Robust Guaranteed Cost Satisfactory Fault-Tolerant Control with Regional Poles Constraints

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Summary
The problem of robust guaranteed cost satisfactory fault-tolerant control with regional poles constraints against actuator failures is investigated for a class of continuous-time systems with uncertainties which do not satisfy matching conditions. The state-feedback controller is designed to guarantee the closed-loop system satisfying the pre-specified regional pole index, H_∞ norm-bound constraint on disturbance attenuation and having the quadratic cost performance simultaneously, for all admissible value-bounded uncertainties and possible actuator failures. Thus, the resulting closed-loop system can provide satisfactory stability, transient property, H_∞ performance and cost performance despite of possible actuator faults. A simulative example shows the validity of the proposed method.

Key words:
fault-tolerant control, actuator failures, poles constraints, guaranteed cost control

1. Introduction

As science and technology develops, the reliability and security of complex systems becomes more important. Consequently, the problem of fault-tolerant control has received considerable attention in recent years (see e.g., [1~4] and the references therein). In actual implementation, systems are often required having certain transient characteristics even in tolerant faults cases, which can be described by regional closed-loop poles constraints (see e.g., [5~8]). However, most existing fault tolerant control results (see e.g., [9~11]) did seldom take such performance requirements into account.

Therefore, it is our motivation to investigate the robust H_∞ guaranteed cost satisfactory fault-tolerant control problem with poles constraints for a class of uncertain continuous-time systems subject to actuator failures. In view of possible actuator failure as well as value-bounded uncertainties existing in both the state and control input matrices, taking the transient property, robust behaviour on H_∞ performance and quadratic cost performance requirements into consideration, we first derive the existence conditions of robust H_∞ guaranteed cost satisfactory fault-tolerant state-feedback control law with regional poles constraints. Then by LMI technique, a convex optimization problem is formulated to find the corresponding controller. Furthermore, the consistency of the performance indices mentioned earlier is discussed for fault-tolerant control. Finally, simulative example is provided to illustrate the validity of the proposed method.

Notations: R^n and R^{n*m} denote, respectively, the n-dimensional Euclidean space and the set of n×m real matrices. The superscript 'T' denotes matrix transposition. I and 0, respectively, are unit matrix and zero-matrix of appropriate dimensions. $diag\{\cdots\}$ denotes a block-diagonal matrix. The notation $P>0$ (respective, $P\geq 0$ and $P<0$), for $P\in R^{n*n}$ means that the matrix P is real symmetric positive definite (respectively, nonnegative definite and negative definite).

2. Problem Formulation

Consider an uncertain continuous-time system described by the following state equation:

$$\begin{aligned}\dot{x}(t) &= (A+\Delta A)x(t)+(B+\Delta B)u^f(t)+D\omega(t)\\ z(t) &= Cx(t)\end{aligned} \tag{1}$$

where $x(t)\in R^n$ is the state vector, $u^f(t)\in R^p$ is the control input from the actuator that may be fault, $z(t)\in R^k$ is the controlled output, $\omega(t)\in R^q$ is the input disturbance and $\|\omega(t)\|_2\leq\beta$, A,B,C and D are known real constant matrices with appropriate dimensions denoting the nominal system(1), ΔA and ΔB are unknown matrices representing parameter uncertainties in the state matrix and input matrix, respectively.

For the simplicity of research and without loss of general, we suppose the states are available for state-feedback.

$$u(t)=Kx(t) \tag{2}$$

where $K\in R^{p*n}$ is the feedback gain matrix.

For the control input, the following failure model in [10] is adopted for this study:

$$u^f(t)=Mu(t) \tag{3}$$

$$M=diag\left[m_1,m_2,\cdots,m_p\right] \tag{4}$$

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where M denotes the actuator faults function matrix, $0 \le m_{il} \le m_i \le m_{iu}$, $m_{il} < 1$, $m_{iu} \ge 1$, $i = 1, 2, \cdots, p$.

Remark 1: In the above fault matrix M, if $m_i = 1$, it corresponds to the normal case $u^f(t) = u(t)$. If $m_i = 0$, outage of actuator control signal occurs. If $0 < m_{il} < m_i < m_{iu}$, $m_{il} < 1$, $m_{iu} \ge 1$ and $m_i \ne 1$, the corresponding actuator would be in partial failure case. Hence, let $u^f(t)$ denote the control input vector both in normal and actuator failures cases for fault-tolerant control research in this paper.

The decomposition of fault function M is given below with a similar manner in [6], which will be used for our main results. Define

$M_0 = diag[m_{01}, m_{02}, \cdots, m_{0p}]$, $J = diag[j_1, j_2, \cdots, j_p]$, $|L| = diag[|l_1|, |l_2|, \cdots, |l_p|]$, where $m_{0i} = (m_{il} + m_{iu})/2$, $j_i = (m_{iu} - m_{il})/(m_{iu} + m_{il})$, $l_i = (m_i - m_{0i})/m_{0i}$.

So, we then have

$$M = M_0(I + L) \qquad |L| \le J \le I \tag{5}$$

Consider the state-feedback control law(2), and then the faulty closed-loop system is given by

$$\begin{aligned} \dot{x}(t) &= (A_C + \Delta A_C)x(t) + D\omega(t) \\ z(t) &= Cx(t) \end{aligned} \tag{6}$$

where $A_C = A + BMK$, $\Delta A_C = \Delta A + \Delta BMK$.

The cost function associated with system (1) is

$$J = \int_0^\infty \left[x^T(t)Qx(t) + (u^f(t))^T Ru^f(t) \right] dt \tag{7}$$

where $Q = Q^T > 0$, $R = R^T > 0$ are given weighting matrices. Consider possible actuator faults (3), then the cost function is

$$J = \int_0^\infty \left[x^T(t)\left(Q + (MK)^T R(MK)\right)x(t) \right] dt \tag{8}$$

Definition 1: For system(1), if there exists state-feedback controller, such that the faulty closed-loop system (6) will meet the following indices constraints simultaneously,

(a) The closed-loop poles lie within the circular region $\Phi(q,r)$, $\Phi(q,r)$ denotes the disc with the centre $-q + j0$ and the radius r, where r and q are known constants with $q > r > 0$,

(b) The H$_\infty$ norm bound γ,

(c) The closed-loop value of the cost function (8) exists an upper bound satisfying $J \le J^*$,

then for all admissible uncertainties and possible faults, the given indices, circular pole index $\Phi(q,r)$, cost function performance $J^* > 0$ and H$_\infty$ norm bound $\gamma > 0$ are said to be consistent, state-feedback controller $u(t) = Kx(t)$ is said to be robust H$_\infty$ guaranteed cost satisfactory fault-tolerant controller with regional poles constraints and J^* is said to be a guaranteed cost.

Now, the satisfactory fault-tolerant control problem considered in this paper is stated in the following.

Problem: For the system (1) with actuator failure, given the index $\Phi(q,r)$, H$_\infty$ norm bound $\gamma > 0$ and the cost function (8), determine a control law $u(t) = Kx(t)$ so that the closed-loop system satisfies criteria (a), (b) and (c) simultaneously.

3. Main Results

3.1 Conditions for the existence of robust H$_\infty$ guaranteed cost satisfactory fault-tolerant controller with regional poles constraints

In this subsection, we will derive the sufficient conditions for the existence of state-feedback satisfactory fault-tolerant control to satisfy the requirements on (a), (b) and (c) simultaneously.

In the proof of our main results, we will use the following lemmas.

Lemma 1: Given any symmetric matrix Y, and constant matrix R_1, R_2 with appropriate dimension, if Σ is a diagonal matrix satisfies $|\Sigma| \le U$, where U is a given diagonal matrix, then

$$Y + R_1\Sigma R_2 + R_2^T\Sigma^T R_1^T < 0$$

If and only if there exists positive constant $\beta > 0$ such that

$$Y + \beta R_1 U R_1^T + \beta^{-1} R_2^T U R_2 < 0$$

Lemma 2: Consider the actuator fault model (3), for any matrix $R = R^T > 0$ and scalar $\varepsilon > 0$, if $R^{-1} - \varepsilon I > 0$ then

$$MRM \le M_0\left(R^{-1} - \varepsilon I\right)^{-1} M_0 + \varepsilon^{-1} M_0 JJM_0 \tag{9}$$

Lemma 3: Consider the system (1) subject to faults, given index $\Phi(q,r)$, if there exists gain matrix K and symmetric positive matrix P such that the following matrix inequality

$$\begin{bmatrix} -P^{-1} & A_C + \Delta A_C + qI \\ \left(A_C + \Delta A_C + qI\right)^T & -r^2 P \end{bmatrix} < 0 \qquad (10)$$

holds for all admissible uncertainties and possible faults, then the system (1) is quadratically d stabilizable.

Theorem 1: Consider the system (1) in fault case and the cost function (8) as well as square integrable disturbance $\omega(t)$, if there exists gain matrix K and symmetric positive matrix P such that the following matrix inequality

$$\begin{aligned}&\left(A_C + \Delta A_C + qI\right)^T P\left(A_C + \Delta A_C + qI\right) - r^2 P + q\gamma^{-2} PDD^T P \\ &+Q + K^T MRMK + qC^T C < 0\end{aligned} \qquad (11)$$

holds for all admissible uncertainties and possible faults, then the faulty closed-loop system is quadratically d stabilizable with an H_∞ norm-bound γ, and the cost function (8) has an upper bound.

Proof: Take $u(t) = Kx(t)$ in the system (1), then the resulting closed-loop system is given by (6). Suppose that there exists a matrix $P > 0$ such that the matrix inequality (11) holds, then

$$\left(A_C + \Delta A_C + qI\right)^T P\left(A_C + \Delta A_C + qI\right) - r^2 P < 0$$

By using the Lemma 3, it is easy to show that the system (6) is quadratically d stabilizable with constraint $\Phi(q,r)$.

To show the H_∞ norm-bound constraint, from the Lyapunov function candidate $V = x^T Px$, its difference is given by

$$\dot{V} = x^T\left(A_C + \Delta A_C\right)^T Px + \omega^T D^T Px + x^T P\left(A_C + \Delta A_C\right)x + x^T PD\omega$$

From the inequality (11), we then have

$$\begin{aligned}&P\left(A_C + \Delta A_C\right) + \left(A_C + \Delta A_C\right)^T P + \gamma^{-2} PDD^T P + q^{-1} Q \\ &+q^{-1} K^T MRMK + C^T C < 0\end{aligned} \qquad (12)$$

Now from the above inequality, it follows that

$$\begin{aligned}\dot{V} &\le x^T\left[\left(A_C + \Delta A_C\right)^T P + P\left(A_C + \Delta A_C\right) + \gamma^{-2} PDD^T P\right]x + \gamma^2 \omega^T \omega \\ &\le -x^T\left[q^{-1} Q + q^{-1} K^T MRMK + C^T C\right]x + \gamma^2 \omega^T \omega\end{aligned}$$

When disturbance $\omega(t) \in L_2[0,\infty)$,

$$\begin{aligned}\hat{J} &= \int_0^\infty \left(z^T z - \gamma^2 \omega^T \omega\right)dt = \int_0^\infty \left(z^T z - \gamma^2 \omega^T \omega + \dot{V}\right)dt - \int_0^\infty \dot{V}dt \\ &\le \int_0^\infty \left[-x^T\left(q^{-1} Q + q^{-1} K^T MRMK\right)x\right]dt - V(\infty) < 0\end{aligned}$$

So $\| z(t) \|_2 \le \gamma \| \omega(t) \|_2$.

For the cost function performance index

$$-V(0) \le \int_0^\infty \dot{V}dt \le \int_0^\infty \left[-x^T\left(q^{-1} Q + q^{-1} K^T MRMK\right)x\right]dt + \gamma^2 \beta^2$$

From the above inequality, it follows that

$$\begin{aligned}J &= \int_0^\infty \left[x^T\left(Q + K^T MRMK\right)x\right]dt \\ &\le qV(0) + q\gamma^2\beta^2 \le qx_0^T Px_0 + q\gamma^2\beta^2 = J^*\end{aligned} \qquad (13)$$

The proof is completed.

Remark 2: Note this upper bound which depends on the initial condition x_0. To remove the dependence on the initial state for the upper bound in (13), suppose x_0 is a random vector with zero mean and covariance matrix equal to identity. In that case, the cost bound in (13) leads to

$$\overline{J} = E\{J\} \le qE\left\{x_0^T Px_0\right\} + q\gamma^2\beta^2 = q\mathrm{Trace}(P) + q\gamma^2\beta^2 = \overline{J}^*$$

Theorem 2: Consider the system (1) in fault case and the cost function (8) as well as square integrable disturbance $\omega(t)$, for the given index $\Phi(q,r)$ and H_∞ norm-bound index γ, if there exists gain matrix K, symmetric positive matrix X and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ such that the following matrix inequality holds, then for all admissible uncertainties and possible faults M, the faulty closed-loop system (6) is quadratically d stabilizable with an H_∞ norm-bound γ, and the corresponding closed-loop cost function (8) satisfies $\overline{J} \le \overline{J}^*$.

$$\begin{bmatrix} \Upsilon_{11} & X\left(A_C + qI\right)^T & X & \left(MKX\right)^T \\ \left(A_C + qI\right)X & -X + \varepsilon_1 aI + \varepsilon_2 bI & 0 & 0 \\ * & * & -\varepsilon_1 I & 0 \\ * & * & * & -\varepsilon_2 I \end{bmatrix} < 0 \quad (14)$$

where
$\Upsilon_{11} = -r^2 X + q\gamma^{-2} DD^T + XQX + XK^T MRMKX + qXC^T CX$.

If matrix inequality (14) has a feasible solution P, K, ε_1 and ε_2, it follows from Theorem 2 that $u(t) = Kx(t)$ is a state-feedback satisfactory fault-tolerant control law with the constraints $\Phi(q,r)$ and (b) for the system (1), and the corresponding closed-loop cost function (8) satisfies $\overline{J} \le \overline{J}^*$.

3.2 Robust H_∞ guaranteed cost satisfactory fault-tolerant controller with regional poles constraints design

Theorem 3: Consider the system (1) and the cost function (8), for the given index $\Phi(q,r)$ and H_∞ norm-bound index γ, if there exists matrix Y, symmetric positive matrix X and scalars $\varepsilon_i > 0 (i = 1 \sim 5)$ such that the following linear matrix inequality

$$\begin{bmatrix} -r^2 X & (AX + BY + qX)^T & \Upsilon_{21} \\ * & -X + \varepsilon_1 aI + \varepsilon_2 bI + \varepsilon_3 BJB^T & 0 \\ * & * & \Upsilon_{22} \end{bmatrix} < 0 \quad (15)$$

holds, where $\Upsilon_{21} = [X \quad Y^T \quad Y^T \quad Y^T \quad D \quad X \quad XC^T \quad Y^T \quad Y^T J]$, $\Upsilon_{22} = diag[-\varepsilon_1 I \quad -\varepsilon_2 I + \varepsilon_4 J \quad -\varepsilon_3 J^{-1} \quad -\varepsilon_4 J^{-1} \quad -q^{-1}\gamma^2 I \quad -Q^{-1} \quad -q^{-1} I \quad \varepsilon_5 I - R^{-1} \quad -\varepsilon_5 I]$. Then for all admissible uncertainties and possible faults M, the faulty closed-loop system (6) with satisfactory fault-tolerant controller $u(t) = Kx(t) = M_0^{-1} Y X^{-1} x(t)$ is quadratically d stabilizable with an H_∞ norm-bound γ, and the corresponding closed-loop cost function (8) is with $\bar{J} \le \bar{J}^*$.

According to Theorem 2 and 3, the consistency of the regional poles constraint, H_∞ performance and cost function indices for fault-tolerant control is deduced as the following optimization problem.

Theorem 4: Given circular pole index $\Phi(q,r)$, suppose the system (1) is robust fault-tolerant state feedback assignable for actuator faults case, then LMI (15) has a feasible solution. Thus, the following minimization problem is meaningful.

$$\min(\gamma): (X, Y, \gamma, \varepsilon_i) \text{ S.t. LMI (15)} \quad (16)$$

Suppose the above minimization problem has a solution $X_L, Y_L, \varepsilon_{iL}, \gamma_L$, and then any index $\gamma > \gamma_L$, LMI (15) has a feasible solution. Thus, the following optimization problem is meaningful.

Theorem 5: Consider the system (1) and the cost function (8), for the given index $\Phi(q,r)$ and H_∞ norm-bound index $\gamma > \gamma_L$, if there exists matrix Y, symmetric positive matrix X and scalars $\varepsilon_i > 0 (i = 1 \sim 5)$ such that the following minimization

$$\min \; q\text{Trace}(J_0) + q\gamma^2\beta^2 \quad (17)$$

S.t. (i) (15)

$$\text{(ii)} \begin{bmatrix} J_0 & I \\ I & X \end{bmatrix} > 0$$

has a solution $X_{\min}, Y_{\min}, \varepsilon_{i\min}, J_{0\min}$, then for all admissible uncertainties and possible faults M, $u(t) = Kx(t) = M_0^{-1} Y_{\min} X_{\min}^{-1} x(t)$ is an optimal guaranteed cost satisfactory fault-tolerant controller, so that the faulty closed-loop system (6) is quadratically d stabilizable with an H_∞ norm-bound γ, and the corresponding closed-loop cost function (8) satisfies $\bar{J} \le q\text{Trace}(J_{0\min}) + q\gamma^2\beta^2$.

Proof: It follows from the Schur complement that the constraint condition (ii) in (17) is equivalent to $J_0 > X^{-1} > 0$. From Theorem 3, we obtain the result of the theorem. The proof is completed.

According to Theorem 1~5, the following satisfactory fault-tolerant controller design method is concluded for the actuator faults case.

Theorem 6: Given consistent indices $\Phi(q,r)$, H_∞ norm index $\gamma > \gamma_L$ and cost function index $\bar{J}^* > q\text{Trace}(J_{0\min}) + q\gamma^2\beta^2$, suppose that the system (1) is robust fault-tolerant state feedback assignable for actuator faults case. If LMI (15) has a feasible solution X, Y, then for all admissible uncertainties and possible faults M, $u(t) = Kx(t) = M_0^{-1} Y X^{-1} x(t)$ is satisfactory fault-tolerant controller making the faulty closed-loop system (6) satisfying the constraints (a), (b) and (c) simultaneously.

4. Simulative Example

In this section, a simple example is presented to illustrate the proposed design method. Consider an uncertain continuous-time system (1) with parameters as follows:

$$A = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 3 & 3 \end{bmatrix}, C = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix}, D = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix},$$

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, a = 0.2, b = 0.3.$$

Suppose the actuator failure parameters $M_l = diag\{0.1, 0.2\}$, $M_u = diag\{1.3, 1.6\}$. Given the pole index $\Phi(1.5,1)$, using the software LMI toolbox in Matlab, it is found that the corresponding LMI (15) is feasible. Then solve the minimization problem (16), it can be obtained that the H_∞ norm index $\gamma > 1.2165$ is consistent with pole index. Choose H_∞ norm index $\gamma = 2$, then solve the minimization problem(17), specify the cost function upper bound index

$\overline{J}^* = 20$. Based on Theorem 6, we can obtain state-feedback satisfactory fault-tolerant controller (SFTC), such that the closed-loop systems will meet given indices constraints simultaneously.

$$K_{\mathrm{SFTC}} = \begin{bmatrix} -0.0298 & -0.0802 \\ -0.0980 & 0.0075 \end{bmatrix}$$

5. Conclusions

Taking the guaranteed cost control in practical systems into account, the problem of satisfactory fault-tolerant controller design with regional poles and H_∞ norm-bound constraints is concerned by LMI approach for a class of uncertain continuous-time systems subject to actuator failures. Attention has been paid to the design of state-feedback controller that guarantees, for all admissible value-bounded uncertainties existing in both the state and control input matrices as well as possible actuator failures, the closed-loop system to satisfy the pre-specified regional pole index, meanwhile the H_∞ index and cost function are restricted within the chosen upper bounds. So, the resulting closed-loop system can provide satisfactory stability, transient property, H_∞ performance and quadratic cost performance despite of possible actuator faults.

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