Topological Analysis between Bodies with Holes

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Summary
The topological relation is one of the most important spatial relations. Analysis of spatial relation has great practical value. The possible topological relations between 3D simple body objects can be analyzed using 9-intersection model. An algorithm is proposed to analyze the topological relations between body objects. It decomposes the complicated objects into simple objects, so it can get the topological relations between complicated body objects such as bodies with holes efficiently.

Key words:
9-intersection model, 3D spatial objects, topological relations, topological analysis, spatial reasoning

1. Introduction

There are many relations between 3D spatial objects, such as metric relations, order relations and topological relations. The most important relation is topological relation, which represents the inherent features of spatial objects that do not change with the observing angle or zooming transformation. The typical topological relations include inside and disjoint. Topological relations have important application value in spatial information query and can improve the accuracy of the input of spatial information [1].

Nowadays the researches mostly focus on the topological analysis between connected region objects in 2D plane. However, the researches on 3D spatial objects especially the body objects with holes are really insufficient. With the improvement of the technology, the conventional 2D topological analysis system can not satisfy the requirements of the developing spatial applications. Spatial topological analysis between 3D spatial objects has become a very important research direction.

In this article we analyze the 8 topological relations between 3D simple body objects by the 9-intersection model. Then we discuss the topological relations between the body objects with holes and the algorithm to compute the topological relation between them quickly.

2. Related Work

Spatial reasoning is a process that people use spatial theory and artificial intelligence to model, describe and represent spatial objects and analyze and process the spatial relations between spatial objects. There are two directions to represent and deduce the topological relations between spatial objects [3]. One is regional connection calculate and the other is 9-intersection model.

RCC (region connection calculation) [4] is a topological model presented by Randell based on Clarke's spatial calculation axiom. It is based on region object and uses a primitive binary tuple C(x, y) which represents that region x and y connects to deduce the topological relations between region objects. The most common model is RCC-5 and RCC-8. The former does not consider the boundaries of spatial objects and the latter considers the relations between boundaries of region objects. RCC-8 [5] includes DC (disconnect), EC (external connect), PO (partial overlap), TPP (tangential proper part of), NTPP (nontangential proper part of), EQ (equal), TPPI (tangential proper part of inverse), NTPPI (nontangential proper part of inverse). All these 8 topological relations are jointly exhaustive and pairwise disjoint.

9-intersection model [6] is a topological representation framework presented by Egenhofer based on the point set topology and 4-intersection model. It describes different topological relations by the intersection situation of interior, boundary and exterior of the spatial objects. The interior, boundary and exterior of two spatial objects can construct 9 different intersections. Each topological relation can be mapped into a different intersection combination. Suppose $A°$ represents the interior of the spatial objects, ∂A represents the boundary of the spatial objects, A^- represents the exterior of spatial objects. There are totally 9 intersection combinations between spatial objects A and B, which are $\partial A \cap \partial B$, $A° \cap B°$, $\partial A \cap B°$, $A° \cap \partial B$, $A^- \cap B^-$, $A^- \cap \partial B$, $A^- \cap B°$, $\partial A \cap B^-$ and $A° \cap B^-$.

On basis of 9-intersection model, Clementini proposed 9-intersection based on the extended dimension. The main improvement is that function DIM() can divide the results of point set more further. If the result of intersection is empty, then we get -1; if the result of intersection is point, then we get 0; if result is line, then

Manuscript received May 5, 2008
Manuscript revised May 20, 2008

we get 1; if the result is region, then we get 2. Thus we can distinguish different topological relations more accurately.

Using RCC model, the compound table of 2D region objects can be deduced [7]. Egenhofer studied the topological relations between regions with holes [8] and the topological reasoning between them [9]. Zhao renliang discussed the topological analysis using the 9-intersection model represented by voronoi figures [10].

Although RCC model can represent the topological relations between region objects, it can only represent topological regions between simple spatial objects and do not suit for 3D spatial objects. 9-intersection model can represent topological relation between spatial point, line, region and body.

Currently the spatial analysis is mainly in 2D plane and is insufficient to 3D spatial objects especially to complicated objects. Zlatanova's work only focuses on simple objects, the more complicated objects and the reasoning of spatial objects are not discussed [11]. Granados discussed the Boolean algorithms of 3D spatial body objects by selective Nef complex [12]. He uses Nef polyhedron that represented by Nef complex to compute the Boolean operations, such as union and intersection. Nef polyhedron is constructed by the half planes by Boolean operations such as intersection, union and difference. The implementation of the algorithm is to represent the selective complex by Nef polyhedron, then the body is mapped into the sphere and deduced by Nef theory. At last the Nef polyhedron is constructed and the result of Boolean operation is get. CGAL implements the algorithm.

3. 9-intersection topological relations

The purpose of this article is to proceed the topological analysis between the complicated 3D spatial objects. Because the 9-intersection model can represent the topological relations between 3D spatial objects well [11], it has been accepted as the basic framework of the representation of topological relations by Open Geospatial Consortium, OGC) and ISO [13]. This article will use the 9-intersection model to analyze topological relation between 3D spatial objects. For convenience, the concepts of basic body object and 9-intersection code is introduced [11].

Definition 1 (9-itnersection topological relation code) Suppose spatial object A and B, and $A°$, ∂A, A^- represent the interior, boundary and exterior of object A. The 9-intersecitons array a matrix R=[$\partial A \cap \partial B$ $A° \cap B°$ $\partial A \cap B°$ $A° \cap \partial B$ $A^- \cap B^-$ $A^- \cap \partial B$ $A^- \cap B°$ $\partial A \cap B^-$ $A° \cap B^-$] in an order. If the intersection is not empty, then 1 is marked; if the intersection is empty, then 0 is marked. Each intersection combination can be represented by a binary number, then the number can be converted into decimal number and it is the 9-intersection relation code.

Each 9-intersection relation code can represent a type of 3D spatial topological relation. For example, if two spatial objects have no intersection points, then the binary code is [000 011 111], to change to decimal number is 031, so R031 can represent the topological relations between this two objects which can be named as *disjoint*.

There are totally 512 topological relations between body objects by 9-intersection model. To get the practical topological relations, we use 25 negative conditions that Zlatanova presented and simplify the 9-intersection topological relations. There are 11 negative conditions out of 25 negative conditions which are related to simple body objects in 3D space. Eight topological relations between simple body objects can be deduced. Just as figure 1 shows.

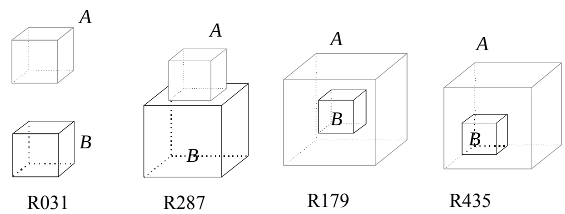
The problem of 9-intersection model is that the redundancy. In theory, there are altogether 512 possible relations. It is difficult to compute all the 9 intersections of the spatial objects, so the 9-intersection model can not be used directly in practice. Some scholars such as Bric [14], Zlatanova [11] and Chen Jun [15] made researches of it. Zlatanova proposes 13 groups of 25 negative conditions. For the spatial objects that specification of OGC defined [13], 69 topological 9-intersection relations and 159 possible topological relations have been deduced and the completeness has also been proved.

4. Topological relations between basic body objects

The purpose of this article is to analyze the topological relations between body objects. Because the 3D objects in real world may be very complicated, it is impossible to discuss the topological relations between them directly. A group of simple 3D spatial objects are defined.

Definition 2 (basic body): A basic body object in 3D space is a convex polyhedron that constructed by n (n>2) connected regions (r_1, r_2, …, r_n). The interior connects and does not contain holes.

The interior, boundary and exterior of the basic body object B can be represented by $B°$, ∂B and B^-.



A

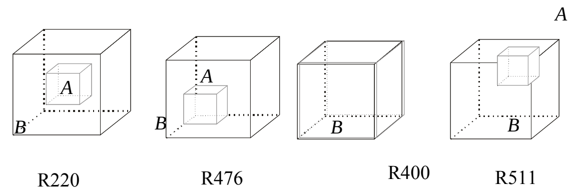


Fig. 1 The 8 possible topological relations between the basic body object *A* (gray) and *B*

There are totally 512 topological relations between body objects by 9-intersection model. To get the practical topological relations, we use 25 negative conditions that Zlatanova presented and simplify the 9-intersection topological relations. There are 11 negative conditions out of 25 negative conditions which are related to simple body objects in 3D space. 8 topological relations between simple body objects can be deduced. Just as figure 1 shows.

Each situation of figure 1 represents a topological relation between bodies. We name them *disjoint*, *meets*, *contains*, *covers*, *inside*, *coveredby*, *equals* and *intersects*. Each topological relation can be represented by a 9-intersection code: R031, R287, R179, R435, R220, R476, R400 and R511.

5. Topological analysis between the basic body and the body with holes

In the previous section, the topological relations between 3D simple body objects are discussed. However, there are more complicated situations in the real world. Because the complicated body objects are constructed by the operations such as union, intersection and supplementary, the basic spatial objects are the basis of the more complicated 3D spatial objects. This article will discuss the topological relations between common spatial complicated body objects which contain one or more holes.

5.1 Related definitions

The body object that contains holes can be looked as one complete basic body object that minuses several simple body objects. So the topological analysis between basic body objects in section 4 can be used to analyze the topological relations between complicated body objects. The body objects containing holes are defined as following:

Definition 3 (body objects containing holes) A body object B with holes is constructed by a basic body object B^* which minuses n (n>0) basic body objects b_i. For $\forall i,j \in \{1,\ldots,n\}$, $i \neq j$, the relation between b_i and b_j is *disjoint*; the relation between b_i and B^* is *inside*. It shows as figure 2 and can be represented as follows.

$$B = B^* - \bigcup_{i=1}^{n} b_i \qquad (1)$$

According to the definition 1, the interior, boundary and exterior of the body object B with holes can be represented as follows:

$$B^\circ = B^{*\circ} - \bigcup_{i=1}^{n} (b_i^\circ \cup \partial b_i)\,, \qquad (2)$$

$$\partial B = \partial B^* \cup (\bigcup_{i=1}^{n} \partial b_i)\,, \qquad (3)$$

$$B^- = B^{*-} \cup (\bigcup_{i=1}^{n} b_i^\circ) \qquad (4)$$

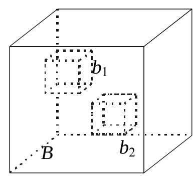


Fig. 2 The body object B containing holes b_1 and b_2

5.2 Topological analysis between basic body object and body object with one hole

Consider the 9-intersection topological relation R between a basic body object B_1 and a body object B_2 containing a hole (n=1). Let B_2 be a body object that is constructed by a basic body B^* which minuses another simple body object b_1, i.e., $B_2 = B^* - b_1$.

Because there are only 8 topological relations between the basic body objects, all the topological relations between B_1 and B^*, B_1 and b_1 can be enumerate. For example, when B_1 *disjoints* B^*, because b_1 is *inside* B^*, B_1 and b_1 is *disjoint*, it can not be the other 7 topological relations. The only figure looks like figure 3(a):

For each topological relation between B_1 and B^*, the number of possible 9-intersection relations between B_1 and b_1 are limited. Because limitation of the length of the article, the proof is omitted here, the conclusion is as following:

(1)If B_1 is *disjoint* with B^*, then B_1 is *disjoint* with b_1.

(2)If B_1 *contains* B^*, then B_1 *contains* b_1.

(3)If B_1 *covers* B^*, then B_1 *contains* b_1.

(4)If B_1 is *inside* B^*, then the topological relation between B_1 and b_1 can be one of the 8 relations.

(5)If B_1 is *covered* by B^*, then the topological relation between B_1 and b_1 can be *disjoint*, *contains*, *covers*, *meets* or *intersects*.

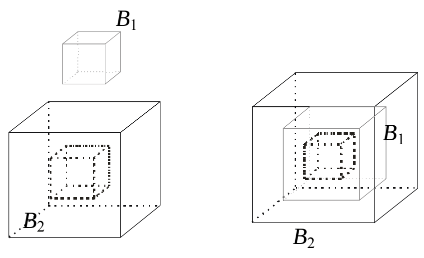
(6)If B_1 *meets* B^*, then B_1 is *disjoint* with b_1.

(7)If B_1 *equals* B^*, then B_1 *contains* b_1.

(8)If B_1 *intersects* B^*, then the topological relation between B_1 and b_1 can be *disjoint*, *contains*, *covers*, *meets* or *intersects*.

To each possible combination of topological relations, consider the 9-intersection relation between B_1 and B_2. For example, when case 1 appears, the relation between B_1 and B^*, B_1 and b_1 are *disjoint*, as figure 3(a) shows. The 9-intersection relation between B_1 and B_2 can be represented by the binary code 000 011 111, or decimal code R031, i.e., *disjoint*.

There may be new 9-intersection relations in the procedure of the analysis. For example, in case 4, when B_1 *contains* b_1 which is like in figure 3(b), the 9-intersection relation can be represented as 011 111 101, i.e., R253. It does not belong to any relations between one of the 8 topological relations of body and body objects. This relation only appears between the basic body and the body with holes. According to the analysis of each possible topological combinations of case 1 to case 8, the topological reasoning table of the simple body B_1 and the body with hole B_2 can be deduced as table 1 shows.



(a) (b)

Fig. 3 Two possible topological relations between the basic body object B_1 (grey) and a body object B_2 with hole

Table 1: The 9-intersection relation deduction table between B_1 and B_2 (B_2= B^*- b_1)

B_1/B^*		D	C		IN		M	E	IT
B_1/b_1		031	179	435	220	476	287	400	511
D	031	D			IN/220	IN/476	M		IT
C	179		C/179	C/435	R253	R509		R433	IT
	435				R509	R509			IT
IN	220				D				
	476				M				
M	287				IN/476	IN/476			IT
E	400				R285				
IT	511				IT	IT			IT

In table 1, D, C, IN, M, E and IT stand for *disjoint*, *covers*, *inside*, *meets*, *equals* and *intersects* respectively. According to table 1, for the simple body object B_1 and the body object B_2 with hole of b_1, we only need to analyze the topological relation r and R^* between B_1 and b_1, B_1 and the outer body B^* of B_2, then the topological relation R between B_1 and B_2 can be computed through the table 1. Because the body contains holes, there are four more new 9-intersection topological relations of R253, R285, R433 and R509 appear during the procedure of the topological reasoning which are represented in table 1 with underlines.

5.3 Topological analysis between basic body objects and body object containing several holes

When there are the body objects with more holes, the body object can be decomposed into several body objects with only one hole, using the table 1 in section 5.2 to analyze the topological relation between them. Then the final topological relation can be computed by union or intersection operations. Here is a theorem as following:

Theorem 1: A simple body object B_1, a body object B_2 containing holes, which is constructed by a simple body B^* that is dug by n (n>0) simple bodies b_i. Let B^* minus a body object b_i and construct a body object B'_i with hole, the topological relation between B_1 and B'_i is R_i, $R_{i1}=\partial B_1 \cap \partial B'_i$, $R_{i2}= B_1^\circ \cap B'^\circ_i$, $R_{i3}=\partial B_1 \cap B'^\circ_i$, $R_{i4}=B_1^\circ \cap \partial B'_i$, $R_{i5}= B_1^- \cap B'^-_i$, $R_{i6}= B_1^- \cap \partial B'_i$, $R_{i7}= B_1^- \cap B'^\circ_i$, $R_{i8}=\partial B_1 \cap B'^-_i$, $R_{i9}= B_1^\circ \cap B'^-_i$, i=1,…,n, then the 9-intersection relation R between B_1 and B_2 is:

$$R=\left[\bigcup_{i=1}^{n}R_{i1}\quad \bigcap_{i=1}^{n}R_{i2}\quad \bigcap_{i=1}^{n}R_{i2}\quad \bigcup_{i=1}^{n}R_{i4}\quad \bigcup_{i=1}^{n}R_{i5}\quad \bigcup_{i=1}^{n}R_{i6}\quad \bigcap_{i=1}^{n}R_{i7}\quad \bigcup_{i=1}^{n}R_{i8}\quad \bigcup_{i=1}^{n}R_{i9}\right] \tag{4}$$

Proof:

According to the definition of the 9-interseciton model, the 9-intersection relation between the basic body object B_1 and the complicated body object B_2 containing n holes is: R=[$\partial B_1 \cap \partial B_2$ $B_1^\circ \cap B_2^\circ$ $\partial B_1 \cap B_2^\circ$ $B_1^\circ \cap \partial B_2$ $B_1^- \cap B_2^-$ $B_1^- \cap \partial B_2$ $B_1^- \cap B_2^\circ$ $\partial B_1 \cap B_2^-$ $B_1^\circ \cap B_2^-$].

According to definition 2 and 3, there are following equations for the body objects B_2 containing n holes:

$$\partial B_2 = \partial B^* \cup (\bigcup_{i=1}^{n} \partial b_i), \tag{5}$$

$$B_2^\circ = B^{*\circ} - \bigcup_{i=1}^{n} (b_i^\circ \cup \partial b_i), \tag{6}$$

$$B_2^- = B^{*-} \cup (\bigcup_{i=1}^{n} b_i^\circ) \tag{7}$$

The 9-intersection relation between the basic body object B_1 and the body object B'_i containing only 1 hole is:

R_i=[$\partial B_1 \cap \partial B'_i$ $B_1^\circ \cap B'^\circ_i$ $\partial B_1 \cap B'^\circ_i$ $B_1^\circ \cap \partial B'_i$ $B_1^- \cap B'^-_i$ $B_1^- \cap \partial B'_i$ $B_1^- \cap B'^\circ_i$ $\partial B_1 \cap B'^-_i$ $B_1^\circ \cap B'^-_i$]

According to definition 2 and 3, there are the following equations.

$$\partial B'_i = \partial B^* \cup \partial b_i, \qquad (8)$$

$$B'^{\circ}_i = B^{*\circ} - b_i^{\circ} \cup \partial b_i, \qquad (9)$$

$$B'^{-}_i = B^{*-} \cup b_i^{\circ} \qquad (10)$$

$$[\bigcup_{i=1}^{n} R_{i1} \ \bigcap_{i=1}^{n} R_{i2} \ \bigcap_{i=1}^{n} R_{i3} \ \bigcup_{i=1}^{n} R_{i4} \ \bigcup_{i=1}^{n} R_{i5} \ \bigcup_{i=1}^{n} R_{i6} \ \bigcap_{i=1}^{n} R_{i7} \ \bigcup_{i=1}^{n} R_{i8} \ \bigcup_{i=1}^{n} R_{i9}\ ]$$

$$=[\bigcup_{i=1}^{n}(\partial B_1 \cap \partial B'_i) \ \bigcap_{i=1}^{n}(B_1^{\circ} \cap B_i'^{\circ}) \ \bigcap_{i=1}^{n}(\partial B_1 \cap B_i'^{\circ}) \ \bigcup_{i=1}^{n}(B_1^{\circ} \cap \partial B'_i) \ \bigcup_{i=1}^{n}(B_1^{-} \cap B_i'^{-}) \ \bigcup_{i=1}^{n}(B_1^{-} \cap \partial B'_i) \ \bigcap_{i=1}^{n}(B_1^{-} \cap B_i'^{\circ}) \ \bigcup_{i=1}^{n}(\partial B_1 \cap B_i'^{-}) \ \bigcup_{i=1}^{n}(B_1^{\circ} \cap B_i'^{-})\ ]$$

$$\bigcup_{i=1}^{n}(\partial B_1 \cap \partial B'_i) = \partial B_1 \cap (\partial B_1{}' \cup \cdots \cup \partial B_n{}')$$
$$= \partial B_1 \cap [\bigcup_{i=1}^{n}(\partial B^* \cup \partial b_i)]$$
$$= \partial B_1 \cap [\partial B^* \cup (\bigcup_{i=1}^{n} \partial b_i)]$$
$$= \partial B_1 \cap \partial B_2 \quad \text{(according to Eq. 5)}$$

$$\bigcap_{i=1}^{n}(B_1^{\circ} \cap B_i'^{\circ}) = B_1^{\circ} \cap (B_1'^{\circ} \cap \cdots \cap B_n'^{\circ})$$
$$= B_1^{\circ} \cap [\bigcap_{i=1}^{n}\left(B^{*\circ} - (b_i^{\circ} \cup \partial b_i)\right)]$$
$$= B_1^{\circ} \cap [B^{*\circ} - \bigcup_{i=1}^{n}(b_i^{\circ} \cup \partial b_i)]$$
$$= B_1^{\circ} \cap B_2^{\circ} \quad \text{(according to Eq. 6)}$$

$$\bigcup_{i=1}^{n}(B_1^{-} \cap B_i'^{-}) = B_1^{-} \cap (B_1'^{-} \cup \cdots \cup B_n'^{-})$$
$$= B_1^{-} \cap [\bigcup_{i=1}^{n}(B^{*-} \cup b_i^{\circ})]$$
$$= B_1^{-} \cap [B^{*-} \cup (\bigcup_{i=1}^{n} b_i^{\circ})]$$
$$= B_1^{-} \cap B_2^{-} \quad \text{(according to Eq. 7)}$$

$$\bigcap_{i=1}^{n}(\partial B_1 \cap B_i'^{\circ}) = \partial B_1 \cap B_2^{\circ}$$

$$\bigcup_{i=1}^{n}(B_1^{\circ} \cap \partial B'_i) = B_1^{\circ} \cap \partial B_2$$

$$\bigcup_{i=1}^{n}(B_1^{-} \cap \partial B'_i) = B_1^{-} \cap \partial B_2^{-}$$

$$\bigcap_{i=1}^{n}(B_1^{-} \cap B_i'^{\circ}) = B_1^{-} \cap B_2^{\circ}$$

$$\bigcup_{i=1}^{n}(\partial B_1 \cap B_i'^{-}) = \partial B_1 \cap B_2^{-}$$

$$\bigcup_{i=1}^{n}(B_1^{\circ} \cap B_i'^{-}) = B_1^{\circ} \cap B_2^{-}$$

So,

$$[\bigcup_{i=1}^{n} R_{i1} \ \bigcap_{i=1}^{n} R_{i2} \ \bigcap_{i=1}^{n} R_{i3} \ \bigcup_{i=1}^{n} R_{i4} \ \bigcup_{i=1}^{n} R_{i5} \ \bigcup_{i=1}^{n} R_{i6} \ \bigcap_{i=1}^{n} R_{i7} \ \bigcup_{i=1}^{n} R_{i8} \ \bigcup_{i=1}^{n} R_{i9}\ ]$$

$=[\partial B_1 \cap \partial B_2 \ \partial B_1 \cap B_2^{\circ} \ \partial B_1 \cap B_2^{\circ} \ B_1^{\circ} \cap \partial B_2 \ B_1^{-} \cap B_2^{-} \ B_1^{-} \cap \partial B_2 \ B_1^{-} \cap B_2^{\circ} \ \partial B_1 \cap B_2^{-} \ B_1^{\circ} \cap B_2^{-}]$

$=R$

(according to the definition of 9-intersection)

According to theorem 1, the detail of the algorithm to compute the topological relation between the body objects B_1 and B_2 containing hole is described as follows:

Algorithm 1: compute the topological relation between basic body object and the body with holes.

Input: the basic body object B_1 and the body object B_2 containing hole. B_2 is constructed by a basic body which minuses n (n>0) basic body objects. Let B^* minus a body object b_i and construct a body B'_i with hole. Suppose the topological relation between B_1 and B'_i is R_i.

Output: the 9-intersection relation R between B_1 and B_2.

Algorithm steps:

(1)Compute the topological relation between the basic body object B_1 and the basic body object B^* and { b_1, b_2,…, b_n}.

(2)According to table 1, compute the 9-intersection relation R_i between the basic body object B_1 and the body object B'_i that contains only one hole.

(3)

$$R=[\partial B_1\cap\partial B_2\ \partial B_1\cap B_2^\circ\ \partial B_1\cap B_2^\circ\ B_1^\circ\cap\partial B_2\ B_1^-\cap B_2^-\ B_1^-\cap\partial B_2\ B_1^-\cap B_2^\circ\ \partial B_1\cap B_2^-\ B_1^\circ\cap B_2^-]$$

$$=[\bigcup_{i=1}^{n}R_{i1}\ \bigcap_{i=1}^{n}R_{i2}\ \bigcap_{i=1}^{n}R_{i3}\ \bigcup_{i=1}^{n}R_{i4}\ \bigcup_{i=1}^{n}R_{i5}\ \bigcup_{i=1}^{n}R_{i6}\ \bigcap_{i=1}^{n}R_{i7}\ \bigcup_{i=1}^{n}R_{i8}\ \bigcup_{i=1}^{n}R_{i9}]$$

(according to the definition of the 9-intersection model)

(4)Return the topological relation R that is represented by the 9-interseciton code.

Let the complexity of body object B_1 be m, the complexity of B_2 be n. The topological intersection condition should be analyzed, and because the length of the article is limited, this article does not discuss the algorithm in step 1 to decide the topological relation between basic body objects. Let the complexity of the body object B_1 be m, the complexity of B_2 is n, the time complexity of algorithm that computes the 9-intersection relation between B_1 and b_i in step 1 is $O(nm\ (n+m)\log\ (n+m))$ [12].

5.4 Topological analysis between body objects containing holes

The discussion above is the analysis of the topological relations between a basis body and a body object with holes. The following is the discussion of the analysis of topological relations between two body objects with holes.

Let there be two body objects with hole, B_1 and B_2, containing m and n holes respectively. And $B_1= B_1^*-b_{11}-\cdots\cdots-b_{1m}$, $B_2= B_2^*-b_{21}-\cdots\cdots-b_{2m}$, in which b_{11} is the body object that is minused from the basic body object, and the hole is constructed by it. B_1^* and B_2^* is the complete body objects that are not minused by the holes.

The center idea of the topological analysis between B_1 and B_2 is: first the topological relations between the construction parts of the two body objects with holes should be computed. Then the topological relations between the construction parts between the construction parts of B_1 and B_2 should be computed. Then the topological relation R between B_1 and B_2 is computed.

Now the detail of the algorithm that compute the topological relation between B_1 and B_2 is as following.

Algorithm 2: compute the topological relation between the body objects containing holes

Input: the body objects B_1 and B_2 containing holes

Output: the 9-intersection topological relation R between B_1 and B_2

Algorithm steps:

(1)Analyze and compute the 9-intersection topological relation between the basic body objects B_1^*, b_{11},…,b_{1m} which construct B_1 and the basic body objects B_2^*, b_{21}, …, b_{2m} which construct B_2.

(2)If there exists a basic body object b_{1i} that it contains B_2^*, then the topological relation between B_1 and B_2 is *contains* (R179) and the result is R=R179.

(3)If (2) does not stand, then algorithm 1 in section 5.3 can be used to compute the topological relations between the basic body objects B_1*, b_{11}, …, b_{1m} which construct body object B_1 and B_2. Let the topological relations be R^*, R_1, …, R_m.

(4)Let $R=[r_1\ r_2\ r_3\ r_4\ r_5\ r_6\ r_7\ r_8\ r_9]$. There are 4 cases.

①R5$\neq\emptyset$;

②Let the 9-intersection topological relation that is computed in step 3 be $R^*=[r_1^*\ r_2^*\ r_3^*\ r_4^*\ r_5^*\ r_6^*\ r_7^*\ r_8^*\ r_9^*]$, $R_1=[r_{11}\ r_{12}\ r_{13}\ r_{14}\ r_{15}\ r_{16}\ r_{17}\ r_{18}\ r_{19}]$, ……, $R_m=[r_{m1}\ r_{m2}\ r_{m3}\ r_{m4}\ r_{m5}\ r_{m6}\ r_{m7}\ r_{m8}\ r_{m9}]$

If exists $r_{i1}\neq\emptyset$ (i=1,…, m), then $r_1\neq\emptyset$, else $r_1=\emptyset$;

If exists $r_{i2}\neq\emptyset$,(i=1,…, m), then $r_2\neq\emptyset$, else $r_2=\emptyset$;

If exists $r_{i3}\neq\emptyset$, (i=1,…, m), then $r_3\neq\emptyset$, else $r_3=\emptyset$;

If exists $r_{i4}\neq\emptyset$, (i=1,…, m), then $r_4\neq\emptyset$, else $r_4=\emptyset$;

If exists $r_{i8}\neq\emptyset$, (i=1,…, m), then $r_8\neq\emptyset$, else $r_8=\emptyset$;

If $r_{i9}\neq\emptyset$, (i=1,…, m), then $r_9\neq\emptyset$, else $r_9=\emptyset$.s

③If r6*$\neq\emptyset$, then r6$\neq\emptyset$; else if ri4*$\neq\emptyset$, (i=1,…, m), then $r_6\neq\emptyset$, else $r_6=\emptyset$.

④If $r_7^*\neq\emptyset$, then $r_7\neq\emptyset$; else if $r_{i4}^*\neq\emptyset$, (i=1,…, m), then $r_7\neq\emptyset$, else $r_7=\emptyset$.

(5)Transfer $R=[r_1\ r_2\ r_3\ r_4\ r_5\ r_6\ r_7\ r_8\ r_9]$ to the 9-intersection relation, then return R.

Because of the length of the article is limited, this article does not describe the algorithm in step 1 to decide the topological relation between the basic body objects in detail. Let the complexity of body object B_1 is m, and the complexity of body object B_2 is n. Then the time complexity of the algorithm in step 1 to compute the 9-intersection topological relation r_i between B_1 and b_i is $(n+m)\log\ (n+m))$ [12]. The time complexity of step 3 and step 4 is both $O(m)$. So the total time complexity of the algorithm is $O(nm(n+m)\log\ (n+m))$.

5.5 Analysis of an example

Now we use a simple example to show how to use the algorithm in section 5.4 to analyze the 9-intersection topological relation. Suppose figure 4 represents a body object B_1 containing a hole b_{11} and a body object B_2 containing 2 holes b_{21} and b_{22}. There are the following equations: $B_2 = B_1^* - b_{11}$, $B_2 = B_2^* - b_{21} - b_{22}$, in which B_1^* and B_2^* are the whole body objects.

According to the algorithm in section 5.4, we can analyze the topological relation between B_1 and B_2.

(1)First the topological relations between b_{11} and B_2^*, b_{11} and b_{21}, b_{11} and b_{22} are *intersects*, *intersects* and *disjoint*. And the topological relations between B_1^* and B_2^*, B_1^* and b_{21}, B_1^* and b_{22} are *intersects*, *intersects* and *disjoint*.

(2)Because there are no *contains* relations, and the topological relation R_m between b_{11} and B_2 is *intersects*, and the topological relation R^* between B_1^* and B_2 is *intersects* according to table 1.

(3) R_m=[111 111 111], R^*=[111 111 111]. According to step 4 of the algorithm, we get R=[111 111 111], i.e., *intersects* (R511).

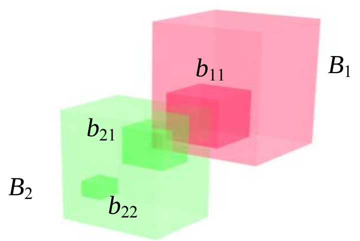


Fig. 4 The body object with hole B_1 and B_2

From the example above, although we simplified the 3D spatial objects, by the basic spatial objects and the Boolean operation in this article, the complicated body objects still can be described, such as a body object with one or more holes. So it has strong representing ability. On the other side, using the algorithms in section 5.4, we only need to decompose the complicated body objects into basic body objects, analyze the topological relations between the basic body objects, then we can get the 9-intersection topological relations between the complicated body objects as figure 4. Therefore the procedure of the topological analysis is greatly simplified. To the other more complicated spatial body objects constructed by several basic body objects, we can also compute the topological relations between them by the same method.

6. Conclusion

The analysis and deduction of the topological relations between spatial objects is the hot spot of the research nowadays. However, there are not many researches on the analysis of topological relations between 3D spatial objects. In order to solve this problem, based on the 9-intersection model, this article analyzes the topological relations between spatial body objects including the body objects with holes, and proposes an easy algorithm to compute the topological relations. The algorithm decomposes the complicated body objects with several holes into several basic body objects, and computes the topological relations between complicated body objects quickly by deduction and Boolean operations including union and intersection. For the systemic work, we are studying the topological relations between complicated points, lines, regions and body objects in 3D space.

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