Analysis of Nonlinear Distortion Using Orthogonal Polynomials HPA Model

Hassan Zareian, and Vahid Tabataba Vakili

Iran University of Science and Technology (IUST), Tehran, Iran

Summary
The OFDM systems are highly sensitive to the nonlinear distortion, introduced by the high power amplifier (HPA). The nonlinear HPA has two effects on the transmission system: the first is the spectral regrowth of the signal which leads to adjacent channel interference (ACI). The second effect is the distortion of the signal in the nominal frequency band which causes inter-symbol interference (ISI). This paper deals with the second phenomenon. In this paper, we propose a new theoretical approach for computing the bit error rate (BER) of the M-QAM-OFDM systems in presence of nonlinear HPA in AWGN channels. This analytical approach is based on HPA modeling using orthogonal polynomials. Theoretical results show closely matching with those obtained by simulations.
Key words:
OFDM, nonlinear distortion, orthogonal polynomials, BER performance, theoretical analysis

1. Introduction

The orthogonal frequency-division multiplexing (OFDM) modulation format has been proposed as the standard for a variety of digital communication applications, such as DVB-T (Digital Video Broadcasting-Terrestrial), DAB (Digital Audio Broadcasting) and for wide-band wireless communication systems [1]. However, it is known that the multi carrier modulated systems such as OFDM are more sensitive to nonlinear distortion due to HPA than single carrier modulated systems [2].

The nonlinear distortion effects in OFDM transmission have been investigated in many papers [2]-[6]. Most of them treat the nonlinear distortion noise as an additive Gaussian process and apply various simulation [2] and analytical [3]-[6] approaches to derive the variance of such a Gaussian noise. In [5], an analytical expression for the output autocorrelation function based on HPA modeling with Bessel series expansion is derived. By using Fourier transformation, the output autocorrelation function can provide information on the power spectral density (PSD) at the HPA output, and at the same time, it allows the analytical calculation of the power of the nonlinear distortion noise.

Manuscript received January 5, 2008
Manuscript revised January 20, 2008

In this paper, we use a novel set of orthogonal polynomials [7] to replace Bessel series expansion in [5] for HPA modeling. The orthogonal polynomials provide an intuitive means of spectral regrowth analysis and enable us to derive a very simple analytical expression for the autocorrelation function and hence the PSD of the nonlinear noise [8]. In this way, it is possible to derive an analytical BER expression for the OFDM systems performance in presence of nonlinear HPA in AWGN channels. With respect to previous investigations, the analytical evaluation proposed here allows an easy and accurate assessment of the performance degradation in OFDM systems as a function of the main system parameter. The performance figure herein considered includes the BER as a function of the output back-off (OBO).

The paper is organized as follows. In Section 2 the orthogonal polynomials and HPA modeling as well as the considered OFDM system model are introduced. In Section 3 the procedure used to analyze the performance effects of the nonlinear distortion is theoretically described. The proposed technique for BER performance of the OFDM system is compared with the previous analytical technique in Section 4. Finally, in Section 5, conclusions are drawn.

2. System Model and Problem Formulation

2.1 HPA Modeling Using Orthogonal Polynomial

Assuming that the baseband equivalent of signal at the input of the HPA is given by

$$x(t)=r(t)e^{j\theta(t)} \tag{1}$$

Then the complex envelope of the HPA output signal can be expressed by

$$y(t) = \left[A(r(t)) e^{j\Phi(r(t))} \right] e^{j\theta(t)} = f(r(t)) e^{j\theta(t)} \tag{2}$$

where $A(\cdot)$ and $\Phi(\cdot)$ model the amplitude modulation / amplitude modulation (AM/AM) and amplitude modulation / phase modulation (AM/PM) distortion, respectively, introduced by the HPA [5]. A complex nonlinear distortion function $f(\cdot)$ which only depends on the envelope of the input signal $x(t)$ is defined by

$$f(r(t)) \triangleq A(r(t)) e^{j\Phi(r(t))} \tag{3}$$

We can model the memoryless HPA input-output relationship by a polynomial [7]

$$y(t) = \sum_{k=0}^{K} a_{2k+1} |x(t)|^{2k} x(t) = \sum_{k=0}^{K} a_{2k+1} \phi_{2k+1}(x(t)) \tag{4}$$

where $\phi_{2k+1}(x(t)) \triangleq |x(t)|^{2k} x(t)$ denotes the conventional polynomial basis function. The polynomial coefficients $\{a_{2k+1}\}$ are generally complex valued, except when there is no AM/PM distortion.

In [7], the orthogonal polynomials basis function for a zero mean, variance σ_x^2 complex Gaussian process $x(t)$ was derived as

$$\psi_{2m+1}(x) = \sum_{k=0}^{m} \frac{(-1)^{m-k}}{\sigma_x^{2k+1}} \frac{\sqrt{m+1}}{(k+1)!} \binom{m}{k} \phi_{2k+1}(x) \tag{5}$$

If the HPA's input-output relationship represented using the orthogonal polynomial model, i.e.

$$y(t) = \sum_{k=0}^{K} \alpha_{2k+1} \psi_{2k+1}(x(t)) \tag{6}$$

It can be shown that [7]

$$R_y(\tau) = \sum_{k=0}^{K} |\alpha_{2k+1}|^2 \left| \frac{R_x(\tau)}{\sigma_x^2} \right|^{2k} \frac{R_x(\tau)}{\sigma_x^2} \tag{7}$$

Taking the Fourier transform of both sides of (7) and assuming spectral symmetry for the input signal, the PSD at the output of the HPA can be obtained by:

$$S_y(f) = \sum_{k=0}^{K-1} |\alpha_{2k+1}|^2 \underbrace{\frac{S_x(f)}{\sigma_x^2} * \cdots \frac{S_x(f)}{\sigma_x^2}}_{2k+1} \tag{8}$$

Therefore, the coefficients in the orthogonal polynomial HPA model have clear meanings in the context of spectral regrowth. By inspecting $|\alpha_1|, |\alpha_3|, |\alpha_5|, \ldots$ we immediately have a sense of the severity of spectral regrowth [8].

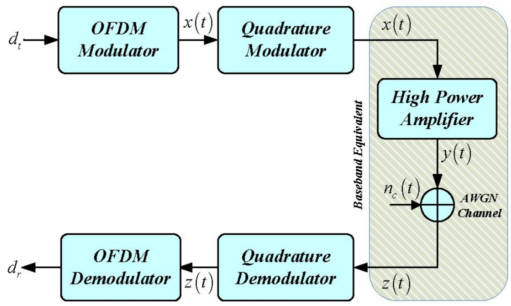

Fig. 1 Simplified block diagram of OFDM system.

2.2 OFDM System

A baseband equivalent scheme of the considered OFDM system can be shown as in Fig. 1. In this paper, we focus on the problem of nonlinear distortion of HPA while the receiver as well as transmitter enjoy perfect IQ balance. The complex baseband OFDM signal $x(t)$ with N subcarriers transmitted during i th time interval $t \in [iT_b, iT_b + T_b]$ may be represented [5]

$$x(t) = \frac{1}{N} \sum_{k=0}^{N-1} d_t(k,i) e^{j(2\pi k/T_b)t} \tag{9}$$

where $d_t(k,i)$ here represents the complex data symbol for the k th subcarrier that are generated at rate $1/T_s$ and $T_b = NT_s$ is the OFDM symbol period.

We will constrain our analysis without loss of generality to the first OFDM symbol, transmitted in the interval $[0, T_b]$. The discrete-time baseband signal $x[m]$ is then generated by taking the inverse discrete Fourier transform (IDFT) of data symbols:

$$x[m] = x(mT_s) = \frac{1}{N} \sum_{k=0}^{N-1} d_t[k] e^{j(2\pi k/N)m} \tag{10}$$

where $d_t[k] \triangleq d_t(k,0)$. The data symbols belong to an alphabet of elements, which depend on the modulation format adopted. For DVB-T standard [10] which employs M-QAM modulation, assuming a signal constellation with distance $2d$ between adjacent symbols, the data symbols belong to Λ set and have the same probability.

$$\Lambda = \left\{ d\left(2m - 1 - \sqrt{M}\right) + jd\left(2n - 1 - \sqrt{M}\right) \right\} \tag{11}$$

where $(m,n) = \left\{1,2,\ldots,\sqrt{M}\right\}$. Moreover, the M-QAM signal mean power is

$$P_A = \frac{2(M-1)}{3} d^2 \tag{12}$$

Due to the Central Limit Theorem, a baseband OFDM signal (when N is large) in (10) can be modeled as a complex Gaussian process with Rayleigh envelope distribution. Hence, by using the Bussgang theorem, amplified baseband signal $y(t)$ can be expressed as [5]

$$y(t) = \kappa_0 x(t) + n_d(t) \tag{13}$$

where the complex coefficient κ_0 and the autocorrelation function $R_{n_d}(\tau)$ of nonlinear distortion noise $n_d(t)$ depend on the AM/AM and AM/PM curves of the HPA and on the back-off to the HPA [6]. The input back-off (IBO) is defined as $IBO = A_s^2/\sigma_x^2$, being A_s^2 the input saturating power of the HPA and σ_x^2 is the mean power of the input signal. The OBO is defined as the ratio between the maximum and the mean output power.

The complex κ_0 attenuation coefficient of the useful component is given by [5]

$$\kappa_0 = \frac{E\left\{y(t)x^*(t)\right\}}{\sigma_x^2} \tag{14}$$

where $E\{\cdot\}$ denotes the statistical expectation operator. It is possible to prove for any nonlinear distortion $f(r)$ function that expression (3), the complex attenuation coefficient κ_0 is obtained by [6]

$$\kappa_0 = \frac{1}{\sigma_x^2} \int_0^\infty f(r) r\, p(r) dr \tag{15}$$

where $p(r)$ represents the Rayleigh probability density function (pdf) of the input envelope. In order to obtain κ_0 and autocorrelation function and hence the power spectral density (PSD) of the nonlinear noise process $R_{n_d}(\tau)$, we use orthogonal polynomial modeling of HPA in (6). It can be easily shown from (2) and (6) that the complex nonlinear distortion function of the HPA is represented by a polynomial

$$f(r) = \sum_{k=0}^{K} \alpha_{2k+1} \sum_{m=0}^{k} \frac{(-1)^{k-m}}{\sigma_x^{2m+1}} \frac{\sqrt{k+1}}{(m+1)!} \binom{k}{m} r^{2m+1} \tag{16}$$

where $(K+1)$ is the number of complex-valued coefficients of $f(\cdot)$. The coefficient κ_0 in (15) can be easily calculated by closed form expression,

$$\kappa_0 = \frac{1}{\sigma_x} \sum_{k=0}^{K} \alpha_{2k+1} \sum_{m=0}^{k} (-1)^{k-m} \frac{\sqrt{k+1}}{(m+1)!} \binom{k}{m} \Gamma(m+2) \tag{17}$$

where $\Gamma(\cdot)$ presents the Gamma function. It can be shown that when N is very large, as in DVB-T, the nonlinear distortion noise treat as an additive Gaussian process [6] and its PSD is determined from

$$S_{n_d}(f) = |\kappa_0|^2 S_x(f) - S_y(f) \tag{18}$$

Consequently, we have a complete characterization of both the effects (linear gain and nonlinear distortion noise) produced by the HPA.

The complex signal received through the AWGN channel represented in Fig. 1 is expressed by:

$$\begin{aligned} z(t) &= y(t) + n_c(t) \\ &= \kappa_0 x(t) + n_d(t) + n_c(t) \end{aligned} \tag{19}$$

where $n_c(t)$ is a complex zero mean AWGN with one-sided PSD N_0. The signal $z(t)$ is successively sampled at the rate $1/T_s$, thus the symbols received on the k th subcarrier are expressed, after the discrete Fourier transform (DFT) processing, by

$$\begin{aligned} d_r[k] &= \sum_{m=0}^{N-1} z[m] e^{-j(2\pi m/N)k} \\ &= \kappa_0 d_t[k] + N_d[k] + N_c[k] \end{aligned} \tag{20}$$

The Gaussian noise term $N_c[k]$ obtained by DFT of the thermal Gaussian noise of the receiver has zero mean and variance $\sigma_n^2[k] = \sigma_n^2 = N_0/2$ which is equal in all subcarriers and the term $N_d[k]$ obtained by DFT of the nonlinear distortion noise of HPA at transmitter has zero mean and variance $\sigma_d^2[k]$. It is easy to show, the effect of the nonlinear noise on each OFDM subcarriers is different [6].

In the next sections we will develop a suitable model for the decision variable which will allow an easy performance evaluation in the presence of a nonlinear channel.

3. Analytical BER Performance

In this section, we evaluate the BER performance for the OFDM systems in the presence of nonlinear HPA. In general, the symbol error rate (SER) performance for a M-QAM-OFDM system in AWGN channel is defined as the average of SER performances over all active subcarriers, i.e.

$$SER = \frac{1}{N_a}\sum_{k=1}^{N_a}(SER)_k \tag{21}$$

where N_a represents the number of active subcarriers used to transmit information within the total number N of subcarriers. The theoretical solution to the rectangular M-QAM SER performance is easily determined from [9]

$$SER = 1-\left(1-P_{\sqrt{M}}\right)^2 \tag{22}$$

where $P_{\sqrt{M}}$ are the probability of error of PAM with $\sqrt{M}$ signal points. The decision variable $r[k]$ the k th subcarrier is then obtained by compensating for the attenuation and rotation introduced by the HPA (i.e. κ_0) and can be expressed by

$$\begin{aligned} r[k] &= \left(\frac{1}{\kappa_0}\right)d_r[k] \\ &= d_t[k] + \left(\frac{1}{\kappa_0}\right)\left(N_d[k]+N_c[k]\right) \end{aligned} \tag{23}$$

Defining, $s[k] \triangleq d_t[k]$ as transmitted symbol over the k th subcarrier and $n[k] \triangleq (1/\kappa_0)\left(N_d[k]+N_c[k]\right)$ as the total noise samples, the scaled decision variable $r[k]$ in (23) can be expressed by

$$r[k] = s[k] + n[k] \tag{24}$$

Since the nonlinear and thermal noises are mutually independent, the pdf of the nonlinear-plus-thermal noise is also Gaussian and variance of total noise samples $n[k]$ can be easily evaluated as

$$\sigma_T^2[k] = \left|\frac{1}{\kappa_0}\right|^2\left(\sigma_n^2 + \sigma_d^2[k]\right) \tag{25}$$

To obtain the BER performance, a link between the signal to noise ratio (SNR) per bit $\gamma \triangleq E_b/N_0$ and σ_n^2 is needed. The SNR at the receiver for the k th subcarrier, after the FFT processing, is generally defined as the power ratio between the received signal and the thermal noise, Then SNR per bit can be expressed by

$$\gamma = \frac{1}{N_a \log_2^M}\frac{\sigma_y^2}{\sigma_n^2} \tag{26}$$

where σ_y^2 is the mean power of the HPA output signal. Assuming a signal constellation with distance $2d$ between adjacent symbols, the symbol error probability $P_{\sqrt{M}}$, can be expressed by [9]

$$P_{\sqrt{M}} = 2\left(1-\frac{1}{\sqrt{M}}\right)Q\left(\frac{d}{\sigma_T}\right) \tag{27}$$

Therefore, according to (22) SER for the k th subcarrier is can expressed by

$$(SER)_k = 1-\left(1-2\left(1-\frac{1}{\sqrt{M}}\right)Q\left(\frac{d}{\sigma_T[k]}\right)\right)^2 \tag{28}$$

For the HPA input signal with the mean power $\sigma_x^2 = N_a P_A$ in the M-QAM-OFDM system, it is easy to show

$$d^2 = \frac{\sigma_x^2}{N_a}\frac{3}{2(M-1)} \tag{29}$$

Finally, from (23) and assuming a Gray-coded signal set, BER can be written as

$$BER = \frac{1}{\log_2^M}\left(\frac{1}{N_a}\sum_{k=1}^{N_a}(SER)_k\right) \tag{30}$$

4. Numerical Results

In order to confirm of the proposed analytical approach, the BER performance comparison between analytical results and computer simulation is demonstrated in this section.

The OFDM signal for simulations is similar to the DVB-T standard [10], 1705 active subcarriers makes use of the 2K-Mode with 4,16 and 64-QAM modulations for each subcarriers. To obtain the following numerical results, we have considered the Bessel series expansion HPA model described in [6]. For each IBO values, after extracting the orthogonal polynomial coefficients of the HPA with an order $K = 12$, it is easily possible to to predict spectral regrowth present at the HPA output by (8) and to obtain complex attenuation coefficient value in (17). Consequently, PSD of nonlinear distortion noise is determined from (18).

Fig. 2 shows the theoretical PSD results and simulations at the output of the HPA for different OBO values. In the simulations, the signal PSD is estimated by means of periodogram, as the average of the PSD, computed by DFT, of the interpolated (by four) signal in each OFDM symbol interval. The PSD estimated by simulations prove the validity of the theoretical approach adopted in this work.

The complex attenuation coefficient values obtained through analytical and simulation methods for different IBO are listed in Table 1. A perfect agreement exists between the values computed in (17) and the values derived by simulation in (14).

Fig. 3 compares the analytical and the simulated BER performances for the M-QAM-OFDM system with different OBO values as well as the ideal HPA case. The comparison between the theoretical curves and the simulated points confirm the correctness of the analytical approach. However, the computer calculation time to obtain the OFDM system BER performance has been dramatically reduced.

Table 1: List of the complex attenuation coefficient values obtained through analytical and simulation methods.

Back-Off		The complex attenuation coefficient κ_0	
Input	Output	Analytical	Simulation
1 dB	1.9 dB	0.7885-j0.3406	0.7891-j0.3401
4 dB	3.3 dB	1.0138-j0.2840	1.0116-j0.2852
7 dB	5.4 dB	1.1837-j0.1563	1.1833-j0.1573
10 dB	8.0 dB	1.2596-j0.0562	1.2594-j0.0559
13 dB	10.9 dB	1.2778-j0.0185	1.2777-j0.0186

5. Conclusion

In this paper, we have developed a new analytical approach for description of the effects of nonlinear distortion on the performance of an OFDM system. By modeling nonlinear distortion function of HPA by orthogonal polynomials, it is easily possible to determined the output correlation function and hence the PSD of the nonlinear noise. These results have been used to propose the analytical performance evaluation of OFDM systems in presence of nonlinear HPA in AWGN channels. In contrast to previous work using Bessel series expansion, the analytical evaluation proposed here is easy and accurate. The accuracy of our formulas was verified by computer simulations.

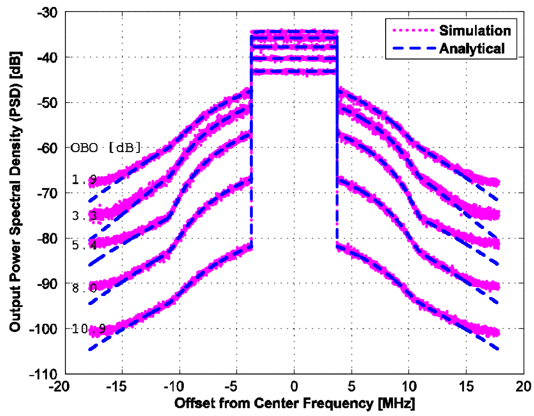

Fig. 2 Comparison between analytical and simulation results for PSD of the OFDM signal, at the output of HPA for different OBO values.

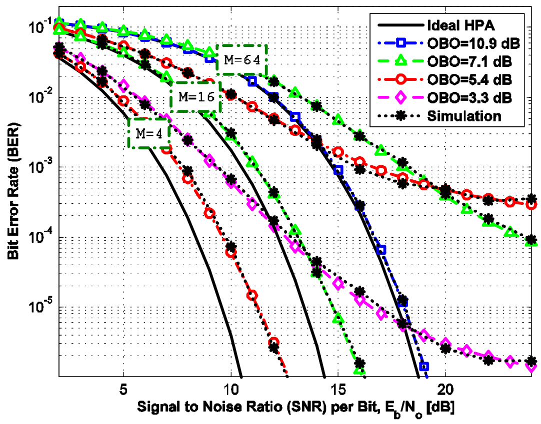

Fig. 3 Comparisons between analytical and simulation BER results for M-QAM-OFDM system with and without nonlinear HPA.

References

[1] R. V. Nee and R. Prasad, OFDM for wireless multimedia communications. Boston: Artech House, 2000.
[2] G. Santella and F. Mazzenga, "A hybrid analytical - simulation procedure for performance evaluation in M-QAM-OFDM schemes in presence of nonlinear.
[3] E. Costa, M. Midrio, and S. Pupolin, "Impact of amplifier nonlinearities on OFDM transmission system performance," IEEE Commun. Lett., vol. 3, pp. 37–39, Feb. 1999.
[4] D. Dardari, V. Tralli, and A. Vaccari, "A theoretical characterization of nonlinear distortion effects in OFDM systems," IEEE Trans. Commun.,vol. 48, pp. 1755–1764, Oct. 2000.

[5] P. Banelli and S. Cacopardi, "Theoretical analysis and performance of OFDM signals in nonlinear AWGN channels," IEEE Trans. Commun.,vol. 48, pp. 430–441, Mar. 2000.
[6] P. Banelli, G. Baruffa, and S. Cacopardi, "Effects of HPA non Linearity on frequency multiplexed OFDM signals," IEEE Trans. Broadcast., vol. 47, no. 2, pp. 123–136, June 2001.
[7] R. Raich, and G. T. Zhou, "Orthogonal polynomials for complex Gaussian processes," IEEE Trans. Signal Processing, vol. 52, pp. 2788-2797, Oct. 2004.
[8] R. Raich, and G. T. Zhou, "Theory and applications of orthogonal polynomials for Gaussian input," in Proc. IEEE Workshop on Statistical Signal Processing, pp. 97-100, Sept. 2003.
[9] J. G. Proakis, Digital Communications, 4th ed. New York: McGraw-Hill, 2001.
[10] ETSI EN 300 744 V1.4.1, Digital Video Broadcasting (DVB); Frame structure, channel coding and modulation for digital terrestrial television, Jan. 2001.

Hassan Zareian was born in Tehran, Iran, on February 26, 1974. He received the B.S. degree in electrical and the M.S. degree in electrical engineering, with emphasis on communication theory from the Iran University of Science and Technology (IUST), in 1997 and 1999, respectively. Since April 2002, he has been pursuing the Ph.D. degree in electrical engineering at IUST. His research interests include nonlinear distortions, broadcasting, and wireless communications.

Vahid Tabataba Vakili received the B.S. degree from Sharif University of Technology, Tehran, Iran, in 1970, the M.S. degree from the University of Manchester, Manchester, U.K., in 1973, and the Ph.D. degree from the University of Bradford, Bradford, U.K., in 1977, all in electrical engineering. In 1985, he joined the Department of Electrical Engineering, Iran University of Science and Technology, Tehran. In 1997, he was promoted to Associate Professor. He has served as the Head of the Communications Engineering Department and as the Head of postgraduate studies. His research interests are in the areas of mobile cellular systems, interference cancellation for CDMA systems, and space–time processing and coding.