural networks when electrons are moving in asymmetric electromagnetic fields [20], so we must consider the activations vary in space as well as in time. There has been some works devoted to the investigation of the stability of neural networks with reaction-diffusion terms, which are expressed by partial differential equations, for example, see [20-26] and references therein. To the best of our knowledge, few authors have studied the stability of impulsive BAM neural network model with both time-varying delays and reaction-diffusion terms.

In this paper, we consider the delayed BAM neural network with reaction diffusion terms and present a modified stability criterion.

2. Model description and preliminaries

In this paper, we consider the following model:

$$\frac{\partial u_i}{\partial t}=\sum_{k=1}^{m}\frac{\partial}{\partial x_k}(\alpha_{ik}\frac{\partial u_i}{\partial x_k})-a_iu_i(t,x)+\sum_{j=1}^{p}p_{ji}f_j(v_j(t-\tau_{ji}),x)+I_i,x\in\Omega, \tag{1}$$

$$\frac{\partial v_j}{\partial t}=\sum_{k=1}^{m}\frac{\partial}{\partial x_k}(\beta_{jk}\frac{\partial v_j}{\partial x_k})-b_jv_j(t,x)+\sum_{i=1}^{n}q_{ij}g_i(u_i(t-\sigma_{ij}),x)+J_j,x\in\Omega. \tag{2}$$

The boundary conditions are the following:

$$\frac{\partial u_i}{\partial l}=(\frac{\partial u_i}{\partial x_1},\frac{\partial u_i}{\partial x_2},\cdots,\frac{\partial u_i}{\partial x_m})^T=0,x\in\Omega, \tag{3}$$

$$\frac{\partial v_j}{\partial l}=(\frac{\partial v_j}{\partial x_1},\frac{\partial v_j}{\partial x_2},\cdots,\frac{\partial v_j}{\partial x_m})^T=0,x\in\Omega, \tag{4}$$

where $i=1,2,\cdots,n,\ j=1,2,\cdots,p,\ t\ge 0;\ a_i>0,\ b_j>0$ are all constants; Ω is a compact set with smooth boundary $\partial\Omega$, and $mes\Omega>0$ in space $\Re^m$; n, p are the number

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of neurons; $u_i(t,x)$, $v_j(t,x)$ are the states of the ith neurons and the jth neurons; τ_{ji} σ_{ij} are the transmission delays; $f_j(v_j(t-\tau_{ji},x)), g_i(u_i(t-\sigma_{ij},x))$ are the input-output function of the jth neurons and the ith neurons ; I_i, J_j are the external inputs; p_{ji}, q_{ij} are the connection weights; $\alpha_{ik} \geq 0$, $\beta_{jk} \geq 0$ are the transmission diffusion operator.

The initial conditions associated with (1-4) are assumed to be of the forms.

$$u_i(s,x)=\varphi_i(s,x), -\tau\leq s\leq 0, v_j(s,x)=\psi_i(s,x), -\sigma\leq s\leq 0,$$

where $\tau = \max_{1\leq i\leq n; 1\leq j\leq p} \tau_{ji}, \sigma = \max_{1\leq i\leq n; 1\leq j\leq p} \sigma_{ij}$, we assume that the input-output functions g_i, f_j possess the following properties.

(**H**) There are constants $\alpha_i > 0$ and $\beta_j > 0$ such that

$$|g_i(u,x)-g_i(v,x)|\leq \alpha_i |u-v|,\ |f_j(u,x)-f_j(v,x)|\leq \beta_j |u-v|$$

for any $u,v\in\Re, x\in\Omega,\ i=1,2,\cdots,n, j=1,2,\cdots,p.$

For $u = (u_1(t,x), u_2(t,x), \cdots, u_r(t,x))^T$, we denote $\| u_i(t,x) \| = [\int_\Omega | u_i(t,x) |^2\ du]^{\frac{1}{2}}$.

Definition 1 The equilibrium $(u^*, v^*)^T$ of model (1)-(4) is said to be globally exponentially stable, if there exist constants $\varepsilon > 0$ and $K > 1$ such that

$$\| u_i(t,x) - u_i^* \|^r \leq KMe^{-\varepsilon t}, \| v_j(t,x) - v_j^* \|^r \leq KMe^{-\varepsilon t}$$

for all $t \geq 0$, in which $r \geq 1$, and

$$M = \sup_{-\tau\leq s\leq 0} \sum_{i=1}^{n} \| \varphi_i^* - u_i^* \|^r + \sup_{-\sigma\leq s\leq 0} \sum_{j=1}^{p} \| \psi_j^* - v_j^* \|^r .$$

Definition 2 [27]. A map H: $\Re^n \rightarrow \Re^n$ is a homeomorphism of $\Re^n$ onto itself, if $H\in C^0$, H is one-to-one, H is onto and the inverse map $H^{-1} \in C^0$.

Lemma 1 [27]. Let $H: \Re^n \rightarrow \Re^n$ be continuous. If H satisfies the following conditions:

(i) $H(x)$ is injective on $\Re^n$.

(ii) $\| H(x) \| \rightarrow \infty$ as $\|x\| \rightarrow \infty$.

Then H is a homeomorphism.

3. Main result

Theorem 1. Under assumption (**H**), model (1)-(4) has a unique equilibrium point, which is globally exponentially stable, if there exist constants $r>0$, $s > 0$ ($r^{-1} + s^{-1} = 1$), $d_q > 0 (q = 1,2,\cdots,n + p)$ such that

$$a_i d_i - s^{-1}\sum_{j=1}^{p} d_i | p_{ji} | \beta_j - r^{-1}\sum_{j=1}^{p} d_{n+j} | q_{ij} | \alpha_i > 0, \quad (5)$$

$$b_j d_{n+j} - s^{-1}\sum_{i=1}^{n} d_{n+j} | q_{ij} | \alpha_i - r^{-1}\sum_{i=1}^{n} d_i | p_{ji} | \beta_j > 0 \quad (6)$$

for $i=1,2,\cdots,n, j=1,2,\cdots,p.$

Proof. We shall prove this theorem in two steps.

Step1: We will prove the existence and uniqueness of the equilibrium point.

Consider the following functions associated with model (1)-(2)

$$h_i(z) = -a_i u_i + \sum_{j=1}^{p} p_{ji} f_j(v_j) + I_i, \quad (7)$$

$$h_{n+j}(z) = -b_j v_j + \sum_{i=1}^{n} q_{ij} g_i(u_i) + J_j, \quad (8)$$

for $i = 1,2,\cdots,n,\ j = 1,2,\cdots,p.$ Definite the map

$$H(z) = (h_1(z),\cdots,h_n(z),h_{n+1}(z),\cdots,h_{n+p}(z))^T . \quad (9)$$

First, we prove that $H(z)$ is an injective map on $\Re^{n+p}$. In fact, if there exist $z \neq \bar{z}$ such that $H(z) = H(\bar{z})$, then

$$-a_i(u_i - \bar{u}_i) + \sum_{j=1}^{p} p_{ji}(f_j(v_j) - f_j(\bar{v}_j)) = 0, \quad (10)$$

$$-b_j(v_j - \bar{v}_j) + \sum_{i=1}^{n} q_{ij}(g_i(u_i) - g_i(\bar{u}_i)) = 0. \quad (11)$$

From (8), by (H_1) and Young's inequality, we have

$$0=\sum_{i=1}^{n} d_i sign(u_i-\bar{u}_i)\,|u_i-\bar{u}_i|^{r-1}[-a_i(u_i-\bar{u}_i)$$

$$+\sum_{j=1}^{p} p_{ji}(f_j(v_j)-f_j(\bar{v}_j))]$$

$$\le \sum_{i=1}^{n}[-a_i d_i+s^{-1}\sum_{j=1}^{p} d_i\,|p_{ji}|\beta_j]\,|u_i-\bar{u}_i|^r$$

$$+\sum_{j=1}^{p}(r^{-1}\sum_{i=1}^{n} d_i\,|p_{ji}|\beta_j)\,|v_j-\bar{v}_j|^r. \qquad (12)$$

Similarly,

$$0\le \sum_{j=1}^{p}[-b_j d_{n+j}+s^{-1}\sum_{i=1}^{n} d_{n+j}\,|q_{ij}|\alpha_i]\,|v_j-\bar{v}_j|^r$$

$$+\sum_{i=1}^{n}(r^{-1}\sum_{j=1}^{p} d_{n+j}\,|q_{ij}|\alpha_i)\,|u_i-\bar{u}_i|^r. \qquad (13)$$

From (12) and (13), we get

$$0\le \sum_{i=1}^{n}(-a_i d_i+s^{-1}\sum_{j=1}^{p} d_i\,|p_{ji}|\beta_j+r^{-1}\sum_{j=1}^{p} d_{n+j}\,|q_{ij}|\alpha_i)\,|u_i-\bar{u}_i|^r$$

$$+\sum_{j=1}^{p}(-b_j d_{n+j}+s^{-1}\sum_{i=1}^{n} d_{n+j}\,|q_{ij}|\alpha_i+r^{-1}\sum_{i=1}^{n} d_i\,|p_{ji}|\beta_j)\,|v_j-\bar{v}_j|^r. \quad (14)$$

In view of (5) and (6), we have

$$\vartheta_1=\min_{1\le i\le n}[a_i d_i-s^{-1}\sum_{j=1}^{p} d_i\,|p_{ji}|\beta_j-r^{-1}\sum_{j=1}^{p} d_{n+j}\,|q_{ij}|\alpha_i]>0,$$

$$\vartheta_2=\min_{1\le j\le p}[b_j d_{n+j}-s^{-1}\sum_{i=1}^{n} d_{n+j}\,|q_{ij}|\alpha_i-r^{-1}\sum_{i=1}^{n} d_i\,|p_{ji}|\beta_j]>0.$$

From (14), we get

$$0\le -(\vartheta_1\sum_{i=1}^{n}|u_i-\bar{u}_i|^r+\vartheta_2\sum_{j=1}^{p}|v_j-\bar{v}_j|^r).$$

Hence, $u_i=\bar{u}_i$ and $v_j=\bar{v}_j$ ($i=1,2,\cdots;n,\ j=1,2,\cdots;p$), which is a contradiction. So, the map H is an injective on $\Re^{n+p}$.

Second, we shall prove that $\|H(u)\|\to\infty$ as $\|u\|\to\infty$. Since

$$h_i(z)-h_i(0)=-a_i u_i+\sum_{j=1}^{p} p_{ji}(f_j(v_j)-f_j(0)),$$

$$h_{n+j}(z)-h_{n+j}(0)=-b_j v_j+\sum_{i=1}^{n} q_{ij}(g_i(u_i)-g_i(0)).$$

Using (H_1) and Young's inequality, we have

$$\sum_{i=1}^{n} d_i[h_i(z)-h_i(0)]sign(u_i)\,|u_i|^{r-1}$$

$$\le \sum_{i=1}^{n}[-a_i d_i+s^{-1}\sum_{j=1}^{p} d_i\,|p_{ji}|\beta_j]\,|u_i|^r+\sum_{j=1}^{p}(\sum_{i=1}^{n} r^{-1} d_i\,|p_{ji}|\beta_j)\,|v_j|^r$$

and

$$\sum_{j=1}^{p} d_{n+j}[h_{n+j}(z)-h_{n+j}(0)]sign(v_j)\,|v_j|^{r-1}$$

$$\le \sum_{j=1}^{p}(-b_j d_{n+j}+s^{-1}\sum_{i=1}^{n} d_{n+j}\,|q_{ij}|\alpha_i)\,|v_j|^r+\sum_{i=1}^{n}(\sum_{j=1}^{p} r^{-1} d_{n+j}\,|q_{ij}|\alpha_i)\,|u_i|^r.$$

Hence,

$$\sum_{i=1}^{n} d_i[h_i(z)-h_i(0)]sign(u_i)\,|u_i|^{r-1}$$

$$+\sum_{j=1}^{p} d_{n+j}[h_{n+j}(z)-h_{n+j}(0)]sign(v_j)\,|v_j|^{r-1}$$

$$\le \sum_{i=1}^{n}(-a_i d_i+s^{-1}\sum_{j=1}^{p} d_i\,|p_{ji}|\beta_j+r^{-1}\sum_{j=1}^{p} d_{n+j}\,|q_{ij}|\alpha_i)\,|u_i|^r$$

$$+\sum_{j=1}^{p}(-b_j d_{n+j}+s^{-1}\sum_{i=1}^{n} d_{n+j}\,|q_{ij}|\alpha_i+r^{-1}\sum_{i=1}^{n}\sum_{j=1}^{p} d_i\,|p_{ji}|\beta_j))\,|v_j|^r$$

$$\le -\vartheta_1\sum_{i=1}^{n}|u_i|^r-\vartheta_2\sum_{j=1}^{p}|v_j|^r\le -\vartheta(\sum_{i=1}^{n}|u_i|^r+\sum_{j=1}^{p}|v_j|^r),$$

where $\vartheta=\min(\vartheta_1,\vartheta_2)$. Thus

$$\sum_{i=1}^{n} d_i\left|h_i(z)-h_i(0)\right|\,|u_i|^{r-1}+\sum_{j=1}^{p} d_{n+j}\left|h_{n+j}(z)-h_{n+j}(0)\right|\,|v_j|^{r-1}$$

$$\ge \vartheta(\sum_{i=1}^{n}|u_i|^r+\sum_{j=1}^{p}|v_j|^r).$$

Let $d=\max(\max_{1\le i\le n}\{d_i\},\max_{1\le j\le p}\{d_{n+j}\})$, $\|\cdot\|_1$ is defined as $\|z\|_1=\sum_{i=1}^{n+p}|z_i|$, by Holder's inequality, we can get

$$\| H(z)-H(0) \|_1 \geq \frac{\vartheta}{2(n+p)} (\sum_{i=1}^{n} | u_i |^r + \sum_{j=1}^{p} | v_j |^r)^{\frac{1}{r}}.$$

According to the classical results in functional analysis, for any two different vector norms $\|\cdot\|_1$ and $\|\cdot\|_2$ defined on $\Re$, they are equivalent in sense that there exist two positive constants c_1 and c_2 such that $c_1 \| z \|_2 \leq \| z \|_1 \leq c_2 \| z \|_2$. So, there exists some suitable constant c_1 such that

$$\| z \|_r = (\sum_{i=1}^{n} u_i |^r + \sum_{j=1}^{p} v_j |^r)^{\frac{1}{r}} \geq c_1 \| z \|_1.$$

Thus

$$\| H(z)-H(0) \|_1 \geq \frac{c_1 \vartheta}{2(n+p)} \| z \|_1.$$

Hence

$$\| H(z)-H(0) \|_1 \to \infty \text{ as } \| u \|_1 \to \infty.$$

Which directly implies that $\| H(z) \|_1 \to \infty$ as $\| u \|_1 \to \infty$. So, From Lemma 1, we know that $H(z)$ is a homeomorphism on $\Re^{n+p}$, thus model (1)-(2) has a unique equilibrium point.

Step2: We prove that the unique equilibrium point of model (1)-(2) is globally exponentially stable.

Suppose that $(u_1^*, \cdots, u_n^*, v_1^*, \cdots, v_p^*)^T$ is an equilibrium point of system (1)-(2), $z(t,x)$ is any solution of (1)-(2). Let $U_i(t,x) = u_i(t,x) - u_i^*, V_j(t,x) = v_j(t,x) - v_j^*$. It is easy to see that system (1)-(2) can be rewritten to the following form:

$$\frac{\partial U_i(t,x)}{\partial t} = \sum_{k=1}^{m} \frac{\partial}{\partial x_k} (\alpha_{ik} \frac{\partial U_i(t,x)}{\partial x_k}) - a_i U_i(t,x)$$

$$+ \sum_{j=1}^{p} p_{ji} [f_j(V_j(t-\tau_{ji},x)+v_j^*) - f_j(v_j^*)], \quad (15)$$

$$\frac{\partial V_j(t,x)}{\partial t} = \sum_{k=1}^{m} \frac{\partial}{\partial x_k} (\beta_{ik} \frac{\partial V_i(t,x)}{\partial x_k}) - b_j V_j(t,x)$$

$$+ \sum_{i=1}^{n} q_{ij} [g_i(U_i(t-\sigma_{ij},x)+u_i^*) - g_i(u_i^*)], \quad (16)$$

Multiplying both sides of (15) with $2U_i(t,x)$ and integrating over Ω, we obtain

$$\frac{d}{dt} \int_{\Omega} U_i^2(t,x) dt = \int_{\Omega} 2U_i(t,x) \frac{\partial U_i(t,x)}{\partial t} dt$$

$$= 2 \int_{\Omega} U_i(t,x) \sum_{k=1}^{m} \frac{\partial}{\partial x_k} (\alpha_{ik} \frac{\partial U_i(t,x)}{\partial x_k}) dx$$

$$-2a_i \int_{\Omega} U_i^2(t,x) dx + 2 \sum_{j=1}^{p} p_{ji} \int_{\Omega} U_i(t,x)$$

$$\times [f_j(V_j(t-\tau_{ji},x)+v_j^*) - f_j(v_j^*)] dx.$$

By (H) and Cauchy's inequality, we have

$$\frac{d}{dt} (\| U_i(t,x) \|)^2 \leq 2 \int_{\Omega} U_i(t,x) \sum_{k=1}^{m} \frac{\partial}{\partial x_k} (\alpha_{ik} \frac{\partial U_i(t,x)}{\partial x_k}) dx$$

$$-2a_i (\| U_i(t,x) \|)^2 + 2 \sum_{j=1}^{p} p_{ji} |\beta_j \| U_i(t,x) \| \| V_j(t-\tau_{ji},x) \|.$$

Applying the boundary condition (3)-(4), from the proof of [22] we know that

$$\int_{\Omega} U_i(t,x) \sum_{k=1}^{m} \frac{\partial}{\partial x_k} (\alpha_{ik} \frac{\partial U_i(t,x)}{\partial x_k}) dx \leq 0.$$

Thus, we obtain

$$\frac{d}{dt} \| U_i(t,x) \| \leq -a_i \| U_i(t,x) \| + \sum_{j=1}^{p} p_{ji} |\beta_j \| V_j(t-\tau_{ji},x) \|. \quad (17)$$

Similarly, we have

$$\frac{d}{dt} \| V_j(t,x) \| \leq -b_j \| V_j(t,x) \| + \sum_{i=1}^{n} q_{ij} |\alpha_i \| U_i(t-\sigma_{ij},x) \|. \quad (18)$$

From conditions (5) and (6), there exist a small constant $0 < \varepsilon < \min_{1 \leq i \leq n, 1 \leq j \leq p} (a_i, b_j)$ such that

$$d_i[-a_i + r^{-1}\varepsilon + s^{-1} \sum_{j=1}^{p} p_{ji} |\beta_j] + e^{\varepsilon\sigma} r^{-1} \sum_{j=1}^{p} d_{n+j} | q_{ij} |\alpha_i] < 0, \quad (19)$$

$$d_{n+j}[-b_j + r^{-1}\varepsilon + s^{-1} \sum_{i=1}^{n} q_{ij} |\alpha_i] + e^{\varepsilon\sigma} r^{-1} \sum_{j=1}^{p} d_{n+j} | q_{ij} |\alpha_i] < 0. \quad (20)$$

Now consider the Lyapunov functional as the following

$$V(t) = V_1(t) + V_2(t),$$

where

$$V_1(t)=\sum_{i=1}^{m} d_i r^{-1}\{\| U_i(t,x)\|^r e^{\varepsilon t}+\sum_{j=1}^{p}| p_{ji} |\beta_j$$

$$\times\int_{t-\tau_{ji}}^{t}\| V_j(s,x)\|^r e^{\varepsilon(s+\tau_{ji})}ds\},$$

$$V_2(t)=\sum_{j=1}^{p} d_{n+j} r^{-1}\{\| V_j(t,x)\|^r e^{\varepsilon t}+\sum_{i=1}^{n}| q_{ij} |\alpha_i$$

$$\times\int_{t-\sigma_{ij}}^{t}\| U_i(s,x)\|^r e^{\varepsilon(s+\sigma_{ij})}ds\}.$$

Calculating the derivative of the $V(t)$ along solution of (15)-(16) and using Young's inequality, we have

$$\frac{dV_1}{dt}=\sum_{i=1}^{m} d_i\{\| U_i(t,x)\|^{r-1}\frac{d\| U_i(t,x)\|}{dt}e^{\varepsilon t}$$

$$+\varepsilon r^{-1}e^{\varepsilon t}\| U_i(t,x)\|+r^{-1}\sum_{j=1}^{p} p_{ji} |\beta_j[\| V_j(t,x)\|^r e^{\varepsilon(t+\tau_{ji})}$$

$$-\| V_j(t-\tau_{ji},x)\|^r e^{\varepsilon t}]\}$$

$$\le e^{\varepsilon t}\sum_{i=1}^{m} d_i\{[-a_i+\varepsilon r^{-1}+s^{-1}\sum_{j=1}^{p}| p_{ji} |\beta_j]\| U_i(t,x)\|^r$$

$$+e^{\varepsilon\tau}r^{-1}\sum_{j=1}^{p}| p_{ji} |\beta_j\| V_j(t,x)\|^r\}$$

Similarly, we have

$$\frac{dV_2}{dt}\le e^{\varepsilon t}\sum_{j=1}^{p} d_{n+j}\{[-b_j+\varepsilon r^{-1}+s^{-1}\sum_{i=1}^{n} q_{ij} |\alpha_i\| V_j(t,x)\|^r$$

$$+e^{\varepsilon\sigma}r^{-1}\sum_{i=1}^{n}| q_{ij} |\alpha_i\| U_i(t,x)\|^r\}.$$

This implies that

$$\frac{dV}{dt}\le e^{\varepsilon t}\sum_{i=1}^{m} d_i\{[-a_i+\varepsilon r^{-1}+s^{-1}\sum_{j=1}^{p}| p_{ji} |\beta_j]$$

$$+e^{\varepsilon\sigma}r^{-1}\sum_{j=1}^{p} d_{n+j}| q_{ij} |\alpha_i\}\| U_i(t,x)\|^r$$

$$+e^{\varepsilon t}\sum_{j=1}^{p} d_{n+j}\{[-b_j+\varepsilon r^{-1}+s^{-1}\sum_{i=1}^{n}| q_{ij} |\alpha_i]$$

$$+e^{\varepsilon\tau}r^{-1}\sum_{i=1}^{n} d_i| p_{ji} |\beta_j\}\| V_j(t,x)\|^r<0$$

Thus, we have

$$V(t)\le V(0), t\ge 0.$$

Note that

$$V(t)\ge \min_{1\le k\le n+p}\{d_k\}e^{\varepsilon t}r^{-1}\{\sum_{i=1}^{n}\| U_i(t,x)\|^r+\sum_{j=1}^{p}\| V_j(t,x)\|^r\}$$

and

$$V(0)\le r^{-1}\sum_{i=1}^{n} d_i\| U_i(0,x)\|^r+r^{-1}\sum_{i=1}^{n}\sum_{j=1}^{p} d_i| p_{ji}|\beta_j e^{\varepsilon\tau}$$

$$\times\int_{-\tau}^{0}\| V_j(s,x)\|^r e^{\varepsilon s}ds$$

$$+r^{-1}\sum_{j=1}^{p} d_{n+j}\| V_j(0,x)\|^r+r^{-1}\sum_{j=1}^{p}\sum_{i=1}^{n} d_{n+j}| q_{ij}|\alpha_i e^{\varepsilon\sigma}$$

$$\times\int_{-\sigma}^{0}\| U_i(s,x)\|^r e^{\varepsilon s}ds$$

$$\le r^{-1}\max_{1\le k\le n+p}\{d_k\}\{1+\sum_{i=1}^{n}\sum_{j=1}^{p} p_{ji}|\beta_j\tau e^{\varepsilon\tau}+\sum_{j=1}^{p}\sum_{i=1}^{n} q_{ij}|\alpha_i\sigma e^{\varepsilon\sigma}\}M.$$

So

$$\sum_{i=1}^{n}\| U_i(t,x)\|^r+\sum_{j=1}^{p}\| V_j(t,x)\|^r\le MKe^{-\varepsilon t},$$

where

$$K=\frac{\max_{1\le k\le n+p}\{d_k\}}{\min_{1\le k\le n+p}\{d_k\}}\{1+\sum_{i=1}^{n}\sum_{j=1}^{p} p_{ji}|\beta_j\tau e^{\varepsilon\tau}+\sum_{j=1}^{p}\sum_{i=1}^{n} q_{ij}|\alpha_i\sigma e^{\varepsilon\sigma}\}.$$

This implies that the equilibrium of system (1)-(2) is globally exponentially stable. The proof is completed.

Remark 1 Taking r=s=2 in Theorem 1 of this paper, we can directly obtain the Theorem 1 in [23]. If the smooth operators $a_{ik}=b_{jk}=0$, $i=1,2,\cdots,n$, $j=1,2,\cdots,p$, $k=1,\cdots,m$, and $f=g$, then model (1)-(2) turns to the following BAM neural networks in [8] and [9]. It is easy to see that the presented stability criteria in [8] and [9] are also the special cases of Theorem 1 of this paper.

Remark 2 In [3], [4], [7], [8], [9], [10], [12], [14], [15], [23] and [25], the boundedness of the activation functions was required. However, the boundedness of the activation functions was removed.

Remark 3 From the process of the proof of Theorem 1, we know that the method of this paper can be applied to study the stability of the BAM neural networks with both distributed delays and reaction diffusion terms.

4. Conclusions

In this paper, we have presented a modified stability criterion by employing analytic techniques. The given criterion improves and extends some recent results. It should be pointed out that the boundedness of the activation functions has been removed in our result.

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Jianqin Wang was born in 1960. He received the B.S. degrees in Computer Science and Technology in 1988 from Nanchong Teachers College, Sichuan, China. From September 1978 to now, he was with School of Basic Medical, North Sichuan Medical College, Sichuan, China. He is the author or coauthor of 5 journal papers. His current research interests include stability theory of neural networks and chaos synchronization.

Qiankun Song was born in 1964. He received the B.S. degree in Mathematics in 1986 from Sichuan Normal University, Chengdu, China, and the M.S. degree in Applied Mathematics in 1996 from Northwestern Polytechnical University, Xi'an, China. He was a student at refresher class in the Department of Mathematics, Sichuan University, Chengdu, China, from September 1989 to July 1990. From July 1986 to December 2000, he was with Department of Mathematics, Sichuan University of Science and Engineering, Sichuan, China. From January 2001 to June 2006, he was with the Department of Mathematics, Huzhou University, Zhejiang, China. In July 2006, he moved to the Department of Mathematics, Chongqing Jiaotong University, Chongqing, China. He is currently a Professor at Chongqing Jiaotong University. He is the author or coauthor of more than 30 journal papers and one edited book. His current research interests include stability theory of neural networks and chaos synchronization.