A Novel Chaotic Neural Network with Stochastic Noise and Heuristic Mechanism for Minimum Vertex Cover Problem

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Summary
In this paper, we propose a novel chaotic neural network embedded with stochastic simulated annealing noise and a heuristic mechanism to solve minimum vertex cover problem. The proposed network can make a global search with the affection of stochastic noise and obtain a chaotic search by the chaotic dynamics. The stochastic noise with simulated annealing is able to find a global optimum solution if the annealing process is carried out sufficiently slowly. For increasing the network convergence speed and degree, a heuristic mechanism on vertex degree is introduced to modify the convergence trend. The proposed network is tested on a large number of random graphs. The simulation results show that the proposed algorithm is effective and better than some other works in solving minimum vertex cover problem.

Key words:
Chaotic neural network, Stochastic noise, Heuristic mechanism, Minimum vertex cover problem

1. Introduction

The minimum vertex cover problem of an undirected graph G=(V,E) where V is the set of vertices and E denotes the set of edges, consists in finding the smallest subset V' $\subseteq V$ such that $\forall\langle i, j\rangle \in E$, we have $i \in V'$ or $j \in V'$ (or both). V' is said to be a vertex cover of G [1][2]. The minimum vertex cover problem is of central importance in computer science. It is very intractable [2]. It is a NP-complete problem proved by Karp in 1972 [3]. This problem has been widely studied by many researchers for its important practical application in many fields, especially in multiple sequence alignments for computational biochemistry [4]. Despite researchers have presents many algorithms for the problem, no tractable algorithm is known for solving it due to its complexity. Furthermore, few parallel algorithms have proposed to solve the problem. A kind of possible parallel algorithms for solving such optimization problems are neural network algorithms, especial the Hopfield-type neural network proposed by Hopfield and Tank. [5]

The Hopfield-type neural networks have widely used to solve combinatorial optimization problems since it was proposed. But the Hopfield-type neural network is easy to get in local minima, which is its fatal disadvantage. In order to overcome the shortcoming and increase the searching ability of the Hopfield-type neural network, some researchers presented their improved methods based on mathematic theories and problem specialties. In 1990, based on the chaotic dynamics in neural network, Aihara et al. proposed a chaotic neural network (CNN) to solve optimization problems [6]. Furthermore, Chen and Aihara presented a new chaotic neural network by introducing an annealing negative self-feedback into the motion function of CNN, named transiently chaotic neural network (TCNN) in 1995 [7]. Subsequently, influenced by the CNN and TCNN many neural network algorithms were put forward. Although a neural network with chaotic dynamics will have an efficient searching ability, it has completely deterministic dynamics and is not guaranteed to settle down a global optimum no matter how slowly the annealing parameter is reduced [8]. It is well-known that stochastic simulated annealing (SSA) tends to find a global optimum if the annealing process is carried out sufficiently slowly. In 1996, Ball and Mlynski proposed a stochastic neural network based on integration of Langevin equation of neurons motion for solving combinatorial optimization problems [9]. In 1999, Belli et al. disclosed the approximating stochastic process in artificial neural network and showed the essentially stochastic specialty of neural network [10]. Turchetti provided a treatment of the theory and applications of stochastic neural networks and gave some practical applications in the field of artificial intelligence remarkably [11]. However SSA algorithms should cost long time to get the global optimal solution for its search covering the entire solution space, which makes it difficult to be used in practical applications.

In this paper, we proposed a novel chaotic neural network, which not only obtains the chaotic searching ability, but also is introduced a stochastic simulated annealing noise. Moreover, according to characteristic of the minimum vertex cover problem, a heuristic mechanism correlative with the vertex degrees is embedded into our network to modify the convergence trend. The proposed network has the both advantages of CNN and SSA, and it is able to find the global minimum easily in short time for the help of the combined dynamics. Moreover, the heuristic mechanism enhances the convergence speed further. The simulation results show that the proposed network can yield better solutions for random graphs than

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TCNN and SSA for the minimum vertex cover problem. Furthermore, the proposed network can use less time to get convergence with the restriction of the heuristic mechanism. The energy evolution reveals the convergence process and the motive characteristic of our proposed network.

This paper is organized as follows: in the next section, the basic chaotic neural network and TCNN are described. In section 3, we formulate the minimum vertex cover problem for using in our network. In section 4, the novel chaotic neural network with stochastic noise and heuristic mechanism is described clearly. The simulations on some random graphs are tested and shown in section 5, and finally we make the conclusion in section 6.

2. The basic chaotic neural network and TCNN

Aihara et al. proposed an equation of a chaotic neuron model with graded output and exponentially decaying refractoriness as follows [6]:

$$x(t+1) = f\left[A(t) - \alpha\sum_{d=0}^{t} k^d g\{x(t-d)\} - \theta\right] \quad (1)$$

where $x(t)$is the neuron output with an analogue value between 0 and 1 at the discrete time t, t shows the discrete time steps(t=0,1,2,…), f is the output function, $A(t)$is the external stimulation at the time t, g is the refractory function and α , k and θ are the refractory scaling parameter, the refractory decay parameter, and the threshold, respectively. The internal state $y(t+1)$ of the neuron is defined as follows:

$$y(t+1) = A(t) - \alpha\sum_{d=0}^{t} k^d g\{x(t-d)\} - \theta \quad (2)$$

Then Equ(2) can be reduced to different equation as below:

$$y(t+1) = ky(t) - \alpha g[f\{y(t)\}] + \theta(t) \quad (3)$$

where $\theta(t) \equiv A(t) - kA(t-1) - \theta(1-k)$. The transform function f is defined as $f(y)=1/\{1+\exp(-y/\varepsilon)\}$ which is the logistic function with the steepness parameters ε .

Subsequently, Chen and Aihara evolved the chaotic neural network and introduced an annealing negative self-feedback term into the motion function to propose the transiently chaotic neural network (TCNN)[7]. The motion function of TCNN is as follows:

$$y_i(t+1) = ky_i(t) + \alpha[\sum_{j=1, j\neq i}^{n}(w_{ij}x_j + I_i)] - T_i(t)[x_i(t) - I_0] \quad (4)$$

$$T(k+1) = (1-\beta)T(k) \quad (5)$$

TCNN has a powerful searching ability in a whole domain, and its characteristic has been already discussed in many papers. Researchers have proposed many improved algorithms based on analysises of TCNN [12][13].

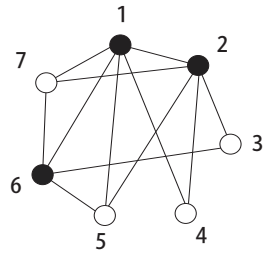


Figure. 1 An example of minimum vertex cover problem with 7 vertexes and 12 edges. The black vertices forms the minimum vertex cover.

3. Problem Formulation

Let $G=(V,E)$ be an undirected graph, V is vertex set and E is edge set. A set $V' \subseteq V$ is a vertex cover of G, if for every edge $(i,j) \in E$, *either* $i \in V'$ or $j \in V'$ or both $i,j \in V'$. A vertex cover is minimal or optimal if it has minimum size, i.e. if there is no vertex cover that has fewer vertexes. The goal of the minimum vertex cover problem is to find a minimum vertex cover. In this problem, $d_{ij}(i=1,2,\cdots,n, j=1,2,\cdots,n)$ is element of the adjacency matrix of graph G, and it has only two values(1 or 0) which express if the connection between vertex #i and #j exists or not. The vertex number and edge number is $|V|=n$ and $|E|=m$. For example, Figure 1 gives a graph with 7 vertexes and 12 edges, where the black vertexes indicate the minimum vertex cover of the graph. The adjacency matrix of Figure 1 is as follows:

$$\begin{vmatrix} 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \end{vmatrix}$$

The state of neuron i can be determined by:

$$v_i = \begin{cases} 1 & \text{if the \#i vertex is in the cover} \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

In this problem, the vertex number determines the size of network. If there are n vertices in the graph, then n neurons are required. As the neurons have only binary values, the size of vertex cover can be expressed by:

$$E_1 = \sum_i v_i \quad (7)$$

If an edge (i,j) is not covered by a cover, then both v_i and v_j are zero. Then the constrained condition can be expressed as:

$$E_2 = \sum_i \sum_j d_{ij} \overline{v_i \vee v_j} \quad (8)$$

where $\vee$ is the logical OR, $\overline{X}$ means the complement of X. The total energy function of minimum vertex cover problem can be described as follows:

$$E = AE_1 + BE_2 = A\sum_i v_i + B\sum_i \sum_j d_{ij}\overline{v_i \vee v_j} \quad (9)$$

Where A and B are coefficients that can modify the neural network efficiency mildly. According to the method of arithmetic conversion, the energy function can be formulated as:

$$\begin{aligned} E &= A\sum_i v_i + B\sum_i \sum_j d_{ij}\overline{v_i \vee v_j} \\ &= A\sum_i v_i + B\sum_i \sum_j d_{ij}\left(1 - v_i \vee v_j\right) \\ &= A\sum_i v_i + B\sum_i \sum_j d_{ij}\left(1 - v_i - v_j + v_i v_j\right) \\ &= A\sum_i v_i + B\sum_i \sum_j d_{ij}\left(v_i v_j - v_i - v_j\right) + B\sum_i \sum_j d_{ij} \end{aligned} \quad (10)$$

So the energy function without the constant term can be described as:

$$E = A\sum_i v_i + B\sum_i \sum_j d_{ij}\left(v_i v_j - v_i - v_j\right) \quad (11)$$

4. The novel chaotic neural network

TCNN has been proved that it has a powerful searching ability for combinatorial optimization problem, but as a chaotic neural network, it has the inherent shortcoming that the chaotic attracting set has a fractal structure and covers only a very small fraction of the entire state space. However, the stochastic annealing method can find a global optimum if the annealing process is carried out sufficiently slowly. When using the stochastic annealing method, the chaotic neural network will have a little slower speed to get convergence, so we embed a heuristic mechanism based on the problem specialty into the motion function of the neural network to increase the convergent speed. The proposed novel chaotic neural network is as follows:

$$\begin{aligned} y_i(t+1) = ky_i(t) + \alpha[\sum_{j=1, j\neq i}^{n}(w_{ij}x_j + I_i)] \\ - T_i(t)[x_i(t) - I_0] - Q_i(t)(R_i^{ran} - D_i) \end{aligned} \quad (12)$$

$$T_i(t+1) = (1-\beta)T_i(t) \quad (13)$$

$$Q_i(t+1) = (1-\rho)Q_i(t) \quad (14)$$

$$R_i^{ran} = random(0 \sim 1) \quad (15)$$

$$D_i = \frac{dg_i}{dg_{biggest}} \quad (16)$$

where the variants are

x_i output of neural i;

y_i internal state of neuron i;

$T_i(t)$ the positive temperature relative with the self-feedback;

$Q_i(t)$ the annealing temperature controlling the stochastic noise;

β, ρ the damping factors for the time-dependent self-feedback term and the stochastic noise($0 \le \beta \le 1$, $0 \le \rho \le 1$;

R_i^{ran} stochastic noise embedded into the motive neuron #i;

D_i The heuristic variants corresponding to the vertex #i degree;

dg_i,$dg_{biggest}$ The degree of vertex #i and the biggest degree in the problem graph.

In the proposed chaotic neural network, the stochastic noise improves the dynamic characteristic of the neuron states, which makes the chaotic neural network obtain both chaotic dynamics and stochastic dynamics. When the neural network has a trend to trap in local minima for the shortcoming of chaotic dynamics, the stochastic dynamics will give a force to drive the network to escape from the local minima. So the neural network has more powerful energy to find a global optimal solution.

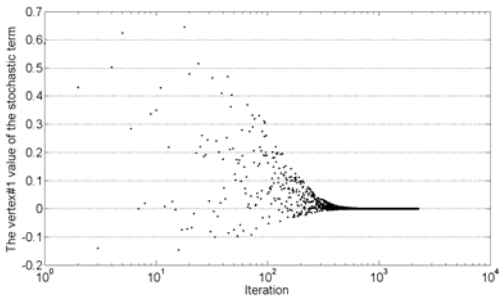
Because vertices have different degrees in the graphs and in vertex cover problem the vertex degrees give a powerful influence to results, Equations (12)(16) utilize the specialty to give a heuristic method for the minimum vertex cover problem. According to the vertex degree, the heuristic mechanism gives different random forces to these vertices with different vertex degrees. From the Equation (12), we can see that when a vertex has more edges connected with other vertices, it will have higher possibility to evolve to 1. Supposing a general situation: there are two neurons #i and #j with different degrees dg_i and dg_j, and they have same internal states at time t. In order to see the convergence trend of neurons, those internal terms are reduced as a sample function $P(x_i,y_i)$. Then the motion function at time t+1 is as follows:

$$y_i(t+1) = P_i(x_i, y_i) - Q_i(t)(R_i^{ran} - \frac{dg_i}{dg_{biggest}}) \quad (17)$$

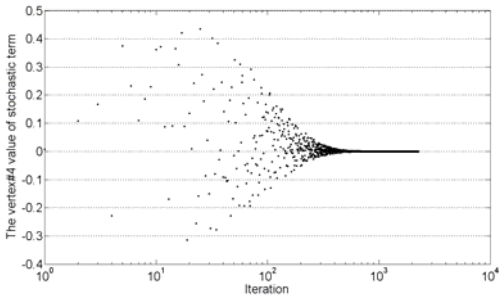
$$y_j(t+1) = P_j(x_j, y_j) - Q_j(t)(R_j^{ran} - \frac{dg_j}{dg_{biggest}}) \quad (18)$$

When just considering the motive trend of neurons, as $dg_i > dg_j$, then there are more chances make the right term $-Q_i(t)(R_i^{ran} - \frac{dg_i}{dg_{biggest}})$ of neuron #i larger than the corresponding term of neuron #j in Equ.(18) and zero. So the neuron #i has a stronger trend to get convergence, which increases the convergence speed evidently. For example, considering a special status with dg_i=$dg_{biggest}$ and dg_j=0, the neuron #i always has a positive noise to help itself reach state 1; similarly neuron #j always has a negative noise to trend state 0. Figure 2 shows the

evolutions of the stochastic term when using the proposed neural network to solve the problem of Figure 1. It is obviously that vertex #1 has much more positive noises than negative noises in the influence of the heuristic method, which means it has more chances to get convergence to 1 and be a vertex of minimum vertex cover. The vertex #4 has a less average distribution of the noise because it only has two edges and the parameters D_i is 0.4. Then vertex #1 has more possibility than vertex #4 to be a vertex of minimum vertex cover.



(a)



(b)

Figure 2. The stochastic noise in vertex #1 and #4 when using our proposed neural network to solve the problem of Figure 1. (a) the evolution of vertex #1. (b) the evolution of vertex #4.

On the minimum vertex cover problem, the neuron output should be limited in binary values to match the energy request. However, in order to grasp the characteristic of particular tuning, the output function used in our proposed neural network is still a sigmoid function:

$$x_i(t) = \frac{1}{1+e^{-y_i/\varepsilon}} \tag{19}$$

where ε is the steepness parameter of the output. For corresponding to the energy request, a restricting function is introduced as follows:

$$x_i(t) = \begin{cases} 1 & if \quad x_i(t) \geq (0.5+X_{sh}) \\ 0 & if \quad x_i(t) < (0.5-X_{sh}) \end{cases} \tag{20}$$

$$X_{sh} = (1-\delta)X_{sh} \tag{21}$$

The constant δ is a damping factor controlling the limited range and X_{sh} is the threshold value of output. With the evolution of the neural network, the outputs are separated to binary values eventually. The proposed network is simulated on some random graphs to verify its efficiency, and the simulation results shown in section 5 displays that it has better ability in solving the combinatorial optimization problem.

5. Simulation Results

In order to assess the effectiveness of the proposed chaotic neural network, extensive simulations were implemented in *C++* on normal PC(Pentium4 1.8GHz) to a number of randomly generated graphs of various sizes. The parameter values used in the simulations were:

$k=0.9$, $\delta=0.01$, $\alpha=0.0015$, $\varepsilon=0.0025$, $T(0)=0.25$, $Q(0)=1.5$, $\beta=0.002$, $\rho=0.002$

Other parameters are set as those in *TCNN*.

All the tested graphs are randomly generated, thus, any specific algorithm based on special subclass graph characteristics may fail. The random graph is defined in terms of two parameters, *n* and *d*. The parameter *n* specifies the number of vertices in the graph; parameter $d(1>d>0)$, is the edge density. Then the number of edges o the random graph will be $|dn(n-1)/2|$. We generate 14 graphs from n=20 to 300 to test our proposed network. For getting the general simulation results, every graph is tested in 100 times and the average results are summarized. For comparing the efficiency of our network, the basic chaotic neural network and TCNN are also preformed in the same situation. The simulation results are shown in Table 1. The columns “Vertex”, “Den” and “Edge” express the number of vertex, the edge density and the edge count. Similarly, the columns “Cover” and “Time” denote the average value of minimum vertex cover and spending time.

From Table 1, we can get some experimental observations as follows:

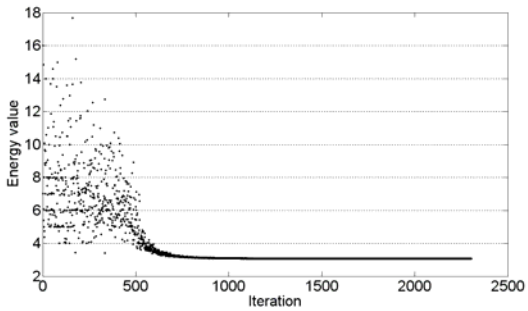
1. Although sometimes the proposed network uses more time to get a solution, the solution is usually better than those of the other algorithms (CNN and TCNN). Generally, the proposed network is superior in the aspects of time spending and solution quality.
2. In some low density graphs, the proposed network displays an efficient searching ability to get the global optimal solutions obviously.
3. The embedded stochastic noise does not give more computing burden to the proposed neural network. Usually, the proposed network is able to get an efficient result with less time than TCNN.

For explaining the network evolution, we plot in Figure. 3 the total energy *E* before the effect of the restricting function Equ.(20) (21) when using the proposed neural network on the problem of Figure 1 with different parameters. The process of energy evolution exposes the complex dynamics of our proposed network. In Figure

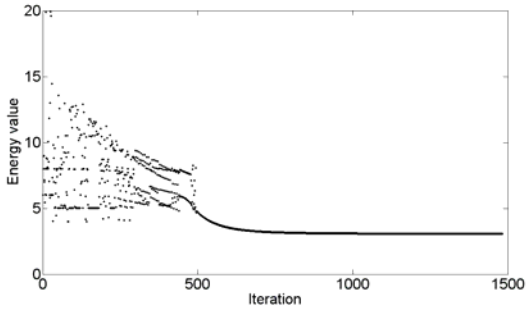
3(a), if we used a low annealing speed of the stochastic term in the proposed network, the combinatorial dynamics is so complex that the network needs a little long time to get convergence and find the optimal solution. In Figure 3(b), when the network is set with a high annealing speed on the stochastic term, the heuristic mechanism exposes its influence clearly and the network dynamics enter a limited orbit to reach saturate states quickly. It is verified that the proposed network can be controlled flexibly on the aspect of network dynamics and the heuristic mechanism gives an effective force to impel the network to get convergence.

Table 1. The simulation results for the random graphs.

Graph			Chaotic Neural Network		TCNN		Proposed network	
Vertex	Den	Edge	Cover	Time	Cover	Time	Cover	Time
20	0.15	28	8	<0.01	8	<0.01	8	<0.01
20	0.25	47	13	<0.01	13	0.01	13	0.01
50	0.15	183	29	0.01	29	0.02	29	0.02
50	0.25	306	34	0.02	33	0.02	33	0.03
100	0.15	742	77	0.14	75	0.23	73	0.18
100	0.25	1237	85	0.16	82	0.27	82	0.21
150	0.15	1676	130	0.34	121	0.43	118	0.47
150	0.25	2793	137	0.40	132	0.52	130	0.45
200	0.15	2985	175	0.73	165	0.98	165	0.86
200	0.25	4975	182	0.69	179	1.42	179	0.98
250	0.15	4668	224	0.98	217	2.58	217	2.05
250	0.25	7781	233	1.21	229	2.26	226	1.84
300	0.15	6727	268	1.96	264	3.45	262	3.89
300	0.25	11212	289	2.37	275	3.88	272	4.02



(a)



(b)

Figure 3. The energy evolution when using the proposed network to process the graph of Figure 1. (a). ρ =0.002. (b). ρ =0.015.

6. Conclusion

We have proposed a novel chaotic neural network with stochastic noise and heuristic mechanism for solving the minimum vertex cover problem. The proposed network has double dynamics characteristic of external stochastic noise and internal chaotic dynamics. The characteristic makes the proposed network obtain high ability to find the optimal solution and escape from the local minima. Furthermore, a heuristic mechanism corresponding with the vertex degree is introduced into the proposed neural network, and it impels the network to get convergence quickly. The proposed neural network was tested on a large number of random graphs to verify it efficiency. The simulation results indicated that the proposed neural network is better than other chaotic neural networks and can find optimal or near optimal solutions efficiently.

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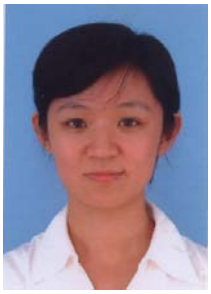

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