Zero Vectors Combinatorial Code Family for Spectral Amplitude Coding (SAC-OCDMA)

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Summary
Although multiple access interference (MAI) problems existing in optical code division multiple access (OCDMA) systems can be solved by electrical subtraction, phase induced intensity noise (PIIN) resulting from the phase incoherent of overlapping still remains and completely deteriorates the system performance. It is therefore desirable to alleviate MAI and PIIN effects in designing OCDMA system. We propose a new method to construct codes with zero in-phase cross correlation namely Zero Vector Combinatorial Code (ZVCC) based on combination of specific vectors with combinatorial theories. Simplicity of code construction, flexibility of choosing the number of users and weights, make our proposed system strong candidate for future networks. In addition, transmitter side and receiver side are proposed. Detailed examples are given on how to pick the best code among different options based on specific parameters. The results show that the proposed code is effective to reduce the MAI, PIIN and cost, while maintaining a good signal to noise ratio and bit error probability. A comparison study between the proposed code and the existing codes such as Hadamard and MDW has been carried out.

Key words:
Optical Code Division Multiple Access (OCDMA), Multiple Access Interference (MAI), Zero Vector Combinatorial Code (ZVCC), Optical Communication, Direct Recovery Scheme (DRS)

1. Introduction

A major challenge in devising optical communication systems is to fully exploit the vast bandwidth provided by optical fiber channels. The successful of long-span fiber optic communication systems has shifted the focus of optical network to shorter-span metropolitan and local area domains [1]. There is not much difference between multiple access and multiplexing techniques. Multiple access allows communication media to be shared between different users. It represents one of the most essential functions of access networks, while multiplexing is combination of signals into single transmission signal.

The three basic multiple access techniques are Wave Division Multiple Access (WDMA), Time Division Multiple Access (TDMA) and Code Division Multiple Access (CDMA). TDMA is a technology that allows multiple users to access a channel by allocating time slots to each user within each channel. WDMA is a technology allowing multiple users to access a channel by allocating wavelength or frequency to each user within each channel. TDMA and WDMA have a limited bandwidth for every user.

Optical Code Division Multiple Accesses (OCDMA) [1-4] has gained more attention in recent years due to its success in wireless communication systems. This is because it allows multiple users to access the channel simultaneously with low latency through assignment of unique code sequence [3]. An OCDMA system suffers from different noises such as shot noise, thermal noise, dark current and multiple access interference (MAI) from other users. Among these noises, MAI is considered as the dominate source. Therefore, intelligent design of code sequence is important to reduce the contribution of MAI to the total received power [4]. Although MAI can be cancelled by balance detection scheme, a phase induced intensity noise (PIIN) arising from spontaneous emission of broad band source, inherently remains. It is then important to design codeword such that the effect of MAI and PIIN of the total received power is reduced. In OCDMA systems, minimization of cross correlation to a very small value is an advantage. Systems using Codes with zero cross correlation have less noise, which results in reducing the hardware complexity. In this work, Zero Vector Combinatorial codes (ZVCC) is proposed.

This paper is organized as it follows. In section II, optical spectral code division multiple access is reviewed. In section III, codes construction and development of all theoretical studies is presented. The advantages of ZVCC compared to other OSCDMA codes, is reported in section IV. In section V, the performance analysis of the new proposed system is done, and finally, conclusions are given in section VI.
2. Optical Spectrum Code Division Multiple Access (OSCDMA)

Optical CDMA systems can be divided into major categories. The first is coherent systems, where knowledge of the phase and amplitude is needed in order to achieve successful detection. The second is incoherent systems, where the performance of the system depends mainly on the amplitude. OSCDMA is an incoherent broadband light source which contains N users with optical transmitters and receivers as shown in Fig. 1 [5][6].

OSCDMA system contains encoders and decoders which can be designed by using any kind of optical filtering technology. Spectral Amplitude Coding (SAC) was introduced to eliminate the MAI existing in conventional OCDMA systems. SAC systems use complementary detection technique to recover the original signals. This balance receiver is used as a part of the receiver which filters the incoming signals. For unmatched transmitters half of transmitter spectral components will match the direct filter and the other half will match the complementary filter. The output of the balance receiver represents the difference between the two parts, with unmatched channels being cancelled, while the matched channel is demodulated.

It is possible to design codes with the full orthogonality in the incoherent spectral intensity OCDMA system, since there is a subtraction between two photo detectors. In this system, the signature sequence is spread across different wavelength with each chip occupying different wavelength [5][6]. The advantage of OSCDMA is it does not need synchronization as the chip spreads in frequency and not in time.

3. CODE CONSTRUCTION

Let \( V \) be a vector space of dimension \( n \). The set \( \{v_1, v_2, ..., v_n\} \) is a basis of \( V \) if the set spans \( \{v_i\} \) where \( i=1, ..., n \) are linearly independent and if any element of \( V \) can be written as a combination of the \( v_i, i=1, 2, ..., n \).

Let \( R \) denotes the field of real numbers. The space of all \( n \)-tuples of real numbers forms an \( n \)-dimensional vector space over \( R \) denoted by \( R^n \). An element \( x \) of \( R^n \) can be written as a column vector:

\[
x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}
\]

(1)

Let \( e_i \) denote the element of \( R^n \) having ‘0’ at all lines except the \( i^{th} \) line.

\[
\{e_1, e_2, ..., e_n\} \text{ is a basis of } R^n \text{ called the standard basis.}
\]

For \( n=2 \)

\[
e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\]

(2)

Let \( x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \) an element of \( R^2 \)

\[
x= \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ x_2 \end{pmatrix}
\]

(3)

Therefore, we have

\[
x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} x_1 \\ 1 \end{pmatrix} = x_1e_1 + x_2e_2
\]

(4)

For \( n=3 \)

\[
e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad e_3 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}
\]

(5)

Where \( \{e_1, e_2, e_3\} \) is a basis of \( R^3 \).

Fig. 1: OCDMA Network
Let \( x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \) an element of \( \mathbb{R}^3 \).

Therefore, we obtain
\[
\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = x_1 e_1 + x_2 e_2 + x_3 e_3
\]  
(6)

Let \( N \) be the number of users and \( L \) the code length. We can write the code in a matrix form corresponding to the code word of \( N \) users. We obtain \( N \times L \) matrix with two dimensions as shown in Fig.2.

Matrix \( (N \text{ users'} \times L \text{ length}) \)

\[
\begin{array}{c|ccccccc}
\text{User1#} & 1 & 0 & 0 & . & . & . & 0 \\
\text{User2#} & 0 & 0 & 0 & . & 1 & . & . \\
\vdots & . & . & 1 & . & . & . & . \\
\text{UserN#} & 0 & 1 & 0 & . & . & . & 0 \\
\end{array}
\]

Fig.2: Matrix of ZVCC

The construction of ZVCC is based on the following mentioned: to have (shortest) Zero Cross Correlation (ZCC), we have to have one ’1’ in each column. This means for ZCC any column is an element of the standard basis of \( \mathbb{R}^n \). Given \( N \) and the weight \( W \), we can generate all possibilities of ZVCC having length \( L = N \times W \) by getting all the permutations of the vectors \( e_1, e_2, \ldots, e_n \) with repetitions of each vector \( W \)-times

A. Elimination of redundant patterns

Permutation has been done for the first vector to produce patterns of these vectors. Symmetric patterns appeared for the same weight and number of users i.e., swapping between the locations of users’, the first user become second user or the last user and so on, while for asymmetric no such duplicate, so we have to find way to remove the symmetric patterns. Fig. 3 shows the flowchart step to construct code pattern for both cases.

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\[ C_{\text{pos}} = \frac{(W \times N)!}{(W! \times N!)} \]  

(8)  

where \( W \) represent the invert of \( W \) (i.e., the number of zero). By applying this equation, it is clear that it depends on the weight to get better Signal to Noise Ratio (SNR), nevertheless, its decrease the number of possibilities and the length remain the same. Let us investigate the following example.  

\[ N=2 \]
\[ W=2 \]
\[ L= N \times W = 2 \times 2 = 4. \]
\[ W = L-W = 4-2 = 2 \]

Therefore  

\[ C_{\text{pos}} = \frac{(W \times N)!}{(W! \times W!)} = \frac{(2 \times 2)!}{(2! \times 2!)} = 6 \]

We obtain the codes patterns possibilities as shown in (9), (10), (11), (12), (13), and (14).

\[
\begin{align*}
Hz_1 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \\
Hz_2 &= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \\
Hz_3 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \\
Hz_4 &= \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \\
Hz_5 &= \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \\
Hz_6 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}
\end{align*}
\]

(9)  

(10)  

(11)  

(12)  

(13)  

(14)  

From the above codes patterns possibilities, we observe that there is a symmetric between (9) and (12), (10) and (14), (11) and (13). Combination of any two symmetric equations produce one pattern, hence, when we swap the position of user1# and user2#, to reduce the numbers of symmetric patterns we use the following equation:

\[ N_c = \frac{(W \times N)!}{N \left( \frac{W!}{N!} \right)^N} \]  

(15)  

Where \( N_c \) is number of patterns after modification. For the example of \( (N=2, W=2) \) after applying Eq. (15) we obtain:

\[ N_c = \frac{(2 \times 2)!}{2 \left( \frac{2!}{2!} \right)^2} = 3 \]

Cpos became 3 instead of 6 after applying the new equation, hence, we obtain Hz1, Hz2, and Hz3.

### 4. COMPARISON AND EVALUATION

In recent year, many codes have been proposed such as modified double weight (MDW) code [5], modified frequency hopping (MFH) [10] and zero cross correlation (ZCC) [11]. All these codes suffer from various limitations one way or another, the code construction is complicated (e.g., MFH and ZCC) or the cross correlation is not ideal. However, the code construction not only depends on a cross correlation properties, the length plays an important role we have to consider as well. Long length is a disadvantage since the code is subject to either very wide band source or narrow filter bandwidths are required.

ZVCC has long length and this is considered as disadvantage, we have to make a tradeoff between the length and multiple access interference (MAI), because a MAI is a dominant source of the noises. In MFH code, although the code length is shorter compared to ZVCC, the cross correlation always equal to unity and this contributes to phase induce intensity noise (PIIN), while in ZVCC always the cross correlation equals to zero which eliminates the effect of PIIN. In table 1 ZVCC shows flexibility in terms of choosing the number of users and the weight due to the use of a novel method in code construction which generates many possibilities from a given number of users and weights.

<table>
<thead>
<tr>
<th>code</th>
<th>existence</th>
<th>weight</th>
<th>cross correlation</th>
<th>Code length</th>
</tr>
</thead>
<tbody>
<tr>
<td>MFH</td>
<td>K=Q^m</td>
<td>Q+1</td>
<td>( \lambda = 1 )</td>
<td>N= Q^m+Q</td>
</tr>
<tr>
<td>MDW</td>
<td>K=n</td>
<td>Even</td>
<td>( \lambda = 1 )</td>
<td>N=3n+8/3{\sin (Nii/3)}^2</td>
</tr>
<tr>
<td>ZCC</td>
<td>K=2^m</td>
<td>2^m-1</td>
<td>( \lambda = 0 )</td>
<td>C=2^m</td>
</tr>
<tr>
<td>Hadamard</td>
<td>K=2^m-1</td>
<td>2^m-1</td>
<td>( \lambda = 2^{m+2} )</td>
<td>N= 2^m</td>
</tr>
<tr>
<td>ZVCC</td>
<td>Any number</td>
<td>option</td>
<td>( \lambda = 0 )</td>
<td>L=WxN</td>
</tr>
</tbody>
</table>

| Table 1: SAC-OCDMA codes comparison |
5. DIRECT RECOVERY SCHEME

In OCDMA system, intelligent code design is desired to reduce the MAI contribution. Therefore, a receiver side which plays an important role should be addressed as well. In conventional SAC-OCDMA, balance detection scheme is use to differentiate between the wanted and the unwanted code sequences. Hence, in Direct Recovery Scheme (DRS), no subtractors are needed. At the receiver side, direct filters are using and this significantly eliminate the effect of MAI since only the wanted signal will be filtered out. DRS have been proposed as structure of transmitter and receiver for ZVCC code family.

Fig. 4, shows the procedure of encoding the signal in transmitter side then coupled to the fiber media, at the receiver side, the receiver signals are splitting into equally part followed by filter and photodiode, finally the signals are recovered by the intended users. Due to all these, phase induce intensity noise (PIIN) could be eliminated which provides a possible way of achieving greater performance.

6. CODE SELECTION FROM Cpos

Since we have large numbers of Cpos in code construction, in order to differentiate between these possibilities, simulation results are acquired by using a software tool named VPI transmission maker. For Hz1, Hz2, Hz3, Hz4, Hz5, and Hz6, the number of users is 2, the weight is 2, the length is 4 and Cpos is 6. After eliminates the symmetric patterns became 3. To choose one family among six families, a simulation work has been done respectively for each family started from Hz1 up to Hz6 using direct recovery scheme (transmitter part and receiver part) to reproduce the signal for intended user and the results can be show as follows:

Fig. 5: Eye diagram of ZVCC at 10 km

Fig. 6: Eye diagram of ZVCC at 50 km
In Fig.5, Fig.6, and Fig.7, a different fiber length has been chosen to observe the increasing length in system performance in terms of eye diagram patterns. In Figs 5, and Fig.6, ZVCC is adopted with the following parameters: length of fiber varies from 10 up to 50 and data rate is 2.5Gbps. It is shown that, the system improves and the effect of MAI is eliminated while in Fig 7 the effect of dispersion in optical fiber is unavoidable because the distance is quite long. The simulation results obtained from Hz2 and Hz3 is similar as compared to Hz 1 when using the same parameters, therefore, we can take any code randomly from Cpos. Further improvement could be achieved by increasing the data rate up to 10Gbps.

7. SYSTEM PERFORMANCE

In communication system maximum Signal to Noise Ratio (SNR) yields better Bit Error Rate (BER), these two completely affect the system performance. In SAC-OCDMA system good design of the code plays an important role by keeping up the value of interference from simultaneous users as small as possible. In ZVCC, in order to compute the SNR, we used the same method described in [7]. Since the value of cross correlation is zero we compensate this value in the SNR [7], the SNR of ZVCC can be expressed as follows

\[
\text{SNR} = \frac{P_e e^{BR}}{L} \left[ (N - l) + W^2 \right] + \frac{P_{sr} B^2}{2 \Delta V} \left[ (N - l) + W^2 \right]^2 \frac{4 K_B T_{\text{tr}} B}{R_L}
\]

(16)

BER = \( P_e = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{\text{SNR}}{8}} \right) \) (17)

Where L is the code length, W is the code weight, N the number of users, \( \Re \) is photodiode responsively, \( P_e \) is the effective power of a broad-band source at the receiver, \( e \) is an electronic charge, B is the electrical equivalent noise band-width of the receiver, \( K_B \) is the Boltizmann’s constant, \( T_{\text{tr}} \) the absolute temperature of receiver noise, \( R_L \) the load resistance and \( \Delta V \) the optical source bandwidth. \( \text{erfc} \) is a complementary error function express as follows

\[
\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-t^2} dt
\]

(18)

In literature reviews, there have been many codes proposed for SAC-OCDMA such as Hadamard [8] and modified double weight (MDW) code [9] and zero cross correlation (ZCC) [11]. However, our proposed code families have more flexibility of generating tremendous option of code patterns. In this work, the code performance has been done, the system parameters are: \( \Delta V = 3.75 \, \text{THz} \); \( B = 80 \, \text{MHz} \); \( T_{\text{tr}} = 300 \); \( R_L = 1030 \, \Omega \); bit-rate 155 Mbps; photodiode efficiency 0.6 and central wavelength 1550 nm.

In Fig.8 and Fig.9, a different weight values have been chosen to observe the increase of the weight on the performance. It is shown that, the performance of the system improves when the weight increases which increases the SNR. In Fig 9, the parameter taken: \( w = 6 \), it is clear that, the effect of MAI and PIIN is reduced when the weight increases regardless for the number of users.

![Fig.8. BER versus the number of simultaneous users](image_url)
8. Conclusion

In this work, construction methods for new code families with zero cross correlation for SAC-OCDMA system, ZVCC has been proposed. The properties and theoretical development of new family have been proved. New design for transmitter side and receiver side using direct filters to recovery and reproduce the original data namely direct recovery scheme (DRS) has been examined. The new transmitter has been analyzed using ZVCC taking into consideration the effect of noises such as thermal noise and shot noise. The study showed that, compared with a former code such as Hadamard and MDW better performance due to elimination of the effect of phase induce intensity noise.

Moreover, the way of constructing ZVCC is very flexibly in choosing the number of users and the weight. It has shown that, a lot of possibilities for code generation can be obtained and no difference between them in terms of system performance.

I. References