A Study on Network partition Detection Relevant to Ad-hoc Networks: Connectivity Index Approach

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Summary

The issue of network partitioning is an important aspect of any network design and its study is more relevant to mobile ad-hoc networks (MANETs). MANETs are highly vulnerable to network partition due to the dynamic change in the topology. Very often the network will partition and remerge, affecting the performance of routing protocols. This paper introduces the application of Connectivity Index (*CI*) concept to detect the partition (having two clusters) of MANET.

Key words:

Connectivity Index, Cluster, MANET, Direct Sum, Complete Product, Network Partition

1. Introduction

temporary, Ad-hoc networks are decentralized, distributed self-organizing networks capable of forming a communication network without relying on any fixed infrastructure. The nodes of the network communicate each other over wireless channels. All nodes can function, if needed, as relay stations for data packets to be routed to their final destination. In other words, adhoc networks allow for multi-hop transmission of data between nodes outside the direct radio reach of each other. MANETs have been widely used for tactical military communication systems. The United States Defense Advanced Research Project Agency (DARPA) has sponsored projects such as the Near-Term Digital Radio (NTDR) system to control infantry, armor, and artillery units in battle field scenarios where no communication infrastructure exists.

The study of network topology plays an important role in designing routing protocols. The failure of set of links or nodes in the underlying network can cause the network to break away into two or more components or clusters. As a result of this, nodes within a cluster can communicate each other, but there is no communication across the nodes in different cluster. This phenomenon is called network partitioning, visualized as graph partitioning. Needless to say mobility of nodes is the main cause for the network partition. Very often the network will partition and remerge. Due to this, nodes from the different clusters try to route the packets without success and hence tries to explore new routes, which will be unsuccessful. These result in lot of route requests and route reply packets triggered from the nodes, which ultimately bring down the performance of the network due to congestion. Besides, network partition causes a serious consequence in the performance of adhoc networks which encourages distributed and client server applications. In this kind of applications, specified task is distributed among the nodes of the network and they exchange the information in order to complete it. But on the occurrence of partition, they fail to do the intended operations, which require data from other cluster.

Based on the discussion so far, it is apparent that there is a need to detect network partition. It is important at what stage the partition is detected: whether it is before (predetection) or after it happens (post-detection.) If the partition is predicted before it happens, then routing algorithms can adopt to reduce the congestion, which is the aftereffect of the partition. These algorithms are devised at the cost of more processing and computation which reduces the performance of the network and also power constraints arise. In the post detection approach, the partition is detected immediately after it occurs. These methods do not over utilize the scarce resources. but due to network partition, till it is detected the performance of the routing protocols and network throughput reduces due to congestion. Hence there should be a trade off in selecting and devising both the methods to detect network partition. In this paper an attempt is made to device a post network partition detection algorithm using CI.

2. Related Work

Several researchers have proposed the mobility prediction schemes with reference to pre-detection and post-detection. The detection algorithms were designed

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for individual and group mobility models. Based on the movement of individual mobility of the nodes [1], an attempt was made to predict future availability of wireless links and alternate paths to improve the performance of routing algorithms. The pre detection of network partition for the group mobility models [2] is based on the analysis of the topology changes due to the mobility of the nodes. By analyzing the pattern of movements, the possible network partition predicted by applying the data clustering algorithms.

In this paper we have introduced *CI* and spectral graph theory concepts to post detect network partition for certain class of network topologies. The paper is organized as follows: in part 3, some important definitions of graph theory relevant to the work are given. Part 4 presents detailed analytical analysis on network partition detection using *CI*. Part 5 presents the simulation algorithm. In part 6 simulation results are analyzed. Conclusions are given in Part 7.

3. Definitions

1. A graph G is a triplet consists of Vertex Set V(G), Edge Set E(G) and a relation that associates with each edge, two vertices. An edge between two nodes *i* and *j* is represented as (i, j) and using the usual notation can be written as

 $E(G) \subseteq \{(i, j) \mid \forall i, j \in V \text{ and } (i, j) = (j, i)\}$

The graph G can also be denoted by G(V, E), to explicitly show the vertex and edge sets.

2. Vertex/Edge Adjacency: Two vertices are said to be *adjacent* to each other, if there exist an edge between them. Two edges are said to be *adjacent* to each other, if the one of the end vertex of the edges is same.

3. Direct Sum of Two Graphs: Direct sum of two graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ is denoted by $G_1 \oplus G_2$ where $V_1 \cap V_2 = \phi$ is the graph G(V, E) for which $V = V_1 + V_2$ and $E = E_1 + E_2$

4. Complete Product of Two Graphs: The complete product of two graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ is denoted by $G_1 \otimes G_2$ is the graph obtained from $G_1 \oplus G_2$ by joining every vertex of $G_1(V_1, E_1)$ with every vertex of $G_2(V_2, E_2)$. Total number of links in

complete product of two graphs is $|E_1| + |E_2| + |V_1| |V_2|$

5. Direct Sum of two Clusters: Cluster is a graph. Union of two clusters C_1 and C_2 is the direct sum of two clusters and a link is added between two arbitrary vertices u and v, where $u \in C_1$ and $v \in C_2$.

6. *Complete Product of Tow Clusters*: is same as the complete product of two graphs.

7. Energy of a Graph: Let A be the adjacency matrix of a graph G. Let $\sigma(A) = \{\lambda_1, \lambda_2, \dots, \lambda_n\}$ be the set of ordered eigenvalues of A. Then the energy E_{π} of the graph is defined as sum of absolute values of all the

eigenvalues of A, that is $E_{\pi} = \sum_{i=1}^{\infty} |\lambda_i|$

4. Mathematical Analysis for Network Partition Detection using Connectivity Index

In this section we investigate the post detection of network partition by using *CI* and spectral graph theory concepts. For the purpose of study, wireless network is formed by the union of two connected sub-networks called *clusters* in two different ways: 1. *Direct Sum of two clusters*. 2: *Complete Product of two clusters*. The expression for *CI* is derived with respect to pre and post network formations and after network partition. Figures 1 and 2 illustrate the network formation and partition. In this section, a few lemmas related to the network partition detection are stated and proved

Let G(p,q) be a graph with p nodes and q links

Lemma 1: The *CI* of regular graphs is $\frac{p}{2}$.

Lemma 2: Let $G_1(p_1, q_1)$, $G_2(p_2, q_2)$ be two and

 $G(p, q) = G_1 + G_2 + (u, v) | u \in G_1, v \in G_2$ is a graph obtained by the direct sum of two graphs along with a link between two vertices u, v from G_1 and G_2 respectively. Then

$$CI(G) = CI(G_1) + CI(G_2) - \sum_{i=1}^{d(u)} \frac{1}{\sqrt{d(u_i)}} \left(\frac{1}{\sqrt{d(u)}} - \frac{1}{\sqrt{d(u)} + 1} \right) - \sum_{j=1}^{d(v)} \frac{1}{\sqrt{d(v_j)}} \left(\frac{1}{\sqrt{d(v)}} - \frac{1}{\sqrt{d(v)} + 1} \right) + \frac{1}{\sqrt{(d(u) + 1)}} \left(\frac{1}{\sqrt{d(u)}} - \frac{1}{\sqrt{d(u)} + 1} \right) + \frac{1}{\sqrt{(d(u) + 1)}} \left(\frac{1}{\sqrt{d(u)}} - \frac{1}{\sqrt{d(u)}} \right) + \frac{1}{\sqrt{(d(u) + 1)}} \left(\frac{1}{\sqrt{d(u)}} - \frac{1}{\sqrt{d(u)}} \right) + \frac{1}{\sqrt{(d(u) + 1)}} \left(\frac{1}{\sqrt{d(u)}} - \frac{1}{\sqrt{d(u)}} \right) + \frac{1}{\sqrt{(d(u) + 1)}} \left(\frac{1}{\sqrt{d(u)}} - \frac{1}{\sqrt{d(u)}} \right) + \frac{1}{\sqrt{(d(u) + 1)}} \left(\frac{1}{\sqrt{d(u)}} - \frac{1}{\sqrt{d(u)}} \right) + \frac{1}{\sqrt{(d(u) + 1)}} \left(\frac{1}{\sqrt{d(u)}} - \frac{1}{\sqrt{d(u)}} \right) + \frac{1}{\sqrt{(d(u) + 1)}} \left(\frac{1}{\sqrt{d(u)}} - \frac{1}{\sqrt{d(u)}} \right) + \frac{1}{\sqrt{(d(u) + 1)}} \left(\frac{1}{\sqrt{d(u)}} - \frac{1}{\sqrt{d(u)}} \right) + \frac{1}{\sqrt{(d(u) + 1)}} \left(\frac{1}{\sqrt{d(u)}} - \frac{1}{\sqrt{d(u)}} \right) + \frac{1}{\sqrt{(d(u) + 1)}} \left(\frac{1}{\sqrt{d(u)}} - \frac{1}{\sqrt{d(u)}} \right) + \frac{1}{\sqrt{(d(u) + 1)}} \left(\frac{1}{\sqrt{(u)}} - \frac{1}{\sqrt{(u)}} \right) + \frac{1}{\sqrt{(u)}} \left(\frac{1}{\sqrt{(u)}} - \frac{1}{\sqrt{(u)}} \right$$

where the vertices $u_i \in G_1(V)$ and $v_i \in G_2(V)$ and u is adjacent to v.

Proof:

Let $CI(G_1)$, $CI(G_2)$ be the connectivity indices of graphs G_1 and G_2 respectively. By the definition of CI, the expressions which includes the degrees of u and v are affected in CI of G_1 and G_2 respectively and is denoted by $CI'(G_1)$ $CI'(G_2)$ respectively and are expressed as

$$CI(G) = CI'(G_1) + CI'(G_2) + \frac{1}{\sqrt{(d(u)+1)(d(v)+1)}}$$

where $CI'(G_1)$ and $CI'(G_2)$ are expressed as

$$Cl(G) = Cl(G) - \sum_{i=l,(u,v)\in E(G_{1}),v\in G_{1}}^{d(u)} \frac{1}{\sqrt{d(u) d(v_{i})}} + \sum_{i=l,(u,v)\in E(G_{1}),v\in G_{1i}}^{d(u)} \frac{1}{\sqrt{(d(u)+1) d(v_{i})}}$$

and

$$CI(G_{2})=CI(G_{2}) - \sum_{i=l,(u,v)\in E(G_{1}),v\in G_{1}}^{d(u)} \frac{1}{\sqrt{d(u) d(v_{i})}} + \sum_{i=1,(u,v)\in E(G_{1}),v\in G_{1i}}^{d(u)} \frac{1}{\sqrt{(d(u)+1)d(v_{i})}}$$

$$\therefore CI(G) = CI(G_1) + CI(G_2) - \sum_{i=1}^{d(u)} \frac{1}{\sqrt{d(u_i)}} \left(\frac{1}{\sqrt{d(u)}} - \frac{1}{\sqrt{d(u)+1}} \right)$$
$$- \sum_{j=1}^{d(v)} \frac{1}{\sqrt{d(v_i)}} \left(\frac{1}{\sqrt{d(v)}} - \frac{1}{\sqrt{d(v)+1}} \right) + \frac{1}{\sqrt{(d(u)+1)}(d(v)+1)}$$

This Lemma is very significant in the eve of network partition detection. Here $CI(G) < CI(G_1) + CI(G_2)$ implies that the connectivity Index of the connected network is less than that of partitioned network with two clusters. This helps to detect the network partition.

Lemma 3: Let $G_1(p_1, q_1)$, $G_2(p_2, q_2)$ be two regular graphs of degrees r_1 and r_2 respectively and $G(p, q) = G_1 \cup G_2 + (u, v) | u \in G_1, v \in G_2$ Then

$$CI(G) = \frac{p_1}{2} + \frac{p_2}{2} - \frac{r_1}{r_1} - \frac{r_2}{r_2} + \sum_{i=1}^2 \sqrt{\frac{r_i}{1+r_i}} + \sqrt{\frac{1}{(1+r_1)(1+r_2)}}$$

Proof: Let, $CI(G_1)$ and $CI(G_2)$ be the connectivity indices of graphs G_1 and G_2 respectively. By the definition of CI, the expressions which includes the degrees of u and v are affected in CI of G_1 and G_2 respectively.

That is $CI(G_1)$ is expressed as

$$C(G) = \frac{P_1}{2} - \sum_{i=l,(u,v)\in E(G_i),v\in G_i}^{r_1} \frac{1}{\sqrt{d(u)d(v_i)}} + \sum_{i=l,(u,v)\in E(G_i),v\in G_i}^{r_1} \frac{1}{\sqrt{(d(u)+1)d(v_i)}}$$

Since G_1 is regular each vertex has degree r_1

$$\therefore CI(G_{1}) = \frac{p_{1}}{2} - \sum_{i=1,(u,v)\in E(G_{1}),v\in G_{1}}^{r_{1}} \frac{1}{\sqrt{r_{1}.r_{1}}} + \sum_{i=1,(u,v)\in E(G_{1}),v\in G_{1}}^{r_{1}} \frac{1}{\sqrt{(r_{1}+1).r_{1}}}$$

Similarly $CI(G_2)$ is expressed as

$$\therefore CI(G_2) = \frac{P_2}{2} - \sum_{i=1,(u,v) \in E(G_1), v \in G_{2i}}^{r_2} \frac{1}{\sqrt{r_2 \cdot r_2}} + \sum_{i=1,(u,v) \in E(G_1), v \in G_{2i}}^{r_2} \frac{1}{\sqrt{(r_2 + 1)r_2}}$$
$$\therefore CI(G_2) = CI(G_1) + CI(G_2) + \frac{1}{\sqrt{(r_1 + 1)(r_2 + 1)}}$$

Hence

$$CI(G) = \frac{p_1}{2} + \frac{p_2}{2} - \frac{r_1}{r_1} - \frac{r_2}{r_2} + \sum_{i=1}^2 \sqrt{\frac{r_i}{1+r_i}} + \sqrt{\frac{1}{(1+r_1)(1+r_2)}}$$

Lemma 4: Let $G_1(p_1,q_1), G_2(p_2,q_2)$ be two complete graphs And $G(p,q) = G_1 \cup G_2 + (u,v) | u \in G_1, v \in G_2$

Then the

$$CI(G) = \frac{p_1}{2} + \frac{p_2}{2} - 2 + \sum_{i=1}^2 \sqrt{\frac{p_i - 1}{p_i}} + \sqrt{\frac{1}{p_1 p_2}}$$

Proof: Since $G_1(p_1, q_1)$ and $G_2(p_2, q_2)$ are regular graphs of degree $r_1 = p_1 - 1$ and $r_2 = p_2 - 1$ respectively. By applying lemma 3 for the values of r_1 and r_2 , we arrive at the result.

Lemma 5: Let $G_1(p_1,q_1), G_2(p_2,q_2)$ be two regular graphs of degrees r_1 and r_2 respectively and $G(p,q)=G_1\otimes G_2$ is the complete product of two graphs such that G is regular. Then

$$CI(G) = \frac{p_1 + p_2}{2}$$

Proof: Since $G_1(p_1, q_1)$, $G_2(p_2, q_2)$ are two regular graphs of degrees r_1 and r_2 respectively and their complete product generates graph G(p,q) with number of points $p = p_1 + p_2$ and each vertex of

 $G_1(p_1,q_1)$ is associated with every vertex of $G_2(p_2,q_2)$.

Therefore every vertex of $G_1(p_1,q_1)$ and $G_2(p_2,q_2)$ has degree of $r_1 + p_1$ and $r_2 + p_2$ respectively and then *G* can be regular if and only if $r_1 + p_1 = r_2 + p_2 = n$, where *n* is any positive integer. Hence, by Lemma 1, $CI(G) = \frac{p_1 + p_2}{2}$.

We can conclude from this lemma that there is no change in the value of *CI* during network formation and partition. Hence it is not possible to detect the partition of these kinds of regular networks formed by the complete product of two regular clusters.

Lemma 6: Let $G_1(p_1,q_1), G_2(p_2,q_2)$ be two graphs and $G(p,q)=G_1 \otimes G_2$ is the complete product of two graphs. Then

$$CI(G) = \sum_{((u,v) \in E(G_1))} \frac{1}{\sqrt{(d(u) + p_2)(d(v) + p_2)}} + \sum_{((w,x) \in E(G_2))} \frac{1}{\sqrt{(d(w) + p_1)(d(x) + p_1)}}$$

+
$$\sum_{y \in V(G_1), z \in V(G_2)} \frac{1}{\sqrt{(d(y) + p_2)(d(z) + p_1)}}$$

Proof: By the definition of complete product of two graphs, each node of G_1 has association with every node of G_2 . Hence the degree of every node of G_1 and G_2 is

increased by p_2 and p_1 respectively. That is if $u \in G_1, v \in G_2$ and d(u), d(v) are the degrees of u and u respectively. Then their degrees are increased to $d(u) + p_2$ and $d(v) + p_1$ respectively. By substituting these values, we obtain the result.

From this lemma, it is apparent that $CI(G) \ge CI(G_1) + CI(G_2)$ which implies that the connectivity Index of the connected network is greater than or equal to that of partitioned network with two clusters. This helps to detect the network partition.

Lemma 7: Let $G_1(p_1, q_1)$, $G_2(p_2, q_2)$ be two graphs and $G(p,q) = (G_1 + G_2) + (u, v)$, where $u \in G_1 v \in G_2$ is a clustered graph obtained by the union of two graphs along with a link between two vertices u, v from $G_1(p_1, q_1)$ and $G_2(p_2, q_2)$ respectively. Then G(p,q) - (u, v) is a disconnected graph partitioned into two components $G_1(p_1, q_1)$ and $G_2(p_2, q_2)$. Then delta change in the *CI* of the resultant graph is

$$\begin{split} \sum_{i=1}^{d(u)} \frac{1}{\sqrt{d(u_i)}} & \left(\frac{1}{\sqrt{d(u)}} - \frac{1}{\sqrt{d(u)+1}}\right) + \sum_{j=1}^{d(v)} \frac{1}{\sqrt{d(v_j)}} & \left(\frac{1}{\sqrt{d(v)}} - \frac{1}{\sqrt{d(v)+1}}\right) \\ & -\frac{1}{\sqrt{(d(u)+1)}(d(v)+1)}, \text{ where } (u, u_i) \in E(G_1) \text{ and} \\ & (v, v_i) \in E(G_2) \end{split}$$

Proof: We know that from Lemma2

$$CI(G) = CI(G_1) + CI(G_2) - \sum_{i=1}^{d(u)} \frac{1}{\sqrt{d(u_i)}} \left(\frac{1}{\sqrt{d(u)}} - \frac{1}{\sqrt{d(u)+1}} \right)$$
$$- \sum_{j=1}^{d(v)} \frac{1}{\sqrt{d(v_i)}} \left(\frac{1}{\sqrt{d(v)}} - \frac{1}{\sqrt{d(v)+1}} \right) \text{ where }$$
$$(u, u_i) \in E(G_1) \text{ and } (v, v_i) \in E(G_2)$$
(1)

$$CI(G - (u, v)) = CI(G_1) + CI(G_2)$$
(2)

From equations 1 and 2, we arrive at

$$\therefore CI(G) - CI(G(p,q) - (u,v)) = \sum_{i=1}^{d(u)} \frac{1}{\sqrt{d(u_i)}} \left(\frac{1}{\sqrt{d(u)}} - \frac{1}{\sqrt{d(u)+1}} \right) + \sum_{j=1}^{d(v)} \frac{1}{\sqrt{d(v_j)}} \left(\frac{1}{\sqrt{d(v)}} - \frac{1}{\sqrt{d(v)+1}} \right) - \frac{1}{\sqrt{(d(u)+1)}(d(v)+1)}$$
(3)

Hence delta change in the CI after the network partition of two clusters is

$$\sum_{i=1}^{n(u)} \frac{1}{\sqrt{d(u_i)}} \left(\frac{1}{\sqrt{d(u)}} - \frac{1}{\sqrt{d(u)+1}} \right) + \sum_{j=1}^{n(v)} \frac{1}{\sqrt{d(v_i)}} \left(\frac{1}{\sqrt{d(v)}} - \frac{1}{\sqrt{d(v)+1}} \right) - \frac{1}{\sqrt{(d(u)+1)}(d(v)+1)}$$

5. Algorithm

In order to study and demonstrate the correctness of the lemmas stated in the previous section, we have simulated the relevant aspects of *clustered* ad-hoc network using MATLAB. The simulation process for the both kinds of network formation and partition detection is similar. The following procedure describes various steps that needs to be executed to simulate network formation (direct sum, complete product) and partition

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The simulation process starts with two minimal 1connected clustered networks with N_1 and N_2 as number of nodes in clusters G_1 and G_2 respectively (which are fixed for each simulation). Let G is a network obtained by the union of two clusters G_1 and G_2 . Then progressively, fully connected network is built. The methodology adopted is:

- 1. Compute the parameters: *CI*(G),eigenvalue2 and graph energy
- 2. Choose two nodes u and v randomly, where $u \in V(G_1)$, $v \in V(G_2)$ and then add the link between them.
- 3. Compute energy of a graph, eigenvalue2 and connectivity index.
- 4. Progressively add the links in each cluster.
- 5. Partition the network by dropping the link (u, v) between two clusters and compute the parameters mentioned in step 1 and restore back the link.
- 6. At iteration number equal to *j* (random), drop a link in each cluster. Go to step 3
- 7. If the resultant cluster topologies are not fully connected then go to step 3.
- 8. Plot the graphs.
- 9. Step 4 and 5 are executed in order to demonstrate the post detection of network partition by using CI. Step 6 is executed in order to study the effect of dropping of two links from each cluster which do not cause network partition.

6. Simulation Results

Network formation and partition simulation is carried out on various size clusters from 10 nodes to 50 nodes. Alternatively connected network and partitioned network are simulated and the parameters mentioned in the procedure are computed during the course of simulation. A separate simulation is carried out for each kind of network formation and the analysis of the simulation results are done in the next sub sections.

6.1 Direct Sum of Two Clusters

Here the two clusters of size 20 nodes each are simulated and network is formed by the direct sum of two clusters. Besides this a link is formed between two arbitrarily choose n nodes belongs to each of the clusters to ensure connected network. (Figure-1 describes it). Common link between two clusters are dropped and again restored in turn to ensure that the network is partitioned and connected back: the parameters are computed. This process is repeated till the network is fully connected. At some point of simulation we deliberately dropped tow links from each of the clusters other than the common link and the parameters are computed. Figures-3 and 4 demonstrates the network formation and partition process. In Figure-3 higher peak values of CI signify network partition and lower peak values signify connected network partition. In the case of arbitrary link drops from each cluster with network connected, there is a small dip in the low peak value. These simulation results are in line with the results of lemma2. Figure-4 describes the similar pattern for the other parameter eigenvalue2.

6.2 Complete Product of Two Clusters

Similar simulations are carried out on complete product of two clustered networks of 20 nodes each. (Figure-2 describes it). The simulation results are inline with the statement that "The *CI* of the connected network is greater than that of partitioned network."(Lemma6). In Figure-5 lower peak values of *CI and graph energy* signify network partition and higher peak values signify

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that network is connected. The lower peak values of eigenvalue2 indicate connected network and higher peak values signify network partition.

Finally these simulation results successfully demonstrate detection of network partition based on previous lemmas for both kinds of network partition.

7. Conclusion

The network partition detection process is an important activity in MANETs. The detection algorithms should be accurate and use limited resources. The study successfully demonstrated the partition detection of the network formed from two clusters using Connectivity Index. Further study is under progress to detect network partition involving more than two clusters.

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(a) Dis-connected Network with Two Clusters of size 10



(b) Direct Sum of two Clusters with a common link

Figure-1



Figure-2 Complete Product of Two Clusters



Figure-3 Demonstration of Network formation by Direct Sum of two Clusters and Partition using CI



Figure-4 Demonstration of Network formation by Direct Sum of two Clusters and Partition using Eigenvalue2



Figure-5 Demonstration of Complete Product of two Cluster Networks and Partition Detection

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