Backstepping Stabilization for a class of SISO Switched Nonlinear Systems with trigonal structure

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Summary

In this paper, we discuss stabilization problem for a class of switched nonlinear systems whose subsystem with trigonal structure. A backstepping switching control design is given. Based on backsteppping approach, stabilizer and a switching law are designed for such systems. The stabilization of the resulting closed-loop systems is proved via common Lyapunov function method.

Key words:

Switched Nonlinear System; Stabilization; Backstepping; Switching Rule.

1. Introduction

The study of hybrid systems in control is motivated by the fundamentally hybrid nature of many modern-day control systems, which are characterized by the interaction of lower-level continuous dynamics and upper-level discrete or logical components. In many of these systems, the continuous dynamics arise from the underlying physical laws such as mass, momentum, and energy conservation, and are usually modeled by continuous-time differential equations. Discrete events, on the other hand, can arise from a variety of sources, including inherent physicochemical discontinuities in the continuous dynamics, controlled transitions between different operating regimes, the use of discrete actuators and sensors in the control system, and the use of logic-based switching for supervisory and safety control. It is well understood at this stage that the interaction of discrete events with even simple continuous dynamical systems can lead to complex dynamics and, possibly, to undesirable outcomes if not appropriately accounted for in the control system design. Even though theory for the analysis and control of purely continuous-time systems exists and, to a large extent, is well-developed, similar techniques for combined discrete-continuous systems are limited at present, primarily due to the difficulty of extending the available concepts and tools to treat the hybrid nature of these systems and their changing dynamics. Motivated by this and the abundance of situations where hybrid systems arise in practice, significant research work has focused on hybrid systems over the last decade.

A switched system is a hybrid system that comprises a collection of subsystems together with a switching rule that specifies the switching among the subsystems. It is well known that different switching law would produce different behavior of system and hence lead to different system performances. For example, for the switched linear system that consists of two stable subsystems, it would be unstable if we apply unsuitable switching rule to this system. Conversely, if the two subsystems are unstable and we adopt suitable switching path, the switched system would be stable. As such, how to design a switching law so that the switched system achieves certain performance is indeed an important and well-motivated problem.

During the last decades, there have been many studies on stability analysis and design for switched systems [1-11]. In [3], it is pointed out that how to construct a switching law that makes a switched system asymptotically stable is one of the three basic problems. There are a lot of papers addressing the topics of stability, quadratic stabilization for switched linear systems. In this paper, we focus on, based on backstepping method, the stabilization problem for a class of switched nonlinear systems whose subsystem with trigonal structure via designing switching strategy and associated state feedback stabilizer. It is assumed that the switching strategy used in this paper is picked in such a way that there are finite switches in finite time. Our goal is to design a switching law $\sigma(t)$ and associated state feedback sub-controllers such that the resulting closed-loop system is asymptotically stable.

The remainder of this paper is organized as follows: stabilization problem is stated in Section 2, while in Section 3 the stabilizer and switching law of switched nonlinear systems is constructed. Section 4 gives stabilizability result. Finally, some conclusions are drawn in Section 5.

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2. System Description and Problem Statement

In this paper, we will consider global stabilization of the following switched nonlinear systems:

$$\begin{cases} \dot{x}_i = x_{i+1} + f_{i\sigma(t)}(\bar{x}_i), & 1 \le i \le n-1 \\ \dot{x}_n = u + f_{n\sigma(t)}(x) \end{cases} \tag{1}$$

where $x = (x_1, x_2, \cdots, x_n)^T \in \mathbb{R}^n$ is the system state, $\bar{x}_i = (x_1, x_2, \cdots, x_i)^T$, $u \in \mathbb{R}$ is the continuous control input. The function $\sigma(\cdot): [0, +\infty) \to \{1, 2, \cdots, N\} \overset{def}{=} \underline{\mathbb{N}}$ denotes the piecewise constant switching law to be designed. Moreover, $\sigma(t) = k$ implies that the k^{th} subsystem is active and the nonlinear functions $f_{ik}(\cdot), 1 \le i \le n, 1 \le k \le N$, are known and smooth and satisfy

$$f_{ik}(0, 0, \cdots, 0) = 0 \quad (1 \le i \le n, 1 \le k \le N) \tag{2}$$

We consider here the stabilization problem, i.e., the control objective is to globally asymptotically stabilize the equilibrium $x = 0$ by designing switching law and stabilizer.

3. Switched Controller Design

The design of stabilizer takes a same procedure as in, e.g., [8], that is, we first design state feedback controller for every subsystem by using backstepping approach, and then design a switching rule.

3.1. Controller of Subsystem Design

Consider the k^{th} subsystem of switched nonlinear system (1)

$$\begin{cases} \dot{x}_i = x_{i+1} + f_{ik}(\bar{x}_i), & 1 \le i \le n-1 \\ \dot{x}_n = u + f_{nk}(x) \end{cases} \tag{3}$$

We design the stabilizer of nonlinear system (3) $u = \alpha_n(x)$ in backstepping way. In this subsection, we assume that the positive integer $k \in \underline{\mathbb{N}}$ be fixed.

Step 1: Let $z_1 = x_1$. From (3),

$$\dot{z}_1 = x_2 + f_{1k}(x_1) \tag{4}$$

Since $f_{1k}(\cdot)$ is a smooth function, in view of (2), we can write

$$f_{1k}(z_1) = z_1 g_{1k}(z_1) \tag{5}$$

where $g_{1k}(z_1)$ is a continuous function with respect to z_1.

Define

$$h_{1k}(z_1) = |g_{1k}(z_1)| \tag{6}$$

We now view x_2 as a virtual control and design it for the following stabilizing function:

$$\alpha_1(x_1) = -nz_1 \tag{7}$$

Define

$$z_2 = x_2 - \alpha_1(x_1) \tag{8}$$

Then, the time derivative of $V_1 = \frac{1}{2} z_1^2$ along to system (4) is given by

$$\dot{V}_1 = z_1 \dot{z}_1 = z_1(x_2 + f_{1k}(x_1)) \tag{9}$$

Computed with (5)-(8) is given by

$$\dot{V}_1 \le -nz_1^2 + z_1^2 h_{1k}(z_1) + z_1 z_2 \tag{10}$$

Step i $(2 \le i \le n-1)$: Let

$$z_1 = x_1 \text{ and } z_j = x_j - \alpha_{j-1}(\bar{x}_{j-1}), \; j = 2, \cdots, i.$$

From (3),

$$\begin{aligned} \dot{z}_j &= x_{j+1} + f_{jk}(\bar{x}_j) - \\ &\sum_{l=1}^{j-1} \frac{\partial \alpha_{j-1}}{\partial x_l}(x_{l+1} + f_{lk}(\bar{x}_l)), \; j = 2, \cdots, i \end{aligned} \tag{11}$$

Setting

$$\begin{aligned} \tilde{f}_{jk}(\bar{z}_j) &= f_{jk}(\bar{x}_j) - \\ &\sum_{l=1}^{j-1} \frac{\partial \alpha_{j-1}}{\partial x_l}(x_{l+1} + f_{lk}(\bar{x}_l)), \; j = 2, \cdots, i \end{aligned} \tag{12}$$

where $\bar{z}_j \overset{def}{=} (z_1, \cdots, z_j)^T$, $\bar{x}_1 \overset{def}{=} x_1$.

By means of $\alpha_{j-1}(0, \cdots, 0) = 0 \; (j = 2, \cdots, i)$ and (2), we have

$$\tilde{f}_{jk}(0, \cdots, 0) = 0, \quad j = 2, \cdots, i \tag{13}$$

So, we can further write

$$\tilde{f}_{jk}(\bar{z}_j) = \sum_{l=1}^{j} z_l g_{jlk}(\bar{z}_j), \quad j = 2, \cdots, i \tag{14}$$

where the functions $g_{jlk}(\bar{z}_j)$, $l = 1, 2, \cdots, j$, are continuous with respect to $\bar{z}_j$. Define

$$h_{jk}(\bar{z}_j) = \frac{j}{4} \max \begin{Bmatrix} g_{j1k}^2(\bar{z}_j), g_{j2k}^2(\bar{z}_j), \\ \cdots, g_{jjk}^2(\bar{z}_j) \end{Bmatrix}, \; j = 2, \cdots, i \tag{15}$$

We now view x_{i+1} as a virtual control and design it for the following stabilizing function:

$$\alpha_i(\bar{x}_i) = -z_{i-1} - (n + 2 - i) z_i \tag{16}$$

Define

$$z_{i+1} = x_{i+1} - \alpha_i(\bar{x}_i) \tag{17}$$

Then, the time derivative of $V_i = \frac{1}{2}(z_1^2 + z_2^2 + \cdots + z_i^2)$ along to system (4) and (11) is given by

$$\dot{V}_i = z_1(x_2 + f_{1k}(z_1)) + \sum_{j=2}^{i} z_j(x_{j+1} + \tilde{f}_{jk}(\bar{z}_j)) \tag{18}$$

Computed with (12)-(17) is given by

$$\dot{V}_i \le -(n + 1 - i) \sum_{j=1}^{i} z_j^2 + \sum_{j=1}^{i} z_j^2 h_{jk}(\bar{z}_j) + z_i z_{i+1} \tag{19}$$

Step n: According to $z_n = x_n - \alpha_{n-1}(\bar{x}_{n-1})$, we have

$$\dot{z}_n = u + f_{nk}(x) - \sum_{l=1}^{n} \frac{\partial \alpha_{n-1}}{\partial x_l}(x_{l+1} + f_{lk}(\bar{x}_l)) \qquad (20)$$

Setting

$$\tilde{f}_{nk}(z) = f_{nk}(x) - \sum_{l=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_l}(x_{l+1} + f_{lk}(\bar{x}_l)) \qquad (21)$$

we can further write

$$\tilde{f}_{nk}(z) = \sum_{l=1}^{n} z_l g_{nlk}(z) \qquad (22)$$

where the functions $g_{nlk}(z), l = 1,2,\cdots,n$, are continuous with respect to z. Define

$$h_{nk}(z) = \frac{n}{4}\max\left\{g_{n1k}^2(z), g_{n2k}^2(z), \cdots, g_{nnk}^2(z)\right\} \qquad (23)$$

We now choose control u as follows:

$$u = -z_{n-1} - 2z_n \qquad (24)$$

The time derivative of $V_n = \frac{1}{2}(z_1^2 + z_2^2 + \cdots + z_n^2)$ along to system (4), (11) and (20) is given by

$$\begin{aligned}\dot{V}_n &= z_1(x_2 + f_{1k}(z_1)) + \sum_{j=2}^{n} z_j(x_{j+1} + \tilde{f}_{jk}(\bar{z}_j)) \\ &\le -\sum_{j=1}^{n} z_j^2 + \sum_{j=1}^{n} z_j^2 h_{jk}(\bar{z}_j)\end{aligned} \qquad (25)$$

By means of Lyapunov stability theory, it follows from $h_{jk}(\bar{x}_j) < 1, j = 1,2,\cdots,n$ that the system (3) is asymptotically stabilize the equilibrium $x = 0$

3.2. Switching Strategy Design

In this subsection, we will employ attenuation domain of each subsystem to design switching rule so that the overall closed-loop system is globally asymptotically stable.

Suppose that

$$\bigcup_{k=1}^{N}\left\{z \in \mathbb{R}^n \;\middle|\; \max_{1\le i\le n} h_{ik}(\bar{z}_i) < 1\right\} = \mathbb{R}^n$$

Then for initial condition

$$z(t_0) = \left(z_1(t_0), z_2(t_0), \cdots, z_n(t_0)\right)^T$$

setting

$$\sigma(t_0) = \arg\min_{1\le k\le N}\left\{\max_{1\le i\le n} h_{ik}(\bar{z}_i(t_0))\right\}$$

where $\bar{z}_i(t_0) = \left(z_1(t_0), z_2(t_0), \cdots, z_i(t_0)\right)^T$, the symbol "$\arg\min$" denotes the index that attains the minimum. If there is one more than such index, we just pick the smallest one.

The first switching time instant is determined by

$$t_1 = \inf\left\{\begin{array}{l} t \ge t_0 \,\middle|\, \text{ there exists a } i \in \{1,2,\cdots,n\} \\ \qquad \text{such that } h_{i\sigma(t_0)}(\bar{z}_i(t)) \ge 1 \end{array}\right\}$$

The corresponding switching index is chosen to be

$$\sigma(t_1) = \arg\min_{1\le k\le N}\left\{\max_{1\le i\le n} h_{ik}(\bar{z}_i(t_1))\right\}$$

Finally, we define the switching time/index sequences recursively by

$$t_{j+1} = \inf\left\{\begin{array}{l} t \ge t_j \,\middle|\, \text{ there exists a } i \in \{1,2,\cdots,n\} \\ \qquad \text{such that } h_{i\sigma(t_j)}(\bar{z}_i(t)) \ge 1 \end{array}\right\}$$

$$\sigma(t_{j+1}) = \arg\min_{1\le k\le N}\left\{\max_{1\le i\le n} h_{ik}(\bar{z}_i(t_{j+1}))\right\}, \quad j = 1,2,\cdots$$

4. Stability Analysis

Theorem 4.1 Suppose the state feedback controller (24) and switching law developed in subsection 3.2 are applied to the switched nonlinear system (1). Then, for any initial conditions, the resulting closed-loop system is globally asymptotically stable at the equilibrium $x = 0$. If the following set formula holds.

$$\bigcup_{k=1}^{N}\left\{z \in \mathbb{R}^n \;\middle|\; \max_{1\le i\le n} h_{ik}(\bar{z}_i) < 1\right\} = \mathbb{R}^n \qquad (26)$$

Proof: Consider the following change of coordinates

$$\begin{cases} z_1 = x_1 \\ z_i = x_i - \alpha_{i-1}(\bar{x}_{i-1}), i = 2,\cdots,n \end{cases}$$

The stabilizers $\alpha_i(\bar{x}_i), i = 1,2,\cdots,n$ developed in subsection 3.1 are applied to switched system (1). Then the resulting system can be written as

$$\begin{cases} \dot{z}_1 = -nz_1 + z_2 + z_1 g_{1\sigma}(z_1) \\ \dot{z}_i = -z_{i-1} - (n+2-i)z_i + z_{i+1} + \\ \qquad \sum_{j=1}^{i} z_j g_{ij\sigma}(\bar{z}_i), 2 \le i \le n-1 \\ \dot{z}_n = -z_{n-1} - 2z_n + \sum_{j=1}^{n} z_j g_{nj\sigma}(z) \end{cases} \qquad (27)$$

Consider the Lyapunov function candidate $V = \frac{1}{2}\sum_{i=1}^{n} z_i^2$.

In view of (6), (15) and (23), the time derivative of V along the trajectories of the system (27) is given by, for any $t \in [t_m, t_{m+1})$

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{n} z_i \dot{z}_i \\ &= -nz_1^2 - \sum_{i=2}^{n}(n+2-i)z_i^2 + \\ &\quad \sum_{i=1}^{n}\sum_{j=1}^{i} z_j z_i g_{ij\sigma(t_m)}(\bar{z}_i) \\ &\le -nz_1^2 - \sum_{i=2}^{n}(n+2-i)z_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{i} z_j^2 + \\ &\quad \frac{1}{4}\sum_{i=1}^{n}\sum_{j=1}^{i} z_i^2 g_{ij\sigma(t_m)}^2(\bar{z}_i) \\ &\le -\|z\|^2 + \sum_{i=1}^{n} z_i^2 h_{i\sigma(t_m)}^2(\bar{z}_i)\end{aligned} \qquad (28)$$

where $\bar{z}_1 = z_1, \bar{z}_n = z, g_{11\sigma(t_m)}(\bar{z}_1) = g_{1\sigma(t_m)}(z_1)$.

Again in view of (11) and the switching law developed in the subsection 3.2, we have $\dot{V}(t)<0$ for every $t\in[t_0,\infty)$. Therefore by Lyapunov stability theorem, the conclusion holds. This is completes the proof. ◊

5. Conclusion

Switching Stability problem for a class of switched nonlinear systems with trigonal structure was investigated in this paper. We construct stabilizer and a switching law for above mention switched systems in backsteppping way. Finally, the stabilizability of the closed-loop systems is proved via common Lyapunov function approach under the fact that the sum of attenuation domain for each subsystem covers with overall state space.

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