Real-Time Interactive Fluid Animation Technique based on Linear Convolution

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Summary
With the recent advancement in computer hardware performance, computer graphics technologies that yield realistic and fluid representation of water, fire, smoke, blaze and special effects have come to play important roles for producing movies, games, advertisements and animation. Although increasing computer processing capabilities enable real-time fluid simulation using complex fluid algorithms, realistic representation of water still requires substantial amounts of computation and production time, prompting needs for studies of various technologies for achieving balance between physical properties and realistic visual effects. This paper proposes a technique for real-time simulation of various physical transformations of water surface according to the interactions between water surface and objects using the height field, linear convolution and bounding sphere. The proposed approach is expected to be applicable in interactive simulations such as games that feature communication with the outside environment.
Key words:
Simulation, Height field, Linear convolution, Water

1. Introduction

One of the ultimate goals of computer graphics(CG) is to implement a virtual world that is indistinguishable from the actual world. The field of fluid simulation is at the forefront of such efforts, and there are various studies being conducted to achieve realistic representation of fluids. With developments in computer hardware since the 1990s, CG algorithms that had been unfeasible for simulating fluids have now been made possible, and the technologies are applied in various fields and used to provide visually realistic representation of fluids such as water, fire, explosion, smoke and fog. Whereas physical movement of fluid involves continuous time and space, the movement has to be simulated according to discrete time and space in a computer graphics system. Although CG fluid representation techniques emphasize visual over physical reality, realistic simulation of various fluid properties requires substantial amounts of calculation as well as system resources for real-time simulation and rendering. Consequently, a trade-off between physical performance and realistic visual quality becomes inevitable when simulating the effects displayed by water, which involves interactions between a particular object and fluid as well as between different types of fluids. In order to perform real-time simulation of movements in an extensive area such as lake or ocean, various fluid properties should be represented to a degree that can represent physical and mathematical properties in real time, and fluids must be expressed by calculating physical approximations that reproduce fluid movements in real time while maintaining visual reality. In other words, various research efforts are necessary to find a balance between physical performance and realistic visual performance. While the interactions between water and objects have been represented with non-real-time fluid simulation that uses physically accurate rules, there have not been many studies on real-time simulation between a static large-scale liquid and objects. This paper proposes a water surface transformation technique using linear convolution and bounding sphere to perform real-time simulation of interactions between water surface and objects.

2. Related Work

In recent several years, studies of physical based simulation technology of various forms of fluids have been increasing. In early 80s, research of expressing sea level was done by re-modeling waves with hermeneutic functions, especially with Height field.
In 1986, Fournier and Reeves showed sea level by Gerstner model which explains that movement of sea level is from circle/oval shaped moving particles[4]. Miller and Pearcel introduced simple particle system model which includes interaction in limited area. Desbrun and Cani published viscous fluid flow simulation using only particle itself, and introduced Smooth Particle Hydro-dynamics[3]. Foster and Metaxas simulate solutions of the full Navier-Stokes equations by using a voxel representation for the volume and marker particles to trace the surface. Thanks to this representation, their algorithm can produce e. g. breaking waves and overturns. Surface tension is simulated by altering pressure gradients. To stabilize the method for large time steps, a damping term has to be

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introduced. And Foster and Metaxas developed complete 3D water simulation with first adoption of grid in graphics[6]. Foster and Fedkiw etc solved Navier-Stokes equation - one of the most famous theory in hydrodynamics - to suitable form for computer animation. And suggested answers to calculating flow of fluids like water. Stam used Semi-Lagrangian technique to solve instability occurred from current calculation of simulations and suggested simulation technique for smoke[16]. Carlson created coupling of fluid simulation and rigid simulation[2]. Terzopoulos applied simple 1D model to their spring, introduced plasticity to animation based on physics[16]. Goktekin added elasticity and related to flexible flow of Water Solver and make possible of fluid animation against non-Newtonian fluids[8]. Kothe and Brackbill studied of applying FLIP(Fluid Implicit Particle) to non-compressible fluid. In addition, they used Material Point Method(Elastic flexible limited element formula) to compressible FLIP[9]. Studies of controlling movement of fluids activated after Foster and Fedkiw suggested solution to controlling flow of fluids by determining speed level to grid cell[5]. After that, Stam and Zoran Popovic controlled smoke simulation by key frame method. Based on physics, Yongning Zhu and Robert Bridson introduced simulation of sand animation which activates similar to that of fluids[17].
Weimer and Warren note that certain multi-grid partial differential solvers can be formulated with help of vector subdivision operations. The method they describe assumes a slow flow, i. e. viscosity dominates inertial behavior, and is therefore not well-suited for water simulation[18].

3. Animation of Water based on Linear Convolution

CG techniques for representing fluids consist of modeling, simulation, reconstruction and rendering. Fluids that are subject to CG representations can be classified into liquids and gases, and liquids can be categorized further into static large-scale liquids such as an ocean and dynamic small-scale liquids[18]. Flow on the surface of large-scale water such as an ocean can be represented by a directional flow such as a wave or a propagation that spans out on a calm lake. The most dynamic movements on a static large-scale liquid are the ripples and waves that occur on the water surface due to fluid-object interactions, and such movements can be simply expressed using Trochoid sinusoidal waves such as sine and cosine. However, since wave models calculated with periodic functions are continuous, it is difficult to recreate turbulence-like rapid transformations. This paper generates movements on water surface by applying the height field and linear convolution.

3.1 Height field

Height field is defined for rectangular regularly spaced grid UxV and associates an elevation h to each position (u, v) in the grid. Height field is a convenient structure to define functions of two parameters, represent real or artificial terrain. Such representation is used to describe geographical data and is called elevation maps. Often height fields are used to represent static terrain models. In such cases algorithms can move significant amount of calculations to pre-processing stage, where intermediate structures are related optimizing run-time performance.

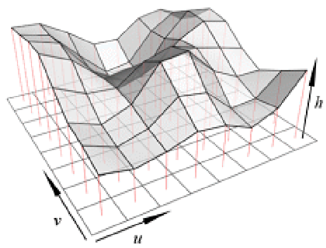

Fig. 1 Height fields

A height field allows easy application and improved processing speed due to its small and simple terrain data structure. Fig. 1 displays the elements of the height field. The height of each peak can be modified to effectively create animation of water surface[16]. Water movement is expressed using a height field based on movement in a specific direction according to time. Since water flow is affected by the heights of surrounding waves at a previous instant, current and previous height fields are necessary to create a natural water surface movement. Accordingly, this study uses two height fields at continuous time points t-1 and t for creating water surface animation.

3.2 Linear Convolution

Convolution is a technique typically used in image processing for domain-based calculations such as sharpening and boundary detection. Convolution uses weighted averages of adjacent values to obtain a value at a particular location[17]. The domain filter deployed in representing a water surface using convolution performs distribution so that the sum of the values becomes 2, and the quality is determined by the number of samples selected.
While 4 sampling guarantees the fastest rendering with a compromised quality, 12 sampling yields a high quality with slow rendering.

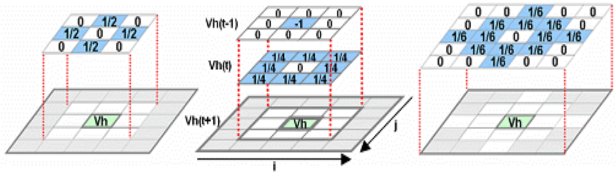

Fig. 2 The domain filter of linear Convolution

In order to create a continuous ripple movement, this paper employed linear convolution that applies an 8 sampling domain filter that has an adequate rendering speed and includes the surroundings of the basis peak, as shown in middle of Fig. 2.

V_h is the height at a specific location calculated by masking the domain filter at the height fields at t and t-1, and the height of the water surface has to be updated as a part of the height field at t+1. Wave motion animation using linear convolution is created by applying the domain filter mask to the height field at t as in Eq.(1), deducting the height value at t-1 and storing it in the height value at t+1. Once linear convolution process is complete for every peak, the height field values are updated.

$$Vh_{(x,z)}(t+1)=\left(\sum_{i=1}^{-1}\sum_{j=1}^{-1}\left(Vh_{(x+i,z+j)}(t)\times C(i,j)\right)\right)-Vh_{(x,z)}(t-1) \quad (1)$$

C(i, j) is the domain filter of linear convolution for generating wave motion animation, and it can be expressed as Eq.(2).

$$C(i,j)=\left\{\begin{matrix} 0, & \text{if } i=j=0 \\ 1/4, & otherwise \end{matrix}\right., \quad i,j\in(-1,0,1)\Bigg\} \quad (2)$$

3.3 Lifetime of Water Wave

When an object is dropped on water surface, it creates a wave, which eventually expires. The lifetime of a water wave can be determined by an element referred to as the damping factor(DF). DF is a positive real number that is multiplied to the height of water yielded by linear convolution. It determines how quickly a wave will expire[15] and is expressed with a real number between 0 and 1. A DF of 0 signifies a wave that disappears instantly at the next instant, and 1 represents a perpetual wave that does not expire. Fig. 3 and Fig. 4 display the variation of the basis water surface height and lifetime changes using DF, respectively.

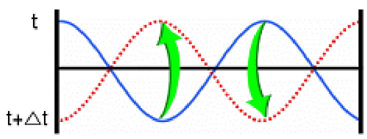

Fig. 3 The variation of the basis water surface height

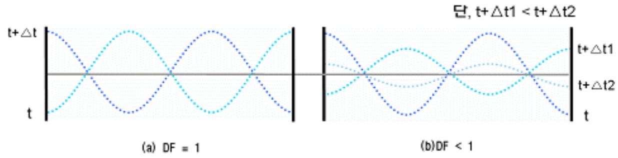

Fig. 4 The variation of lifetime changes using DF,

DF is multiplied to $V_{h(x+z)}$ t+1, the height value obtained by performing linear convolution and incorporated into (Eq. 1) to yield $V_{h(x+z)}$ t+1as written in Eq.(3).

$$Vh_{(x,z)}(t+1)=\left(\left(\sum_{i=1}^{-1}\sum_{j=1}^{-1}\left(Vh_{(x+i,z+j)}(t)\times C(i,j)\right)\right)-Vh_{(x,z)}(t-1)\right)\times DF \quad (3)$$

3.4 Creating a wave motion using a bounding sphere

Water surface displays a set of constant light ripples or a changing pattern as in a wave progressing with force and direction. In order to produce animation of various water surfaces, this study used a bounding sphere to vary the current water surface height field value $V_{h(x,z)}(t)$, generate diverse forms of waves on the water surface and create natural fluid movements. A bounding sphere transforms water surface according to its location and the scope of influence, and it can be constructed by defining the sphere direction(SD), sphere radius(SR) and sphere height ratio(SHR).

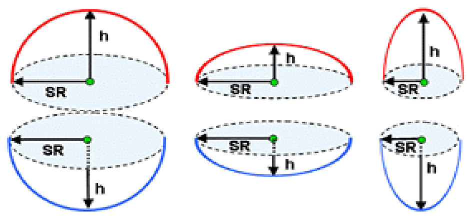

Fig 5. Range of influence of a bounding sphere according to size and height ratio

The distance from (x, z) to (px, pz) can be calculated from Eq.(5), and PH, the height of (px, pz) from Eq.(4).

$$PR=\sqrt{(x-px)^2+(z-pz)^2} \quad (4)$$

$$PH=\sqrt{SR^2-PR^2} \quad (5)$$

The calculated PH is the height of the bounding sphere when it is spherical, and the final $V_{h(px,pz)}(t)$, with SD and SHR incorporated can be written as Eq.(6).

$$Vh_{(px,pz)}(t)=Vh_{(x,z)}(t)+(PH\times SD\times SHR) \quad (6)$$

Various methods of generating wave are necessary for realistic animation of water surface. A water wave can be expressed with a combination of multiple wave motions. In turn, multiple bounding spheres with different characteristics can be used to represent a flow of constant wave flowing in a particular direction on water surface. A water wave created with bounding spheres propagates while generating light ripples around itself. Using these properties, random bounding spheres with different properties can be created at random positions as shown in Fig. 6 to express a water wave.

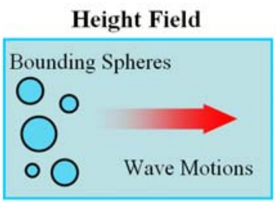

Fig. 6 The bounding spheres of Created at random positions

A combination of bounding spheres can be used as shown in Fig. 7 to represent a constant stream of light ripples or an intense fluctuation such as a wave.

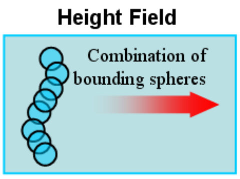

Fig. 7 The combination of bounding spheres

3.5 Propagation of water wave

A wave motion involves a constant propagation with direction and force as in ripples in a lake or waves in a ocean. Accordingly, DT must be adjusted according to the direction of wave motion to apply different DFs based on the properties of the wave. A larger wave interferes with the surface of a smaller wave. Wave propagation can be expressed by adjusting the DF moving in the direction of the wave is greater than that of the new position and applying the uniform BDF to the previous position of the propagating wave.

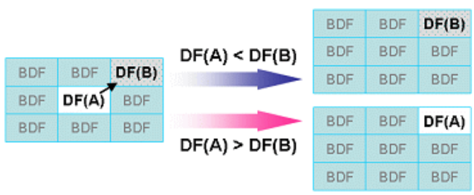

Fig. 8 the DF moving and DF update

Fig. 8 displays the DTs that are updated according to the DF magnitude at each position with the propagation of the wave. Animation of water surface is created by linear convolution that performs calculation by obtaining values adjacent to each peak. Hence even if DF is set at 1.0 to prevent attenuation of the wave, some attenuation will take place due to the DF less than 1.0 as in the case of BDF applied to most part of the water surface. Since such attenuation occurs when DF is greater than BDF, the DF value must be further adjusted, and an add damping factor(ADF) becomes necessary. When a wave is generated by creating a bounding sphere at a particular location, DF is more affected by its radius or cross-section than the sphere's depth. ADF is multiplied to DF to calculate a new DF value when a wave is created to take attenuation into consideration. The ratio of ADF depends on the magnitude of SR of the bounding sphere as shown on the right of Fig. 9.

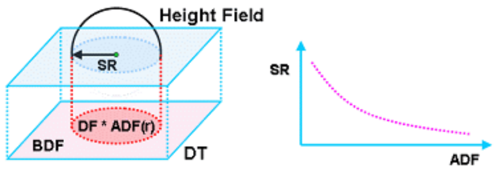

Fig. 9 The relation between SR and ADF

Based on various tests, this study was able to identify an ADF value that minimizes wave attenuation according to the radius of the bounding sphere. A adequate ADF value was obtained based on the depth of the bounding sphere that creates the wave, difference in the direction of propagation and the SR of the bounding sphere. The ADF value was then used with Maple to obtain an n-th order Lagrange interpolating polynomial as shown in Eq.(7).

$$ADF(SR) = \left(\frac{1.987530510}{10^8} SR^9 - \frac{1.914461831}{10^6} SR^8 + \frac{7.58388185}{10^5} SR^7 - \frac{1.634432542}{10^3} SR^6 + \frac{2.128301622}{10^2} SR^5 - \frac{1.745222756}{10} SR^4 + 0.9026511703 SR^3 - 2.831677549 SR^2 + 4.837549162 SR - 2.307788355 \right) \quad (7)$$

The SR range was determined for the bounding sphere to improve the calculation efficiency for producing water animation, and the ADF(SR) according to the radius of the bounding sphere was calculated in advance. Up to this point, a DT was created for applying different DFs according to wave properties to create a water wave propagation, DF was shifted to create a wave motion, and ADF(SR) was applied to account for the attenuation that varies according to SR. Fig. 10 displays a wave propagation created by a bounding sphere in a specific region when DF was shifted from left to right.

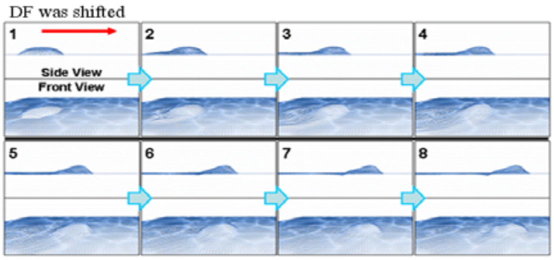

Fig. 10 A wave propagation created when DF was shifted

4. Interaction between water surface and object

Techniques for presenting water vary according to the presence of external influences. A water surface influence by the external environment such as a drop of an object or the terrain of the ocean floor creates new waves or is modified into a new configuration. Since water animation based on linear convolution is able to freely adjust the height fields that comprises a water surface, various shapes of water surface can be created in real time, and a transformed water surface creates new waves as convolution progresses. The interaction between water surface and objects produce diverse effects according to external elements or circumstances.

4.1 Interaction between different wave motions

Wave motions with different characteristics can be attenuated by classifying DFs(Fig. 11), creating a table with DFs that correspond to each peak and producing water surface animation by applying a different DF for each peak as convolution progresses.
Fig. 12 displays how bounding spheres with different DFs are created on the water surface and damping table values are provided.

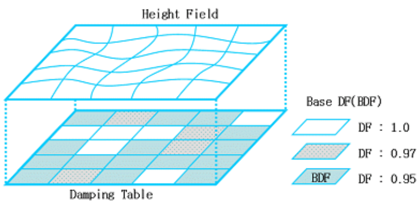

Fig. 11 Classifying damping table

DF values for the domains of bounding sphere A and B were set at 1.0 and 0.97, respectively and the SHR was fixed at 0.8. With a DF of 1.0, bounding sphere A attenuated somewhat due to the lower surrounding DFs, but the wave motion was maintained.

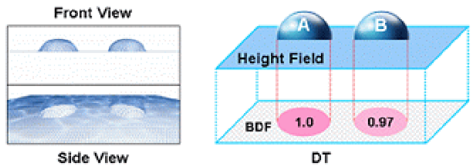

Fig. 12 Bounding spheres with different DFs

With a DF of 0.97, bounding sphere B attenuated faster than A and almost expired at the time indicated by image 8 in Fig. 13.

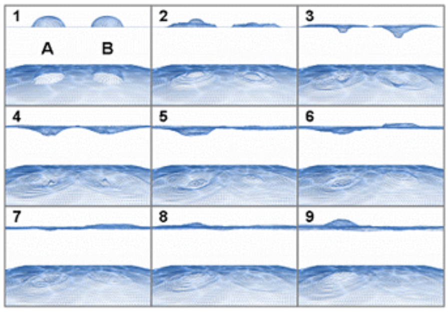

Fig. 13 The attenuation of water surface

4.2 Interaction between fixed object and water surface

An ocean wave is attenuated as it approaches the shore due to the friction with the terrain of the ocean floor. Fig. 14 displays a wave motion affected by the terrain. The interaction with the terrain can be expressed by attenuating the height of the water wave motion while taking into consideration the height of the wave and the depth of the terrain.

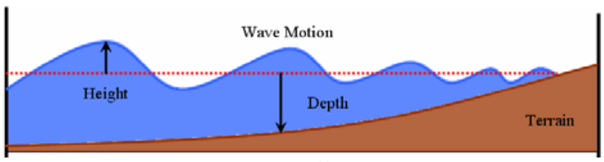

Fig. 14 A wave motion affected by the terrain

The depth ratio is defined by the user and it is a real number value that is multiplied to the wave height and compared with the depth of the terrain to determine the attenuation of the wave. Table1. exhibits wave attenuation with terrain considerations, where the wave height calculated with linear convolution is $V_{h(x,z)}(t+1)$.

Table 1: Wave attenuation with terrain considerations

```
{
 linear convolution processing!
     .....
     if( ( Vh(x,z)(t+1)× Hr) ≥ | Th| )
          Vh(x,z)(t+1)= Vh(x,z)(t+1)× BDF
}
```

T_h is the depth of the terrain and H_r is the ratio of wave height and terrain depth. A dynamic water wave is also affected by collision with the terrain above the water surface.

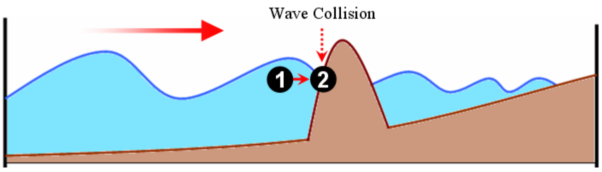

Fig. 15 Affected by collision with the terrain above the water surface

When a wave collides with the terrain as it propagates from position 1 to 2 as shown in Fig. 15, the interaction can be represented by adjusting DF and the height displacement. When the damping and height of wave is shifted, the presence of terrain at the next location can be assessed to predict a collision. When moving DT values as shown in Fig. 16, it should be examined whether there is a terrain that protrudes above the surface in the direction of wave propagation. It there is no terrain, DF is moved and the position prior to the movement is replaced with BDF. As with the case with DF, terrain must be examined before moving a height value. If there is no terrain, the height value calculated with linear convolution is moved and multiplied with the DT value of the new position to obtain the final height value.

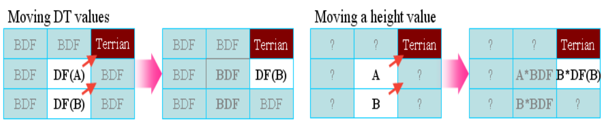

Fig. 16 Moving DT and height values

4.3 Interaction between moving object and water surface

Since a water wave involves a secondary spherical waveform created at the source of a wave according to Huygens' Principle, animation for the wave motion can be created if the height value is modified into the shape of a sphere. When objects of an identical size collide with the water surface, the surface varies according to the weight and velocity of bounding spheres. The position and size of a bounding sphere affects the height field at the current time t, $V_{h(x,z)}(t+1)$. However, since wave motion animation calculates the height at t+1 based on linear convolution by combining the heights at t and t-1, the height field at t-1 should also be adjusted. In order to reflect object's motion properties of the height field at t-1, this paper defined properties such as object's weight and velocity as acceleration factors(AFs) and used them in determining the height field at t-1. When objects of an identical size collide with the water surface, the transformation on the surface varies according to the weight and velocity of bounding spheres. The position and size of a bounding sphere affects the height field at the current time t. However, since wave motion animation calculates the height at t+1 based on linear convolution by combining the heights at t and t-1, the height field at t-1 should also be adjusted. Since the height at t-1 is deducted from the height at t and reflected on the height at t+1, the height at t-1 can be increased or decreased according to object properties to create repulsion of the wave generated. Taking these facts into consideration, this paper defined AFs that are proportional to object's weight and velocity, and used them in determining the height field at t-1.

AF is determined by the characteristics of the basis object and the target object defined by the user. If we denote the basis object as B, target object O, weight w, velocity v, and size(SR) s, AF can be obtained from Eq.(8).

$$AF = \frac{(Ow \times Ov) \div Os}{(Bw \times Bv) \div Bs} - 1 \quad (8)$$

AF modifies the height field at t-1 so that the variation of the water surface according to object movement can be induced during the animation process. When a bounding sphere modifies the current height field $V_{h(x,z)}(t)$, AF is applied according to object properties and the height field at the previous time $V_{h(x,z)}(t-1)$, can be subsequently modified with Eq.(9).

$$Vh_{(x,z)}(t-1) = Vh_{(x,z)}(t) - Vh_{(x,z)}(t) \times AF \quad (9)$$

AF modifies the height field at t-1 so that the variation of the water surface according to object movement can be induced during the animation process. When a bounding sphere modifies the current height field $V_{h(x,z)}(t)$, the height field at the previous time $V_{h(x,z)}(t-1)$ can be subsequently modified with Eq.(10).

$$Vh_{(x,z)}(t-1) = Vh_{(x,z)}(t) - Vh_{(x,z)}(t) \times AF \quad (10)$$

5. Experimental Results

The techniques proposed in this paper were simulated using a system with a 2.6GHz CPU, 1GB memory, and 600Mhz GPU implemented with DirectX SDK 9.0. Simulations of interactions between terrain and water, and between moving objects and water waves were performed on a screen resolution of 800 * 600.

5.1 Interaction between terrain and wave

A height field that has a 1:1 correspondence with the water surface was used to simulate the interaction between a wave propagating with direction and force and the ocean floor terrain. For experimental purposes, a terrain completely submerged under the water surface and another terrain with a ridge protruding like an island were created using gray-scale maps as shown in Fig. 17.

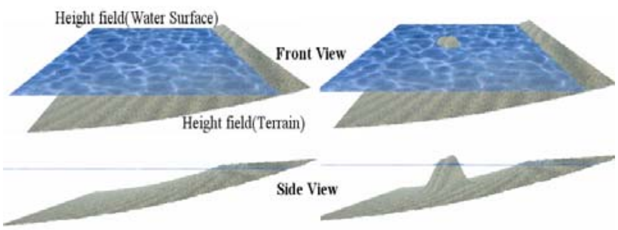

Fig. 17 Two type of terrains

Fig. 18 displays the simulation results with the depth ratio Hr at 10.0, indicating the shape of the wave according to the terrain and the distance to the water surface.

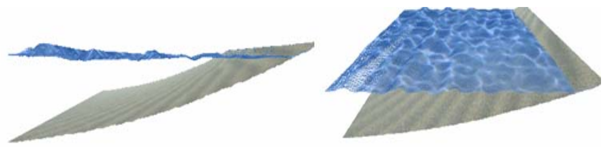

Fig. 18 Affected by the friction with the terrain

Created by a combination of wave motions, an ocean wave is affected by the friction with the terrain based on the established value of Hr and attenuates as it approaches the shore. In addition to being affected by the terrain on the ocean floor, a water wave creates interactions with the terrain above the water surface. When a wave collides with a terrain above the water surface, the wave's DF is not moved and the position of the current wave is replaced with BDF to create a simple representation, which is depicted in Fig. 19. The arrows in Fig. 19 indicate the direction of wave propagation. A-1 illustrates the deformation of the water wave at the moment of collision with the terrain, and A-2 displays how a secondary wave is created by the collision in the opposite direction. B-1 depicts the moment when a wave split according to the terrain as soon as it escapes the terrain, and B-2 exhibits how the split and expired wave is slowly recreated by adjacent waves. Accordingly, such water surfaces can be easily expressed by determining DF's movement.

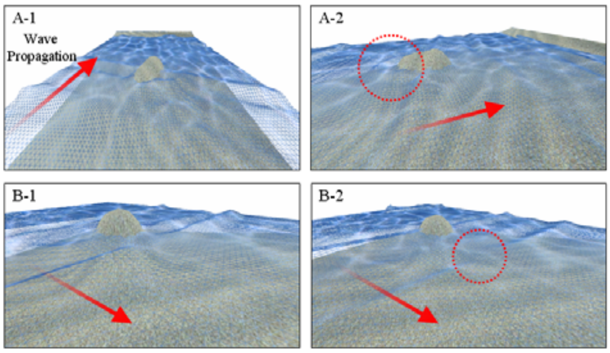

Fig. 19 Interaction between terrain and wave

5.2 Interaction between moving objects and wave

The standard shape of the bounding sphere is a perfect sphere with an SHR of 1.0. In order to represent the an interaction between moving objects and wave, a dynamic object was defined as the bounding sphere with an SR of 5.0, AF of 0 for determining repulsion, SHR of 1.0 and DF of 0.95. Fig. 20 displays the results of simulation conducted without any movement of the water surface to demonstrate in detail the process of a spherical object transforming the water surface. After the height field was modified by the interactions between the water surface and the object in 2, 3, 4 and 5, linear convolution was performed to create the interaction with the water surface.

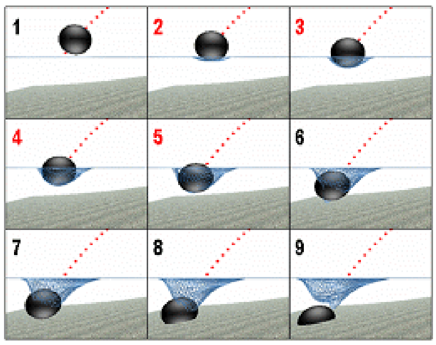

Fig.20 Interaction between moving objects and wave

6. Conclusion

This paper described the animation process of water surface using linear convolution. Waves were created and interactions with the external environments were expressed in real time by defining a bounding sphere model and varying the height of the water surface.

The proposed technique managed to create wave propagation that is difficult to express due to the properties of linear convolution and generated various types of waves by combining wave motions created by bounding spheres.

Furthermore, a wave motion animation was produced by colliding two waves generated with an interval and offsetting the waveform to make it look like that the wave was moving in the direction of propagation. The performance of simulation was determined by the height field and the results were sufficient for real-time simulation. Adding bounding spheres and increasing interactions did not affect the simulation performance significantly, a strong fact that is expected to resolve the shortcomings of limited interactions.

Moreover, the increased productivity in terms of the interaction between water and external environments is expected to create new elements in simulation games with user interactions. In order to express more detailed animation of water surface, studies must be conducted to find ways to create and combine various wave motions by adjusting the combination of bounding spheres and DFs.

The breaking of waves should also be expressed with particles to enhance the perception of reality. Finally, researches should be performed to examine the means of establishing AF according to objects' size, weight, velocity and material properties as well as spatial partition techniques for creating bounding spheres that correspond to objects with complex shapes in order to produce detailed and realistic animation of water surface.

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