A New Wavelet Based Method for Denoising of Biological Signals

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Summary
Wavelet shrinkage denoising methods are widely used for estimation of biological signals from noisy environment. The popular Hard and Soft thresholding filters are commonly used in these methods. In this paper shrinkage method based on a New Thresholding filter for denoising of biological signals is proposed. The efficacy of this filter is evaluated by applying this filter for denoising of ECG signal contaminated with additive Gaussian noise. The performance of this filter is compared with that of Hard and Soft thresholding filters using Mean Square Error (MSE) and Signal to Noise ratio (SNR). Experiments revealed that the New Thresholding filter is significantly more efficient than Hard and Soft filters in denoising the signals. It embodies the features of both Hard and Soft filters. Different qualities of denoising are obtained by varying the parameters of this filter.
Key words:
ECG, Wavelet Transform, Wavelet thresholding, Wavelet shrinkage, Denoising

1. Introduction

Nowadays signals or data are collected at ever-increasing pace by using sensors or computers or instruments. During data acquisition or transmission it is contaminated with noise. Denoising is an important problem that must be addressed before carrying out further analysis of signal or data. This is applicable to biological signals also. The random noises uncorrelated with biological signals can be approximated by additive white Gaussian noise. The effect of denoising of biological signals has direct influence on the sequential job such as malfunction analysis, diagnosis and recognition.

Several methods have been proposed for denoising the signals. Wavelet transform has been proved to be a successful tool for analysis of biological signals because of its good localization properties in time and frequency domain [1][2]. Different wavelet based methods are used for denoising biological signals. Methods based on shrinkage of wavelet coefficients are very popular for estimation of biological signals [3][4][5][6][7]. In these methods noisy biological signal is decomposed into wavelet coefficients by applying wavelet transform. After fixing the threshold using a thresholding rule the coefficients are filtered by using a thresholding filter. In this paper Hypothesis Testing thresholding rule [6] is considered. Denoised signal estimate is obtained by the inverse wavelet transform.

In this paper a New Thresholding filter shrinkage denoising method is proposed. Brief introduction to wavelet shrinkage denoising, Hypothesis Testing thresholding rule, Hard, Soft thresholding filters is presented. The performance of the proposed New filter is analyzed and compared with that of Hard and Soft thresholding filters by using ECG signals contaminated with varying levels of additive white Gaussian noise. MSE and SNR are used as criteria for testing the quality of denoising.

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2. Denoising

2.1. Wavelet Shrinkage Denoising

The noise present in the signal can be removed by applying the wavelet shrinkage denoising method while preserving the signal characteristics, regardless of its frequency content. The algorithm for denoising of signals using wavelet shrinkage method is given below [8]

a. Apply wavelet transform to the noisy signal $X(t)$ and obtain wavelet coefficient matrix w_X of $X(t)$.

b. Find the threshold λ using a thresholding rule. Modify the wavelet coefficients by using a thresholding filter selected and obtain estimate $w_{X\lambda}$ of wavelet coefficients of $X(t)$.

c. Apply inverse wavelet transform to the filtered coefficients and obtain the denoised signal estimate $\hat{X}(t)$.

In this denoising method we have to select a wavelet for forward and inverse transformations. Wavelet Symmlet 8[1] is considered here. The denoising methods differ in the choice of thresholding rules to determine the threshold λ and thresholding filters that determine how the threshold is applied.

2.2 Hypothesis Testing

The thresholding rules determine the threshold levels. In this paper threshold is determined by considering Hypothesis Testing rule [6]. The threshold estimation in this method is independent of thresholding filter used. It calculates level dependant thresholds after performing wavelet transformation on the signal.

Calculation of threshold

Let the wavelet coefficients ω are N_s in number at a particular level and assume that they are normally distributed. Find α -critical value, $v_{N_s}^{\alpha} = \left\{\phi^{-1}\left[\left((1-\alpha)^{1/N_s}+1\right)/2\right]\right\}^2$ where α is error probability parameter. $\phi(\,)$ is cumulative distribution function of standard normal density. Then find the largest of the squared wavelet coefficients at that level, denoted by $\omega_{(N_s)}^2$ and compare it to the above value $v_{N_s}^{\alpha}$. If $\omega_{(N_s)}^2/\hat{\sigma}^2 > v_{N_s}^{\alpha}$ where $\hat{\sigma}$ is an estimate of the standard deviation of noise, $\omega_{(N_s)}$ is retained as signal. Next repeat the process with the square of second largest (in absolute value) wavelet coefficient $\omega_{(N_s-1)}^2$. If $\omega_{(N_s-1)}^2 / \hat{\sigma}^2 > v_{N_s-1}^{\alpha}$, the procedure continues until at some point the p[th] largest (in absolute value) coefficient satisfies $\omega_{(p)}^2 / \hat{\sigma}^2 \le v_p^{\alpha}$. The threshold at that level is then set as $\lambda = \left|\omega_{(p)}\right|$. The recommended value for α is 0.05 [6].

2.3. Thresholding Filters

The noisy wavelet coefficients are filtered by using thresholding filters. The most commonly known Hard and and Soft filters are considered in this paper (Figs 1and 2).

Algorithm for Hard thresholding filter [9]

$$H(\omega,\lambda) = \omega \quad \text{for all } |\omega| > \lambda$$
$$= 0 \quad \text{otherwise}$$

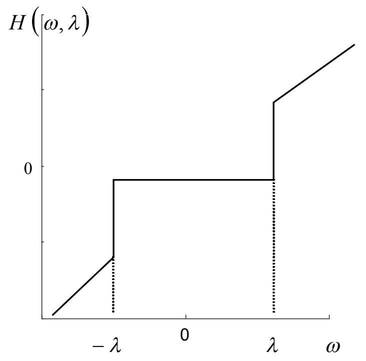

Fig. 1 Hard Thresholding Filter

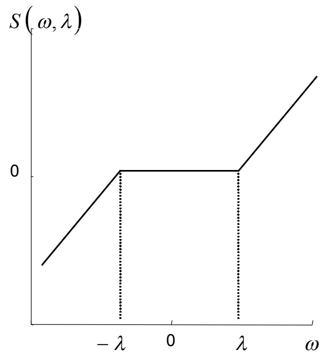

Fig. 2 Soft Thresholding Filter

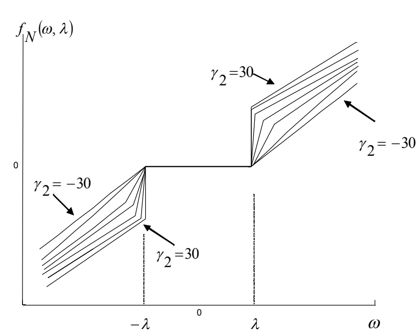

Fig. 3 New Thresholding Filter: $\gamma_1 = 0$

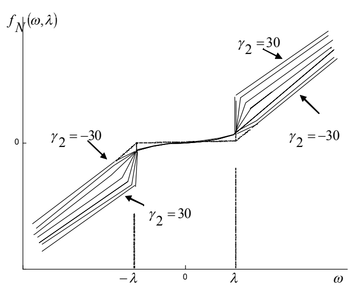

Fig. 4 New Thresholding Filter: $\gamma_1 = 1$

Soft thresholding filter is defined as given below [10]

$$S(\omega, \lambda) = \text{sgn}(\omega) \max(0, |\omega| - \lambda)$$

ω represents detail wavelet coefficients, λ represents the threshold

3. New Thresholding Filter

In this paper we propose a New Thresholding filter for filtering the noisy wavelet coefficients. The proposed New Thresholding filter (shown in Figs 3 and 4) is given as

$$f_N(\omega, \lambda) = \text{sgn}(\omega) \frac{\gamma_1 \omega^2}{5\lambda} \quad for \quad |\omega| \le \lambda \; ; \; 0 \le \gamma_1 \le 1$$

$$= \text{sgn}(\omega) \left[\frac{|\omega|^{\gamma_2 + 1} + (|\omega| - \lambda)^{\gamma_2 + 1}}{|\omega|^{\gamma_2} + (|\omega| - \lambda)^{\gamma_2}} \right] \quad for \quad |\omega| > \lambda \; ; \; \gamma_2 : any \;\; integer$$

γ_1, γ_2 are variable parameters

The behavior of the filter can be varied by varying the parameters of the filter. When $|\omega| > \lambda$ for each input wavelet coefficient this New filter performs contra harmonic filtering operation on the outputs of Hard and Soft filters of that wavelet coefficient. Wavelet coefficients whose absolute values less than threshold are dominated very much by noise. For these input values this filter will give small percentage of these values as output when $\gamma_1 \neq 0$ (Fig. 4) and zero as output when $\gamma_1 = 0$ (Fig. 3). At the threshold when $\gamma_1 = 1$ maximum of twenty percentage of threshold will be obtained as output (Fig. 4). If the value of γ_2 increases in positive direction, keeping $\gamma_1 = 0$ the behavior of the filter approaches that of Hard thresholding filter. Similarly if the value of γ_2 increases in negative direction, keeping $\gamma_1 = 0$ it approaches that of Soft thresholding filter. The proposed filter contains the features of both Hard and Soft thresholding functions. The values of γ_2 at which the New filter behaves as Hard and Soft filters depend upon the signal we considered and these values can be found from experiment. The performance of this filter will be improved if the value of γ_2 increases keeping γ_1 constant. By carefully selecting the values for γ_1 and γ_2 we can get better denoising performance. This filter shows best performance in denoising the signals when $\gamma_1 = 1$ compared to Hard and Soft filters.

4. Simulation Results and Discussion

The results obtained on denoising of ECG signals using Hard, Soft and New Thresholding filters are presented in this section. ECG signals are taken from American Heart Association (AHA) ECG database. ECG signals of sample size 2048 contaminated with additive white Gaussian noise of different values of standard deviation (σ) are simulated. Wavelet decomposition of ECG signal is made up to three levels using Symmlet 8 [1][11]. After fixing the threshold using Hypothesis Testing rule [6] the wavelet coefficients are filtered by using a thresholding filter. The inverse wavelet transform is applied on the resultant coefficients and denoised signal estimate is obtained.

MSE and SNR are used as measure of denoising. They are calculated as given below

$$MSE = (1/n) \left\{ \sum_{i=1}^{n} \left(X(i) - \hat{X}(i) \right)^2 \right\}$$

$$SNR = 10 \log_{10} \left[\sum_{i=1}^{n} X(i)^2 \Big/ \sum_{i=1}^{n} \left(X(i) - \hat{X}(i) \right)^2 \right]$$

n represents no. of samples, $X(i)$ original signal data, $\hat{X}(i)$ denoised signal data, SNR is in dB.

Table 1: Denoising results of ECG X8209 using Hard and Soft thresholding filters

	σ =10		σ =20		σ =30	
	MSE	SNR	MSE	SNR	MSE	SNR
NoisySignal	99.48	20.37	400.30	14.35	898.84	10.82
Hard	44.87	23.85	119.74	19.59	225.02	16.86
Soft	101.71	20.30	268.86	16.08	466.22	13.69

Table 2: Denoising results of ECG X8209 using New Thresholding filter, σ =10

γ_2		-30	-20	-10	0	10	20	30
$\gamma_1 = 0$	MSE	**99.39**	97.75	93.59	58.00	46.23	45.49	**45.16**
	SNR	**20.40**	20.47	20.66	22.74	23.72	23.79	**23.82**
$\gamma_1 = 0.5$	MSE	96.51	94.43	89.47	54.26	42.72	41.26	41.48
	SNR	20.53	20.62	20.86	23.03	24.06	24.22	24.19
$\gamma_1 = 1$	MSE	93.65	90.35	86.69	51.09	39.35	38.70	***38.46***
	SNR	20.66	20.82	21.00	23.29	24.42	24.49	***24.52***

Table 3: Denoising results of ECG X8209 using New Thresholding filter, σ =20

γ_2		-30	-20	-10	0	10	20	30
$\gamma_1 = 0$	MSE	**263.68**	262.56	248.89	152.55	122.21	120.98	**119.39**
	SNR	**16.17**	16.18	16.42	18.54	19.51	19.55	**19.61**
$\gamma_1 = 0.5$	MSE	255.97	250.91	235.58	139.93	111.61	110.90	109.28
	SNR	16.29	16.38	16.66	18.92	19.90	19.93	19.91
$\gamma_1 = 1$	MSE	243.75	243.37	228.04	132.27	105.01	103.11	***101.76***
	SNR	16.51	16.51	16.80	19.16	20.16	20.24	***20.30***

Table 4: Denoising results of ECG X8209 using New Thresholding filter, σ =30

γ_2		-30	-20	-10	0	10	20	30
$\gamma_1 = 0$	MSE	468.35	**466.76**	442.76	272.04	227.29	227.41	**227.76**
	SNR	13.67	**13.69**	13.92	16.04	16.81	16.81	**16.81**
$\gamma_1 = 0.5$	MSE	455.14	446.67	426.87	257.72	209.90	207.31	208.72
	SNR	13.80	13.88	14.08	16.27	17.16	17.21	17.18
$\gamma_1 = 1$	MSE	438.87	434.87	410.43	240.34	197.65	195.59	***194.57***
	SNR	13.95	13.99	14.24	16.57	17.42	17.47	***17.48***

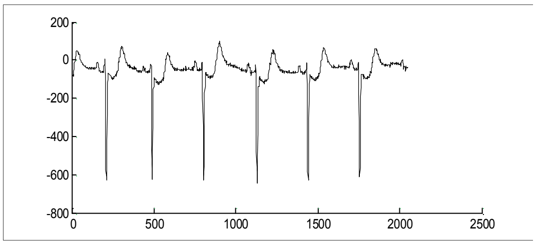

Fig. 5 Original ECG

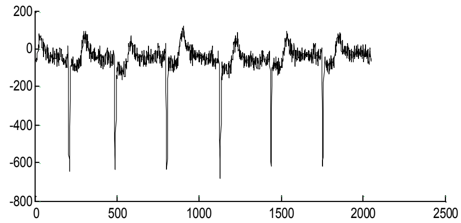

Fig. 6 Noisy ECG

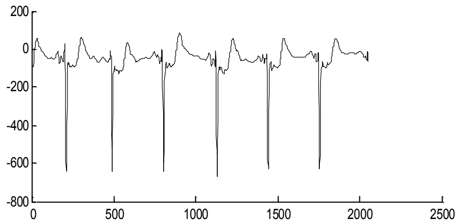

Fig. 7 Denoised ECG using Hard Thresholding Filter

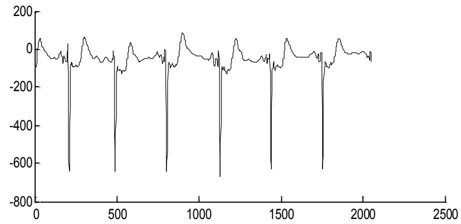

Fig. 8 Denoised ECG using Soft Thresholding Filter

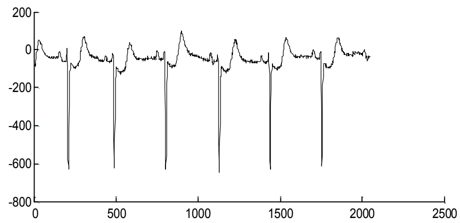

Fig. 9 Denoised ECG using New Thresholding Filter $\gamma_1 = 1$, $\gamma_2 = 10$

The simulation experiment is repeated 100 times and average values of MSE and SNR are found. These experiments are conducted on 50 numbers of ECG signals and found that the results are same. The simulation is implemented in MATLAB environment. Table 1 shows the denoising results of ECG signal X8209 obtained using Hard and Soft thresholding filters for σ=10, 20 and 30. The original and denoised signals X8209 obtained using Hard, Soft and New Thresholding filters for σ=20 are shown in Figs. 5-9. Results of denoising of ECG X8209 for different parameters of New Thresholding filter are reported in tables 2-4. For a noisy signal of σ =10, MSE of 44.87 and SNR of 23.85 are obtained on denoising using Hard thresholding filter and MSE of 101.71 and SNR of 20.30 with Soft thresholding filter (Table1). For New thresholding filter for σ =10, MSE of 45.16 and SNR of 23.82 are found when $\gamma_1 = 0$ and $\gamma_2 = 30$ (Table2). This indicates the New filter behaves as Hard thresholding filter at these values of γ_1 and γ_2 for σ =10. MSE of 99.39 and SNR of 20.40 for σ =10 are obtained for New filter when $\gamma_1 = 0$ and $\gamma_2 = -30$ (Table 2). It shows its working is close to Soft thresholding filter at these values of γ_1 and γ_2 for σ =10. The same behavior of New thresholding filter is noticed for σ =20 and 30. From the results it is observed that for ECG signals keeping $\gamma_1 = 0$, if the values of γ_2 are increased in the positive direction the behavior of New Thresholding filter approaches that of Hard Thresholding filter when $\gamma_2 = 30$ for σ =10, 20 and 30 (Tables2-4). In the negative direction it approaches Soft Thresholding filter when $\gamma_2 = -30$ for σ =10 and 20 (Tables 2 and 3) and $\gamma_2 = -20$ for σ =30 (Table 4). It comprises the features of both Hard and Soft thresholding filters. Different qualities of denoising are obtained for different values of γ_1 and γ_2. The performance of this filter is improved if we increase the values of γ_2 keeping γ_1 constant. It is noticed that when $\gamma_1 \neq 0$ the denoising performance of the New filter superior to Hard and Soft filters is obtained. It is observed that when $\gamma_1 = 1$ the New filter gives the best performance in denoising the ECG signals (values shown italicized in Tables 2-4). The MSE values obtained using Hard, Soft filters (Table 1) and New Thresholding filter (italicized values in Tables 2-4) are plotted against σ in Fig. 10**.** From this figure it is observed that New filter performs superior to Hard and Soft filters.

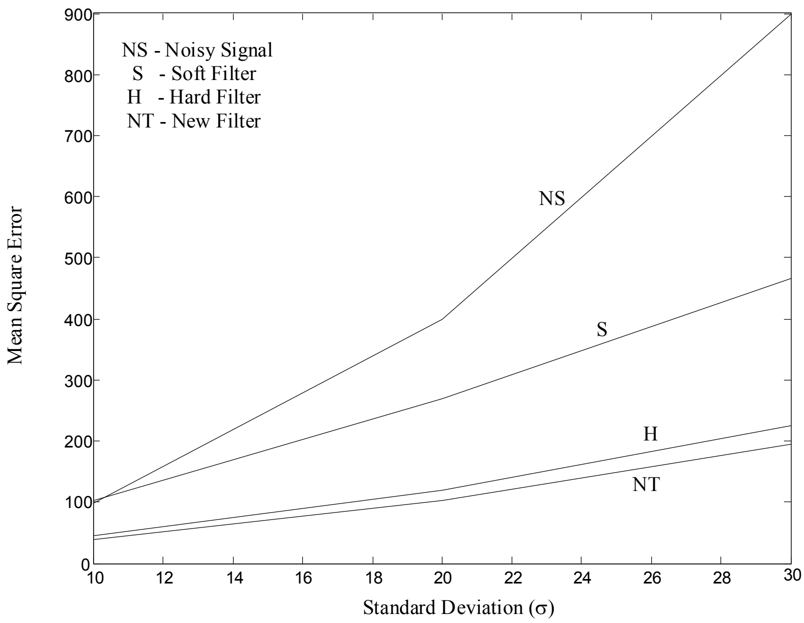

Fig. 10 The MSE performance of Hard, Soft and New Thresholding filters

5. Conclusion

In this paper a New Thresholding filter for wavelet shrinkage estimation of biological signals is proposed. We tested the performance of this filter by using ECG signals. From the simulation results it is noted that the filter has the following features:

a. By varying the parameters γ_1 and γ_2 of the filter, different qualities of denoising can be obtained.
b. Keeping $\gamma_1 = 0$ if the values of γ_2 are increased in the positive direction the behavior of the filter approaches that of Hard thresholding filter and in the negative direction it approaches Soft thresholding filter. It comprises the features of both Hard and Soft filters.
c. If we increase the values of γ_2 keeping γ_1 constant the quality of denoising is improved.
d. It gives better performance than Hard and Soft filters for values of γ_1 other than zero.

This filter can also be used with other thresholding rules like Visu Shrink, False Discovery Rate for estimation of biological signals.

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