An Approach to the Facility Layout Design Optimization

Rekha Bhowmik

Department of Computer Science, Sam Houston State University, Huntsville, Texas 77341, USA

Summary
This paper presents an Iterative Heuristic Algorithm and Branch and Bound Algorithm for optimal location of clusters on different levels. The use of cluster analysis is proposed for grouping highly related departments for both the methods. The vertical location problem is formulated for optimal location problem. Results obtained by both the algorithms are presented and compared in terms of travel cost.

Key words:
Cluster analysis, intergroup adjacency matrix, travel cost, Iterative Heuristic Algorithm, Branch and Bound Algorithm

1. Introduction

The Facility Layout Problem(FLP) is concerned with locating a set of departments, each requiring level area on a given site. The department ratings, based on number of trips, may be represented by an adjacency matrix. The number of trips between pairs of departments can be used to decide on the desirability of locating a department next to each other. Given such order of department, the problem is to find a layout which optimizes a function based on department closeness ratings and distance. There are 2 sub-problems involved in solving FLP: i) find an optimal adjacency matrix subject to certain constraints, and 2) derive a layout from adjacency matrix.

The three-dimensional facilities planning techniques have a number of shortcomings. The location order calculations are not designed to isolate groups of closely related departments and that the location process is only able to optimize the location of a department with respect to the relative locations of previously located departments. Further, the processing time is very high for large datasets and that departments tend to split between levels, which may not be acceptable.

In this paper a three-step Iterative Heuristic technique is proposed for FLPs. In section 2 we present related work. In section 3 we present a three-step heuristic optimization procedure for FLPs. Clustering Analysis and Justification is presented in section 4. In section 5 the allocation of departments to different levels is formulated followed by Branch and Bound Algorithm. An iterative heuristic algorithm for FLP problem is discussed in section 7 followed by Results and Conclusions.

2. Related Work

Unequal area FLP was formulated first. A given region L * W is assumed, where L is the length and W is the width of the region. The objective was to partition the region into departmental sub-regions so as to minimize the total communication cost. Variation of the Quadratic Assignment Problem was presented later. The rectangular layout is split into blocks of equal-area departments. This technique reduced the number of candidate layouts while allowing departments to assume different areas and different shapes.

Exact Mixed Integer Programming formulation was proposed by [1]. The model uses a distance-based objective, but is not based on the discrete representation as in the quadratic assignment problem. [2] formulated a Nonlinear Layout Technique(NLT) based on three constraints. Two constraints are based on structure of the layout, that is, departments may not overlap, and departments may not be located outside the given region boundaries. The third constraint depends on the limits of allowable dimensions of each department. The departments have fixed area and rectangular shapes, but for each department the height and width are optimized using mathematical model. [3] implemented a genetic search for unequal area facility layout and showed how optimal solutions are affected by constraints on permitted department shapes, as specified by a maximum allowable aspect ratio for each department. [4], [5] proposed a modified Mixed Integer Programming-Facilities Layout Planning model by improving perimeter constraint and reported optimal solutions for FLPs with a maximum of eight departments. Anjos[6] improved NLT method by introducing Attractor-Repeller. Sherali et al[7] further improved the MIP-FLP model and provided the approach with up to nine departments. The approach uses polyhedral outer approximation of the area constraints that reduces the errors in department areas. The first model is based on layout problem and the second model is an exact formulation of facility layout problem. Anjos[8] considered one-dimensional facility layout problem, which consists in finding an optimal placement of facilities on a straight line. Lower bound on the optimal value of the space allocation problem is created. They suggest a heuristic procedure which extracts a feasible solution to the one-dimensional space allocation problem. Kaufmann[9] presents cluster analysis to find groups of data. Anjos[10] presented a framework based for facility layout planning based on two mathematical models. The first model finds the starting points for the iterative algorithm. The second model is an

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exact formulation of the facility layout problem. Layouts with relatively little computational effort can be obtained.

We formulate a FLP based on Iterative heuristic Algorithm and Branch and Bound Algorithm to find optimal layouts. We create clusters first and then locate the clusters on multilevel. Layouts are then obtained for each level.

3. Three-Step Iterative Heuristic Technique

In [11] the justification for the use of a three-step procedure for the optimal location of departments on multilevel is presented.

The three steps are:
(i) Clustering technique for identifying groups of highly related departments,
(ii) Exact or efficient algorithm(Branch and Bound) for optimizing the intergroup communication cost, and
(iii) Iterative Heuristic technique for layouts of departments on each level.

In this paper, steps i) and ii) are discussed in detail.

4. Cluster Analysis Approach

Layout techniques have a number of shortcomings. The location order calculations are not designed to isolate groups of closely related departments and that the location process is only able to optimize the location of a department with respect to the locations of pre-located departments. Further, the processing time is high for large problems and that departments tend to split between levels which may not be acceptable[7].

3D layout problem can be mathematically written as:
Minimize Z=

$$= \frac{1}{2} \sum_{l=1}^{f} \sum_{k=1}^{f} \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ik} \, x_{jl} \, t_{ij} \, (h_{kl} \, w + d'_{ij} \, \delta_{kl}) + (1 - \delta_{kl}) \, (d'_{ic} + d'_{jc}) \qquad (1)$$

subject to

$$\sum_{i=1}^{n} x_{il} \, a_i \le A_l \qquad l = 1,2,.., f \qquad (2)$$

$$\sum_{i=1}^{f} x_{il} = 1 \qquad j = 1,2,.., n \qquad (3)$$

where, h_{kl} = absolute value of the vertical distance between k^{th} and l^{th} levels

$$\delta_{kl} = \begin{cases} 0 & for \quad k \neq 1 \\ 1 & for \quad k = 1 \end{cases}$$

d'_{ij}= horizontal distance between the location of departments *i* and *j* when both the departments are on the same level.
d'_{ic} and d'_{jc} = horizontal distance from department *i* and *j* to the circulation point of departments.
This is a mixed integer nonlinear programming problem of very great complexity. The first term of the distance expression represents the weighted vertical travel, the second term represents the horizontal travel when the departments are located on the same level. The variables for the problem are:

where R is the prescribed boundary of the level layout.

$$x_{ik} = 0 \quad or \quad 1 \qquad i = 1,2,...n \qquad k = 1,2,... f$$

$$x^{(i)}, y^{(i)} \qquad i = 1,2,...n$$

$$and \ (x^{(i)}, y^{(i)}) \in R$$

This problem is difficult to solve by any of the mathematical algorithms.
Rewriting the objective function into 3 parts:
Here, the third term represents the sum of intra-level travel costs

$$\frac{1}{2} \sum_{l=1}^{f} \sum_{i=1}^{n} \sum_{j=1}^{n} x_{il} \, t_{ij} \, d'_{ij} \, x_{ij}$$

The first term represents the inter-level vertical travel cost while the second term represents the inter-level horizontal travel cost.

If the departments with high interactions are suitably clustered, it amounts to the vanishing of contributions from the first two terms. For moderate and small values of t_{ij} the departments are not expected to be located on the same level and the contribution from the third term vanishes. Further, for departments located on different levels with moderate values of t_{ij}, the first term will dominate the second. Thus, if the objective function is approximated as the sum of the first and third terms, ignoring the second term, the problem splits clearly into a partitioning problem in the domain of quadratic assignment problem. The use of cluster analysis technique to maximize the adjacency within subsets of departments while minimizing the travel cost between clusters appears to be an extremely good approach. Thus after cluster formation, the layout problem constituting the first term of the objective function is minimized and later the level-wise layout problem is solved by minimizing the third term of objective function with appropriate set of constraints.

Thus, a three step procedure is used to solve the multi-level layout problem, i) Use of clustering technique for identifying groups of highly interrelated departments, ii) Use of an exact or efficient algorithm for minimizing inter-group communication cost, and iii) Use of iterative heuristic algorithms for obtaining layouts of departments at each level based on steps 1 and 2.

In the following section, no discussion of step 3 is presented since the algorithm has been described elsewhere.

4.1. Use of Cluster Analysis

The cluster analysis procedure is a four stage process: In the first stage, the cost of interaction(travel) is specified. The end product of this stage is a dendogram showing the successive fusion of departments, which culminates at the stage where all the departments are in one group. In the second stage, department areas are introduced which splits the single group into clusters of closely associated departments, each of which is small enough to accommodate on a level. We determine the *Intergroup Adjacency Matrix* as discussed in section 5. In the third stage, vertical layout problem is carried out. This is discussed in section 4. The last stage of the process consists of locating departments on different levels using a 2D-layout procedure.

4.2. Intergroup Adjacency Matrix

The procedure for clustering and determining inter-group adjacency matrix involves: i) Develop the *Adjacency Matrix* between pairs of departments. ii) Find the largest number of interaction(travel) between pairs of departments from the adjacency matrix. This is the cluster level to start with the cluster analysis procedure. Choose some cluster level interval. The pairs of departments which fall in this cluster level form a cluster and is designated by some cluster name for the purpose of identification. Decrease the cluster level by the cluster level interval chosen. Find the departments which fall in this cluster level. We go in for the third and subsequent cluster levels by further reducing by the cluster level interval. In this way, the departments falling in a particular cluster level are searched and identified by cluster name. iii) Plot the dendogram, and iv) A search is made in the reverse direction to consider clusters of desired area. If a cluster has an area less than the maximum permissible area per level/level, the identity and size of the cluster are stored in a table. A check is made for the non-repetition of an department. v) Construct an *Intergroup Adjacency Matrix* representing the interaction costs between clusters. Thus, if cluster C_i is obtained by grouping of departments belonging to the set I and C_j is another cluster representing the group of departments belonging to the set J, the element T_{ij} of the *Inter-group Adjacency Matrix* of clusters is:

$$T_{ij} = \sum_{k \varepsilon I} \sum_{l \varepsilon J} t_{kl}$$

4.3. Example

An example has been studied by clustering using clustering algorithm for 3D layout problems. The adjacency matrix for this example has been taken from [8]. 21 departments are to be clustered and the adjacency matrix containing the communication values between departments is known. The communication value , t_{ij} between departments *i* and *j* can be obtained from the adjacency matrix.

At level 1 of the clustering procedure, departments 3 and 4 are fused to form a cluster, since t_{34} is the largest communication value in the adjacency matrix. The number of communication values between this and the remaining 19 departments are obtained. Next largest entry is 182, and so departments 12 and 13 are fused to form a second group. The next largest entry is 151 and so departments 10 and 11 are fused to form the third group. All the groups are designated by cluster names. Since the number of departments for this example is 21, the first group is named as 22. Finally, fusion of the groups takes place to form a single group containing all the 21 departments. The dendogram is thus created.

Since the maximum permissible area per level is 19 units, the groups which have area less than or equal to 19 units are listed. Table 1 shows the identity and size of clusters for three levels. Table 2 shows the intergroup adjacency matrix representing the travel costs between clusters.

Table 1: Identity and size of cluster on three levels

Cluster No.	Department(s) forming the cluster	Cluster Area
1	1, 2, 17 (group number=38)	8
2	3, 4, 20, 21 (group number=28)	4
3	9, 10, 11, 14 (group number=27)	18
4	12, 13 (group number=23)	6
5	5	1
6	6	1
7	7	3
8	8	3
9	15	3
10	16	5
11	18	1
12	19	2

Table 2: Intergroup relationship matrix

No.	Area	1	2	3	4	5	6	7	8	9	10	11	12
1	8	0											
2	4	0	0										
3	18	72	90	0									
4	6	60	265	515	0								
5	1	6	25	0	11	0							
6	1	3	2	2	4	3	0						
7	3	21	18	58	165	0	2	0					
8	3	3	44	135	64	0	3	56	0				
9	3	3	44	135	64	0	3	8	56	0			
10	5	21	18	58	165	0	2	62	8	56	0		
11	1	38	37	3	59	1	0	3	0	0	3	0	
12	2	37	116	20	56	0	0	2	7	7	2	2	0

5. Formulation of the Problem

The vertical layout optimization problem can be written as:

$$C = \frac{1}{2} \sum_{l=1}^{f} \sum_{k=1}^{f} \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ik} \; x_{jl} \; t_{ij} \; d_{kl}$$

where x_{ik} = 1 if the i^{th} department is located on k^{th} level
= 0 otherwise
x_{jl} = 1 if the j^{th} department is located on l^{th} level
= 0 otherwise
t_{ij} = number of interactive trips between the departments i and j
d_{kl} = vertical distance between the k^{th} and l^{th} level
$= | k - l |$
n = number of departments
f = number of levels

The constraints are:

$$\sum_{i=1}^{n} t_{il} \; a_i \le A_l \qquad l = 1,2,.., f \qquad (1)$$

$$\sum_{i=1}^{f} x_{il} = 1 \qquad l = 1,2,.., n \qquad (2)$$

where a_i is the area required for department i and A_l is the available space on level l. Constaint (1) represents the restriction of available space on each level while the constraint of type(2) models the condition that a particular department must be located on any one of the levels[7].

The above problem is a quadratic assignment problem. There exists no reliable exact algorithms which can solve quadratic assignment problem where the number of departments is greater than 12. However, since the number of levels for medium sized problems is usually small and the number of clusters is much smaller than the number of departments, it is possible to attempt an exact solution if a suitable algorithm can be developed. An iterative heuristic algorithm is discussed in section 5 which is simpler and easier to implement.

6. Branch and Bound Algorithm

In this section, the development of a branch and bound algorithm is described.

The branch and bound algorithm proceeds in a sequence of steps. At each step of the algorithm, a partial layout is at hand where a set of departments are assigned to some locations. A lower bound, LB, on the cost of all possible completions of this layout is calculated. If *LB < cost C** of the available layout so far, proceed to allocate a new department. Otherwise, the partial solution is fathomed, the last assignment is prohibited and a new assignment is sought. The method of calculating the LB by solving the candidate problem and the progress of decision tree are elaborated below.

6.1 Calculation of Lower Bound

Assume that the departments belonging to the set I have already been assigned. In particular, department I is assigned to location v_i. A lower bound, LB, on the cost of this partial layout is:

$C_1 + C_2 + C_3$

where C_1 = Fixed interaction cost between already assigned departments
C_2 = Lower bound on the interaction cost between unassigned departments and assigned departments
C_3 = interaction cost among unassigned departments
Cost C_1 is computed as

$$\sum_{l=1}^{f} \sum_{k=1}^{f} \sum_{i \in I} \sum_{j \in I} x_{ik} \; x_{jl} \; t_{ij} \; | k - l |$$

Calculation of $C_2 + C_3$

Initially, consider the cost of locating the i^{th} unassigned department in v^{th} unoccupied location. This is calculated by adding i) fixed cost representing the travel costs between this department in the new location and all the pre-located departments and, ii) lower bound on the cost from other unassigned departments to department *i*. This lower bound is calculated as follows. If K is an unassigned department, the lower bound on the travel cost between departments *i* and *k* is given by l_k

$$l_k \ \min_{k \in K} \ u_{ik} \ |t - v|$$

where *t* is selected from the set of possible levels in which the department *k*(which belongs to the set *K* of departments not located) can be located.

Thus, a lower bound on the cost of assigning the i^{th} department to v^{th} level is given by:

$$b_{iv} = f_{iv} + \sum_{j \in I} u_{ij} \ |v - v_j| + \sum_{k \in K} l_k$$

where, v_j is the level in which the department *j* is located. Hence, $C_2 + C_3$ is obtained by solving the following integer programming problem.

$$C_2 + C_3 = \min \sum_{i \neq I} \sum_{v=1}^{f} b_{iv} \ x_{iv}$$

subject to

$$\sum_{v=1}^{f} x_{iv} = 1$$

and

$$\sum_{i=1}^{n} A_i \ x_{iv} \le A_v^{\ *}$$

where, $v = 1, 2, \ldots, f$

f = number of levels

n = number of departments

$x_{iv} = 0$ or 1

where A_v* is the available area on v^{th} level at this step of optimization. The first group of constraints represent the condition that each un-located department must find a location while the second group of constraints stipulate the level space restriction on each level.

This problem is a zero-one linear programming problem and is fairly easy to solve, from which the lower bound, LB= $C_1 + C_2 + C_3$ can be calculated.

A branch and bound scheme can now be employed. An algorithm for solving the problem is required which has been outlined above. A decision rule for branching from the lowest bound has been employed.

6.2 Branch and Bound Algorithm

The following notations will be used for branch and bound algorithm.

IOP = Initial Optimization Problem

Z_o = Current least upper bound on the optimal solution

$(CP)_i$ = Current candidate problem being explored.

Candidate list = active sub-problems (that are still candidate to be explored).

The branch and bound algorithm can be summarized as follows:

1. Initialize Z_o to be a large positive constant. Set $K=1$. Consider (IOP) to be $(CP)_i$
2. Set $i = 1$
3. Solve the optimization problem $(CPR)_i$. If $(CPR)_i$ has no feasible solution, neither does $(CP)_i$. The minimum value of $(CPR)_i$ is not less than the minimum value of $(CP)_i$
4. Bound $(CP)_i$. Apply an appropriate algorithm to $(CPR)_i$ to bound all solutions emanating from $(CP)_i$
4 If $(CPR)_i$ reveals a feasible solution of *IOP*, go to step 6, otherwise, go to step 8.
5 If $(CP)_i < Z_o$ go to step, otherwise go to step 10
6 Set Z_o equal to the solution value of $(CP)_i$ and go to step 10
7 If the bound calculated in step 4 is less than Z_o, go to step 9, otherwise, go to step 10.
8 Add $(CP)_i$ to candidate list and go to step 10.
9 If all the problems created by the last branch have been explored(bounded and analyzed), go to step 12, otherwise, go to step 11. That is, if $i=K$, go to step 12, otherwise, go to step 11.
10 Explore the next sub-problem among those created by the last branch. That is, let $i = i + 1$ and go to step 3.
12 If the candidate is empty, go to step 15, otherwise, go to step 13.
13. Remove a problem from the candidate list for branching. Label the problem *CP*. Decision rules to “branch from the lowest bound” or “branch from the newest active bound” or a combination of the two are generally used.
14. Branch on *CP*. Partition *CP* into *K* new sub-problems $(CP)_i$, $i= 1, 2,\ldots, K$ and go to step 2.
15. If a feasible solution has not been reached, go to step 17, otherwise, go to step 16.
16. The best feasible solution to date is an optimal solution for *IOP*. Go to step 18.

17. No feasible solution for *IOP* exists, go to step 18.
18. Stop.

Program Development

A program is developed for the branch and bound algorithm. The input consists of:

i) number of clusters
ii) number of levels
iii) maximum number of nodes
iv) number of variables(=number of clusters * number of levels)
v) adjacency matrix containing the interaction trips between clusters
vi) maximum area permitted per level, and
vii) area assigned to each cluster.

Calculate Lower Bound, that is, the interaction cost among already assigned clusters, interaction cost among unassigned clusters, and interaction cost from unassigned clusters to assigned clusters.

The Branch and Bound algorithm proceeds as shown in Fig. 1.

6.3 Tree diagram

An example with number of clusters as 6, and number of levels as 3 is considered for generating the tree diagram. The maximum number of nodes is 100 and the area permitted per level is 11 units. Fig 2 shows the part of decision tree for this problem. The circles are called nodes and represent the set of all possible feasible solutions that can be reached from the node. The number in each circle is the node number and sequentially represents the order in which the branch and bound algorithm is carried out. At each node, the approximate problem is solved to obtain the lower bound and this value is also shown in the decision tree. The lines connecting the nodes are the branches. Nodes 56-58 are terminal nodes at this point because branches do not emanate from them. The branching is done as follows:

Department 1 is allotted to level 1 and the resulting sub-problem has zero as the lower bound. Similarly department 1 is allotted to level 2(node 3) and level 3(node 4). Since node 3 has the smallest lower bound, it is selected first for further branching. With the constraint that department 1 is a candidate for all the three levels, department 2 is allotted to all the three levels. Terminal nodes 2, 4, 5, 6, 7 are still candidates for the optimal solution. The minimum lower bound value among all the candidate nodes provides a lower bound for the problem, 55 at nodes 2 or 4. Branch from node 2 or 4 since it has a smaller lower bound than any other candidate node. Assign department 1 to levels 1, 2 , 3. Node 4 has the least bound. Branch from node 4 and proceed as above.

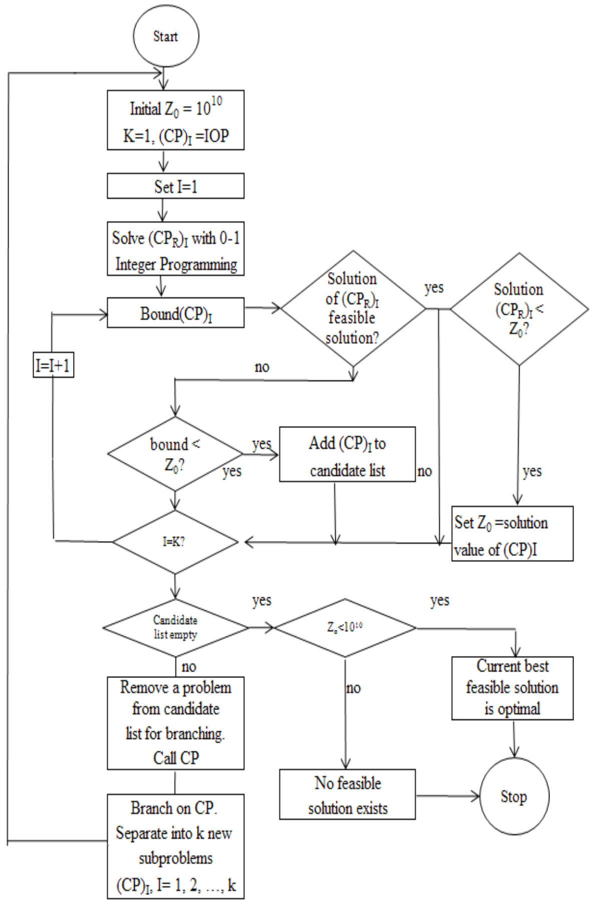


Fig. 1 Bound and Bound Technique

Fig. 2 shows that nodes 67(not shown) and 68 yield a feasible solution with a value of 395 and that no other node has a smaller value than this. Thus, the solutions corresponding to nodes 67 and 68 must be optimal. Departments 3 and 4 are allocated to middle level. Departments 1, 2 remain together and can be allocated to either first or third level and departments 5, 6 can also be allocated to either first or third level.

Departments/Clusters	Levels	Departments/Clusters
5,6	Third	1,2
3,4	Second	3,4
1,2	First	5,6

Minimum cost=395 units

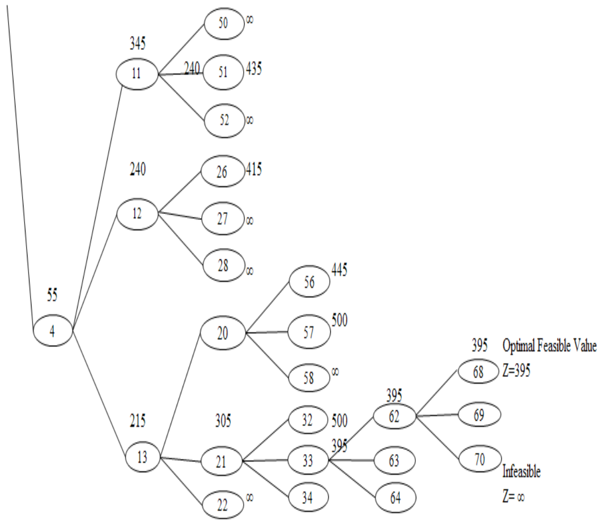


Fig. 2 Tree-Branch and Bound Algorithm(part-of)
Number of levels=3
Number of departments=6
Area/level=11 units

7. Iterative Heuristic Algorithm **for Multilevel Problem**

We discuss below an iterative algorithm for allocation of clusters to minimize vertical communication costs. In each iteration a hierarchical procedure for 2-D location problem is made use of.

The iterative algorithm is described below.

Step 1. Construct an inter-group adjacency matrix representing the communication costs between clusters. Set $i = 1$. Set all d_{ij}'s to unity.

Step 2. Compute the travel cost matrix(Tij) wherein each element of the matrix is given by
$t_{ij} * d_{ij} = T_{ij}$

Step 3. Rank the clusters for location on the basis of travel cost. The cluster having the maximum travel cost with other clusters should be ranked first. The cluster having the largest travel cost with the previously located cluster should be ranked next for location. Thus at any step, the cluster having the maximum sum of travel costs with all the previously located clusters will be ranked next. Hence a complete ordering of clusters can be established.

Step 4. Locate the first cluster at the middle level(mth limitation on the number of levels and available area on each level.

Step 5. As in Step 4, complete the $(m+1)$th level.

Step 6. Repeat Steps 4 and 5 until all the clusters are located.

Step 7. Calculate the vertical distance representing the weighted distance between clusters(or between levels) taking the middle level as datum. Compute the communication cost and print the layout.

Step 8. Perform Steps 2 to 7 until a sufficient number of alternate layouts are available.

Step 9. Select the least cost layout.

8. Results

A program is developed for FLP based on Branch and Bound algorithm and the Iterative Heuristic algorithm. The input consists of number of levels, adjacency matrix containing number of travel trips between clusters, area(in units) of each cluster, and maximum area permitted per level. The program requires to determine pairs of departments forming a cluster, checking if a cluster has been considered for a particular level, and also allocating departments on different levels. An optimal layout design is thus obtained.

Tables 3a, 3b and 3c show the allocation of departments for three, four and five level examples. To test cluster analysis program, layouts were obtained for each level. Table 3a presents the first example with the number of departments as 12 which are located on three levels. The optimal departmentive(cost) of 2094 units and 2135 units was obtained using Branch and Bound algorithm and Iterative Heuristic Algorithm respectively. The second example involved the allocation of 13 clusters to four levels. The cost of locating departments is 3495 units and is shown in table 3b. The third example involved the allocation of 13 clusters on five levels. This is shown in Table 3c.

Table 3a: Representation of Department location on three levels
Number of clusters= 12
Maximum area permitted/ level = 19 sq units

Level	Iterative Heuristic Clusters	Branch and Bound Clusters
Third	$\boxed{1}$,6, 9, 10	$\boxed{1}$, 5, 6, 10,11, 12
Second	$\boxed{2}$, $\boxed{4}$,7, 8, 11, 12	$\boxed{2}$, $\boxed{4}$, 7, 8, 9
First	$\boxed{3}$, 5	$\boxed{3}$

Note: $\boxed{\ }$ represents the clusters
Iterative Heuristic Algorithm, total cost = 2135 units
Branch and Bound Algorithm, total cost = 2094 units

Table 3b: Representation of Department location on four levels
Number of clusters= 13
Maximum area permitted/ level = 14 sq units

Level	Iterative Heuristic Clusters	Branch and Bound Clusters
Fourth	$\boxed{1}$ $\boxed{2}$,6	$\boxed{1}$, 13,
Third	$\boxed{3}$, 13	$\boxed{2}$ $\boxed{4}$,7, 12

Second	$\boxed{4}$,5, 9, 12	5, 8, 10, 11, 13
First	7, 8, 10, 11	$\boxed{3}$, 6

Note: $\boxed{\ }$ represents the clusters
Iterative Heuristic Algorithm, total cost = 3570 units
Branch and Bound Algorithm, total cost = 3495 units

Table 3c: Representation of department location on five levels
Number of clusters= 13
Maximum area permitted/ level = 12 sq units

Level	Iterative Heuristic Clusters	Branch and Bound Clusters
Fifth	$\boxed{1}$	$\boxed{1}$
Fourth	$\boxed{2}$,7, 8, 13,	8, 9, 13,
Third	$\boxed{4}$,11, 12	$\boxed{4}$,11, 12
Second	$\boxed{3}$	$\boxed{3}$
First	5, 6, 9, 10	$\boxed{2}$,5, 6, 7, 10

Note: $\boxed{\ }$ represents the clusters
Iterative Heuristic Algorithm, total cost = 3570 units
Branch and Bound Algorithm, total cost = 3822 units

Table 4 presents the comparison of results obtained by Iterative Heuristic and Branch and Bound Algorithm. The results show that the results obtained by Branch and Bound Algorithm are superior to those obtained by Iterative Heuristic Algorithm. The cost of allocating clusters on different levels is shown in units. Both the methods start by locating the clusters in the middle first, that is, the m^{th} floor.

Table 4: Comparison of Branch and Bound Algorithm and Iterative Heuristic Algorithm

Algorithm	Number of clusters		
	12	13	13
	Number of clusters		
	3	4	5
Branch and Bound	2094.00	3495.00	3822.00
Iterative Heuristic	2135.00	3570.00	3938.00

Tables 5a and 5b compare both Iterative Heuristic algorithm and Branch and Branch algorithm in terms of total cost for locating clusters on three and four level examples. Total cost is the sum of intra-level travel cost, inter-level horizontal cost, inter-level vertical travel cost, and weighted vertical travel cost. For table 5a, the intra-level travel cost is 4412.19 units and the inter-level horizontal cost is 5561.4 units(displayed in Fig. 2). Table 5b shows the total cost which is the sum of intra-level travel cost(=2776.28) units, the inter-level horizontal cost (= 5193.1) units, inter-level vertical travel cost, and the weighted vertical travel cost.

The use of Iterative Heuristic Algorithm for horizontal and vertical movement gives good results as compared to the Branch and Bound Algorithm(which is accurate). In both the cases, the layouts obtained are very practical.

Table 5a: Cost of allocating clusters for three-level example
Number of clusters= 12
Maximum area permitted/ level = 19 sq units

Factor	Vertical travel cost		Weighted vertical travel cost		Total cost	
	I	II	I	II	I	II
2.0	2135	2094	4270	4188	14243.59	14304.9
4.0	2135	2094	8540	8376	18513.59	18492.9
8.0	2135	2094	17080	16752	27053.59	26868.9

I is the Iterative Heuristic Technique for vertical travel cost.
II is the Branch and Bound method for vertical travel cost.

Table 5b: Cost of allocating clusters for four-level example
Number of clusters= 13
Maximum area permitted/ level = 14 sq units

Factor	Vertical travel cost		Weighted vertical travel cost		Total cost	
	I	II	I	II	I	II
2.0	3570	3495	7140	6990	15109.38	17067.5
4.0	3570	3495	14280	13980	22249.38	24057.05
8.0	3570	3495	28560	27960	36529.38	38037.05

I is the Iterative Heuristic Technique for vertical travel cost.
II is the Branch and Bound method for vertical travel cost.

Figure 1a and 1b show the least cost for three-level example using cluster analysis approach and Branch and Bound algorithm respectively.

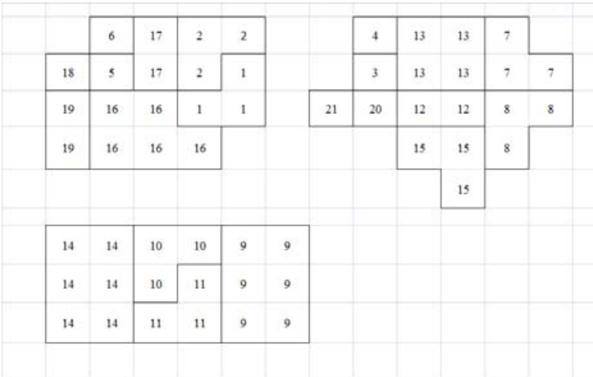
Comparing the total cost by the algorithms involving clustering technique and the corresponding costs by the 3-D algorithm, it is clear that the clustering technique is far superior particularly when the factor for vertical movement is high. Thus the treatment of the 3D communication cost minimization problem as a K-partition problem is essential.

9. Conclusion

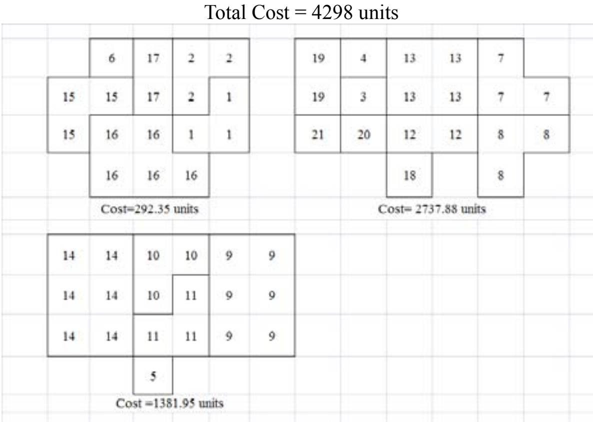
It is interesting to note that the algorithm is extremely efficient and easy to implement. It yields comparable results at a fraction of computing cost. It is ideally suited for solving reasonably medium-sized problems. The Iterative Heuristic algorithm gives reasonably good results at negligible computing cost. For large size problems, it gives very efficient layouts. Because of the closeness of these results, it is postulated that the deviation of these solutions from the exact optimum in large problems will be marginal.

Use of Iterative Heuristic Algorithm after clustering for both horizontal and vertical travel cost yields good results.

The Branch and Bound Algorithm in convention with the new procedure described for the determination of Lower Bound of the candidate problem is suitable for medium-sized assignment problems in facilities planning. The Iterative Heuristic Algorithm gives very good results, close to the exact solution obtained by Branch and Bound Algorithm.



a) Branch and Bound Technique
Total Cost = 4298 units



a) Iterative Heuristic Technique
Total Cost = 4412.19 units

Fig.3 Layouts obtained by "Branch and Bound " and "Iterative Heuristic Technique"
Number of levels = 3
Max area permitted/level=19 units

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Rekha Bhowmik received her B.E., M.E., and PhD degrees, from Indian Institute of Technology, India. Her research interest includes Data Mining, Database Security, Spatial Database, Text Mining, Optimization Techniques. She is a member of IEEE, ACM, ISCA.