Optimization Of Training Sequence Length For Enhancement Of Channel Capacity In Wireless Communications

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ABSTRACT
Channel Capacity is the essential factor of wireless communications. Hence there are wide ranges of theories which strive for the effective usage of channels. Those include implementing diversity at transmitter and receiver ends, frequency allocation so on so forth. In this paper, we have presented a new dimension in increasing the channel capacity. This is done through effective utilization of channel which is possible through an efficient channel estimation strategy. That is channel estimation through training sequence method. The impact of training sequence on the capacity of MIMO system is considered. Based on the ML channel estimator, the estimation-error-involving capacity formula has been derived. We show that the estimation error may decrease the channel capacity significantly, while the optimal training sequences can achieve substantial improvement over the random ones. And in the second result we derived that the estimation-error-involving capacity approaches the ideal capacity as the training length increasing, while a long training length will not bring evident improvement on the capacity, which indicates that a suitable training length should be chosen in practice. Finally we estimate the training sequence length of the diversified values of SNR, number of transmitting and receiving antennas and percentage of ideal capacity. And we conclude that the training sequences very much effect the channel capacity, and by implementing the optimal training sequence length we arrive at the improved channel capacity.

KEY WORDS:
Channel Estimation, Channel Capacity, Training Sequence, Wireless Communication, Algorithms

1.0 INTRODUCTION

Capacity is the most interesting aspect, may be because of its challenges posed on the modern communication engineers. Previously we had many capacity improvement methods while having its own drawbacks. Most of these are due to ISI and Co-channel Interference. This occurs due to improper detection and reframing the actual signals. If this is done perfectly, we could find many solutions for these drawbacks. The perfect solution for this is channel estimation. With these we made an attempt to bring an distinguished relation between training sequences which are used in channel estimation and capacity, in MIMO channels.

Based on the ML channel estimator, the estimation-error-involving capacity formula has been derived. We show that the estimation error may decrease the channel capacity significantly, while the optimal training sequences can achieve substantial improvement over the random ones. The estimation-error-involving capacity approaches the ideal capacity as the training length increasing, while a long training length will not bring evident improvement on the capacity, which indicates that a suitable training length should be chosen in practice.

2.0 CHANNEL ESTIMATION AND DETECTION IN THE BASE STATION

The main blocks in a base station receiver are as shown in below Figure. The channel is the wireless interface between the mobile user and the base station. Many undesirable effects such as interference from other users, delays from multiple paths, fading and noise occur on the signal as it passes through the channel. The detector needs to acquire synchronization with the input signal in order to correctly detect the incoming bit sequence. Hence the parameters of the channel need to be estimated for proper detection. Channel estimation involves estimating and tracking the delays of each users' bits and the channel attenuation over the different paths.

One of the proposed methods for channel estimation is using the Maximum Likelihood method. There has been ongoing related research at Rice using the Maximum Likelihood algorithm for channel estimation. This algorithm is designed to handle time variations in the system, multiple propagation paths, and large number of users with varying level of transmitting power.

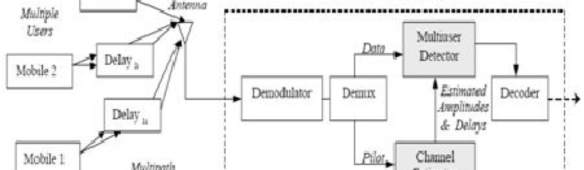

Figure 2 **Simplified view of the base station receiver for uplink**

Manuscript received June 5, 2008.
Manuscript revised June 20, 2008.

In the uplink, since all users are transmitting information, each desired user experiences direct interference from other users (Multiple Access Interference or MAI). Also, signals from users near the base station tend to be stronger and overshadow the signals from users far away from the base station (near-far effect). The optimal multiuser detector was first proposed by Verdbut several sub-optimal schemes have been proposed to reduce the complexity of the algorithm.

This scheme was the iterative multistage method where the inputs of one particular stage are the estimated bits of the previous stage. After interference cancellation, the new estimates, which should be closer to the transmitted bits, are output and fed into the next stage. Ideally at the last iteration stage, the output and the input should be identical if the algorithm converges. Further optimizations have been made on the algorithm, making use of the fact that as the iterations progress, the solution becomes more and more invariant, i.e. more and more elements in the output vector turn out to be the same as the elements in the input vector. The proposed detection algorithm is the Differencing Multistage algorithm, which is based on the above principle.

3.0 CHANNEL ESTIMATION TECHNIQUES

For Channel State Information (CSI) acquisition, three classes of methods are available:

1. **Blind Method** - In this method, estimation of CSI merely is from the received data.

2. **Differential Ones** This approach bypass CSI estimation by differential encoding

3. **Training-based Method** Which estimate CSI by the knowing training sequence.

Among them, the training based ones are the most attractive as they can decouple the demodulation/decoding from the CSI estimation and simplify the structure o the receiver.

3.1 TRAINING SEQUENCE VERSUS BLIND ESTIMATION

In most communication systems, considerable distances separate the transmitter and receiver; therefore, the estimator at the receiver does not have practical access to the transmitted signal that enters the channel. Blind algorithms are those that do not rely upon this knowledge of the transmitted signal. A popular class of blind algorithms are decision directed or decision feedback algorithms. These algorithms rely upon the demodulated and detected sequence at the receiver to reconstruct the transmitted signal. An obvious downfall of these methods is that a decision or bit error at the receiver will cause the construction of an incorrect transmitted signal. In the case of channel estimation, this decision error will introduce a bias in the channel estimate, making it less accurate. In this paper, it is proposed that modifications to standard adaptive algorithms to make them less susceptible to these decision errors.

Although the receiver might not have direct access to the transmitted signal, if the transmitter periodically sends a known training or probe sequence, the receiver can use this training sequence to reconstruct the transmitted waveform. While this method will produce more accurate estimates of the channel during the training interval, these estimates become out of date between these intervals, unlike the continually updated estimates of the blind techniques. Another drawback of training sequence methods is that the training sequence occupies valuable bandwidth, reducing the throughput of the communication system. For example, training sequences account for 22% of GSM's and 9% of IS-54's total bandwidth.

3.2 TYPES OF ESTIMATIONS

There are various types of channel estimation strategies like minimum mean square error (MMSE), Maximum likelihood estimation (MLE), Least Squares estimation (LSE) etc.

3.2.1 Mean Square Error (MSE)

In statistics and signal processing, a **minimum mean square error** (**MMSE**) estimator describes the approach which minimizes the mean square error (MSE), which is a common measure of estimator quality. The term MMSE specifically refers to estimation in a Bayesian setting. In this design we always aim to minimize the MSE of the CSI estimator.

3.2.2 Maximum Likelihood Estimation

Maximum likelihood estimation (MLE) is a popular statistical method used to calculate the best way of fitting a mathematical model to some data. Modeling real world data by estimating maximum likelihood offers a way of tuning the free parameters of the model to provide an optimum fit.

In this paper we considered only the ML one because the MMSE estimator requires the knowledge of SNR which is still need to be estimated in practice.

3.3 ML APPROACH-CHANNEL IDENTIFICATION

The Gaussian likelihood function is established making the hypothesis that unknown data symbols are Gaussian variables, hence uk = N(Tkh,Q). It has been shown that the Gaussian approach yields more accurate channel estimates than the deterministic approach where the unknown data symbols are considered as unknown deterministic disturbances. Adopting the Gaussian hypothesis, we can express (up to a constant term) the negative log likelihood function of the system as

$$-\mathcal{L} = K \ln |\mathbf{Q}| + \sum_{k=1}^{K} (\mathbf{u}_k - \mathbf{T}_k\mathbf{h})^{\mathrm{H}} \mathbf{Q}^{-1} (\mathbf{u}_k - \mathbf{T}_k\mathbf{h}).$$

Relying on the definition of **Q**, the log likelihood can be expressed as a direct function of the unknown parameters **h** and $\sigma 2$. The corresponding ML channel estimate minimizes this expression with respect to **h** and $\sigma 2$. This minimization problem boils down to a computationally demanding (L + 2)- dimensional nonlinear search. To overcome this complexity problem, we propose that the structure of **Q** be disregarded, and ignore the relation that binds it to the parameters **h** and $\sigma 2$. We thus assume that the covariance matrix **Q** of the stochastic term _k can be any symmetric positive definite matrix, regardless of **h** and $\sigma 2$. This hypothesis turns the initial ML problem into a new one. We call the initial problem the parametric ML problem; the problem resulting from the proposed approximations will be called the nonparametric ML problem. The nonparametric ML channel estimate thus maximizes the likelihood function with respect to **h** and **Q** (instead of **h** and $\sigma 2$).

These assumptions transform the parametric ML problem in **h** and $\sigma 2$ into a new optimization problem that is separable in its two variables **h** and **Q**. We exploit this separability property in the next sections in order to solve the optimization problem in a less complex way than the (L + 2)-dimensional nonlinear search of the parametric ML problem. The solution of the nonparametric ML problem differs from the solution of the parametric ML problem. Hence, it is worthwhile to first check the impact of the proposed hypothesis on the accuracy of the resulting ML channel estimates.

4.0 DATA MODEL

We consider a finite impulse response (FIR) convolutive channel of order L : **h** = [h[0] · · · h[L]]T. A burst x[n], n =1, . . . , N, of symbols is transmitted over the channel. Considering that the coherence time of the channel is larger than the duration of the transmitted burst, the received sequence y[n] is the linear convolution of the transmitted sequence with the channel impulse response

$$y[n] = \sum_{i=0}^{L} h[i]x[n-i] + \eta[n]$$

where $\eta[n]$ is the AWGN at the receiver. A total number of K training sequences is inserted in the burst. The kth training sequence, t_k = [t_k[1] · · · t_k [n_t]]T, starts at position n_k : [x[n_k] · · · x[n_k + n_t − 1]]T = t_k. Two possibilities are considered in the text: either the same training sequence is repeated after each block of data (constant-training sequence case), or the training sequence is changed after each block (changing-training-sequence case). Define the vector uk of received symbols that contain a contribution from the k^{th} transmitted training sequence: $\mathbf{u}_k = [y[\mathbf{n}_k] \cdots y[\mathbf{n}_k + \mathbf{n_t} + L - 1]]^{\mathrm{T}}$. It is the sum of a deterministic and a stochastic term

$$\mathbf{u}_k = \mathbf{T}_k\mathbf{h} + \varepsilon_k \quad (2)$$

where $\mathbf{T}_k$ is an (nt + L) × (L + 1) tall Toeplitz matrix with[**t** $\mathbf{T}_k$0 · · · 0]T as its first column and [$\mathbf{t}_k$ [1] 0 · · · 0] as its first row. The stochastic term ε_k is described as

$$\epsilon_k = \underbrace{\left[\begin{array}{ccc|ccc} h_L & \cdots & h_1 & & & \mathbf{0} \\ & \ddots & \vdots & h_0 & & \\ & & h_L & \vdots & \ddots & \\ \mathbf{0} & & & h_{L-1} & \cdots & h_0 \end{array}\right]}_{\mathbf{H}_s(n_t+L)\times(2L)} \mathbf{s}_k + \boldsymbol{\eta}_k$$

where $\mathbf{s}_k$ =[$\mathbf{s}_k$ [1] · · · $\mathbf{s}_k$ [2L]]T =[x[n_k −L] · · · x[n_k −1] $x[n_k + n_t] \cdots x[n_k + n_t + L - 1]]^{\mathrm{T}}$ is the vector of surrounding data symbols, and $\boldsymbol{\eta}k$ = [η[n_k] . . . η[n_k + nt +L − 1]]T is the AWGN term. Assuming that both the noise and the data are white and zero mean (E{ sk [i] sk [j]*} =E{η[i]η[j]*} = 0, ∀i, j, k : i _= j, and E{ $\mathbf{s}_k$ [i]} = E{η[k]} =0), we can say that _k is zero mean. Defining ns as the length of the shortest sequence of data symbols (ns = min$k${nk+1 − (nk + nt − 1)}), we assume ns 2L. This ensures that the sk's are uncorrelated, i.e., E{ sk sHl } = 0 ∀k, l : k _= l. Defining the signal and noise variances as λ2 = E{ sk [i] sk [i]*} and σ2 = E{η[k]η[k]*}, respectively, we can derive the first- and second order statistics of εk

$$\mathrm{E}\{\epsilon_k\} = \mathbf{0}_{(n_t+L)\times 1}$$

$$\mathrm{E}\left\{\epsilon_k\epsilon_k^{\mathrm{H}}\right\} \stackrel{\triangle}{=} \mathbf{Q} = \lambda^2\mathbf{H}_s\mathbf{H}_s^{\mathrm{H}} + \sigma^2\mathbf{I}$$

$$\mathrm{E}\{\epsilon_k\epsilon_l^{\mathrm{H}}\} = \mathbf{0}_{(n_t+L)\times(n_t+L)}\ \forall k,l : k \neq l$$

At present we are aware of data and system model of estimation process through various methods viz. ML, MMSE, LSE etc. The other important part of our motto is knowing about training sequences, its properties and its functionality in various methods.

5.0 TRAINING SEQUENCE:

In most communication systems, considerable distances separate the transmitter and receiver; therefore, the estimator at the receiver does not have practical access to the transmitted signal that enters the channel. Although the receiver might not have direct access to the transmitted signal, if the transmitter periodically sends a known training or probe sequence, the receiver can use this training sequence to reconstruct the transmitted waveform. While this method will produce more accurate estimates of the channel during the training interval, these estimates become out of date between these intervals.

Data (*N* symbols)	Tranining sequence	Data (*N* symbols)

Figure 3 Data format sturcture

5.1 Minimum Number of Training Symbols

For channel estimation there are NT (L + 1) unknowns in every MISO channel, as remarked before. There have to be leastwise as many training symbols as unknowns to estimate the channel and as only the last NP −L training symbols may be used for the estimation, the number of training symbols NP per transmit antenna and per frame has to be at least

NP >NT (L + 1) + L.

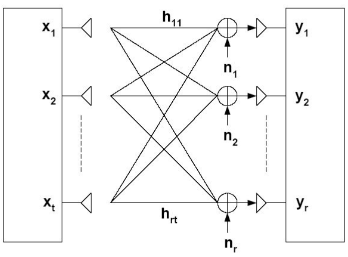

Figure 4 Number Of Transmitting Antennas And Receiving Antennas

5.1.1 Criterion for Optimal Training Sequences

The training sequences should be designed so that the Mean Square Error of the channel estimation is minimized. Such training sequences are called optimal. The energy of the training sequences s2 m(k) = a2 = constant > 0 8k should be constant and equal. The SISO channels of the MIMO channel as well as the elements of the channel impulse responses are assumed to be uncorrelated from each other. Additive white Gaussian noise is assumed. If the matrix S, which contains the training sequences, holds the criterion

$$\left[\mathbf{S}^H\,\mathbf{S} + \frac{\sigma_n^2}{\sigma_h^2}\,\mathbf{I}\right] = \lambda\,\mathbf{I},$$

$$\lambda = \left((N_P - L)\,a^2 + \frac{\sigma_n^2}{\sigma_h^2}\right)$$

5.1.2 Design of Optimal Training Sequences

The matrix S is cyclic. A matrix which holds (34) has orthogonal columns. A cyclic matrix S with orthogonal columns can be constructed by writing a perfect root-of-unity sequence (PRUS) into the first row of S and then filling any next row with the one element right shifted version of the previous row .A root-of-unity sequence s(k) of length N has complex root-of-unity elements with absolute value one and may have P different phases. The elements of a root-of-unity sequence s(k) are of the form s(k) = ej 2_P f(k) with k = 0, ...,N − 1 and 0 _ f(k) _ P, with f(k) as a whole-number sequence. A root-of-unity sequence is said to be perfect if all of its out-of-phase periodic autocorrelation terms are equal to zero. A perfect root-of-unity sequence s(k) can be constructed for any length N by the Frank-Zad off-Chu-sequences

$$s(k) = \begin{cases} e^{j\pi M k^2/N} & \text{for } N \text{ even} \\ e^{j\pi M k(k+1)/N} & \text{for } N \text{ odd} \end{cases}$$

With k=o,1,,,,N-1 M is a natural number greater than zero and needs to becoprime to N.The length N = NT (L+1) of the required perfect root-of unity sequence s(k) is equal to the number of columns of S.The sequence s(k) is written into the first row of S. Any next row is the one element right shifted version of the previous row. The training sequence for each transmit antenna can be extracted from the matrix S.

5.1.3 Required Number of Training Symbols

The MSE related to one of the NT · NR SISO channels of the MIMO channel is given by

$$\mathrm{MSE}_{\mathrm{SISO}} = \frac{1}{N_T}\,\mathrm{MSE}_{\mathrm{MISO}}.$$

To preserve the comparability with a single-input single-output (SISO) system in terms of equal total transmit energy P, the transmit symbols are multiplied by the factor 1 pNT before transmission. The energy of a training symbol is given by

$$s_m^2(k) = a^2 = P_S = \frac{P}{N_T} = \text{konstant} \quad \forall k$$

with the transmit power PS of each transmit antenna and the total transmit power P = 1. The Minimum Mean Square Error of the Least Square channel estimation follows to

$$\text{MMSE}_{\text{SISO,LS}} = \frac{\sigma_n^2}{P}\,\frac{N_T(L+1)}{(N_P - L)}.$$

If the channel matrices fulfill the normalization condition (4), the term P/ σ2n can be replaced by the average signal to noise ratio ρ . The Minimum Mean Square Error (MMSE) of the Least Square channel estimation follows to

$$\text{MMSE}_{\text{SISO,LS}} = \frac{1}{\rho}\,\frac{N_T(L+1)}{(N_P - L)}$$

with the signal to noise ratio as a non-logarithmic value. The required number of training symbols for the Least Square channel estimation depending on the signal to noise ratio (SNR) and the required accuracy of the channel estimate. The required accuracy of the channel estimate is specified by the MMSE. shows lines with the same MMSESISO,LS. For an accuracy of the channel estimate of MMSESISO,LS = 5·10−3 for each of the NT ·NR SISO channels of the MIMO channel, at least NP = 2·NT (L+1)+L training symbols per transmit antenna are required for a SNR of 20dB.

5.1.4 Impact of training sequences on capacity

The impact of training sequence on the capacity of MIMO system is considered. Based on the ML channel estimator, the estimation-error-involving capacity formula has been derived. We show that the estimation error may decrease the channel capacity significantly, while the optimal training sequences can achieve substantial improvement over the random ones. The estimation-error-involving capacity approaches the ideal capacity as the training length increasing, while a long training length will not bring evident improvement on the capacity, which indicates that a suitable training length should be chosen in practice.

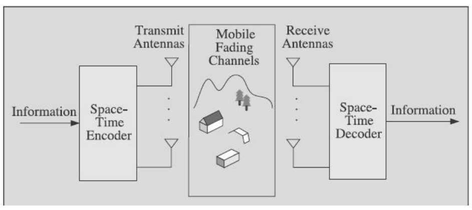

Figure 5 Basic model of MIMO Sytem

The basic model of a MIMO system is illustrated in above figure 5. A finite sequence of information bits is encoded by a space-time encoder and then transmitted across parallel transmit antennas. After the propagation in mobile fading channels, the receive sequence is sent into the space-time decoder to recover the original transmit information. In this paper, we assume the channel is i.i.d. Rayleigh block-fading. The mathematical model for a MIMO system with *M* transmit antennas and *N* receive antennas over a *T* symbol block can be written as

$$\boldsymbol{Y=HX+N}$$

where *T* is also the training length, X is the training sequence, Y is the corresponding received training sequence, H is the CSI matrix and , where CN denotes the complex Gaussian distribution with zero mean and covariance matrix *I*, N is complex Gaussian white noise independent of *X*. The well-known ML CSI estimation is given by

$$\hat{\boldsymbol{H}} = \boldsymbol{YX}^H(\boldsymbol{XX}^H)^{-1}$$

Then CSI-estimation-involving model for the MIMO system can be presented by

$$\boldsymbol{y} = \hat{\boldsymbol{H}}\boldsymbol{x} + (\boldsymbol{H} - \hat{\boldsymbol{H}})\boldsymbol{x} + \boldsymbol{n}$$

where *x* and *y* are the transmitted data and received data respectively. Here we assume x ~ CN(0,σ_P^2) and n ~ ~ CN(0,σ_N^2) where σ_P^2 and σ_N^2 are the power of the transmitted symbol and the noise respectively, consequently the average SNR at each receive antenna can be computed by M σ_P^2/σ_N^2 The sum of the last two terms in the above equation can be regarded as equivalent noise(denoted by $\hat{\boldsymbol{n}}$) and its covariance matrix is given by

$$\begin{aligned}\mathrm{E}\left(\hat{\boldsymbol{n}}\hat{\boldsymbol{n}}^H\right) &= \mathrm{E}\left[\left((\boldsymbol{H}-\hat{\boldsymbol{H}})\boldsymbol{x}+\boldsymbol{n}\right)\left((\boldsymbol{H}-\hat{\boldsymbol{H}})\boldsymbol{x}+\boldsymbol{n}\right)^H\right] \\ &= MN\sigma_N^2\sigma_P^2\left(\boldsymbol{XX}^H\right)^{-1} + \sigma_N^2\boldsymbol{I}\end{aligned}$$

From the above equation we can see that the equivalent noise is usually correlated across the receive antennas. For simplicity, we assume the autocorrelation of the training sequence XX^H is a diagonal matrix such that the noise $\hat{\boldsymbol{n}}$ is spatial uncorrelated, then the *effective* average receive SNR can be computed by

$$\text{SNR}_{est\_err} = \frac{M\sigma_P^2}{M\sigma_N^2\sigma_P^2\mathrm{Tr}\left[\left(\boldsymbol{XX}^H\right)^{-1}\right] + \sigma_N^2}$$

The coherent MIMO channel capacity is given by

$$C = \mathrm{E}\left[\log_2 \det\left(\boldsymbol{I} + \frac{\mathrm{SNR}}{M}\boldsymbol{HH}^H\right)\right]$$

Inserting SNR into capacity it yields

$$C_{est\_err} = \mathrm{E}\left[\log_2 \det\left(\boldsymbol{I} + \frac{\mathrm{SNR}_{est\_err}}{M}\boldsymbol{HH}^H\right)\right]$$

In the high-SNR region, the channel capacity can be approximated as

$$\begin{aligned} C_{est\_err} &\approx K\log_2 \frac{SNR_{est\_err}}{M} + \sum_{i=K'-K+1}^{K'} \mathrm{E}\left[\log_2 \chi_{2i}^2\right] \\ &= K\log_2 \frac{\sigma_P^2}{M\sigma_N^2\sigma_P^2\mathrm{Tr}\left[\left(XX^H\right)^{-1}\right]+\sigma_N^2} + \sum_{i=K'-K+1}^{K'} \mathrm{E}\left[\log_2 \chi_{2i}^2\right] \end{aligned}$$

Where k=min{M,N} and K'=max{M,N} and χ_{2i}^2 is a chi-square random variable with 2i degrees of freedom.

So far we have derived an intuitive relationship between the training sequence and the capacity in. Here we can build up the training sequence design criteria in the capacity sense.

Since the logarithm function is monotonic increasing, the maximization of is equivalent to the minimization of $\mathrm{Tr}[(XX^H)^{-1}]$ which coincides with the existing MSE-based training sequence design criteria. According to, the minimum of is achieved if and only if

$$\boldsymbol{XX}^H = T\sigma_P^2\boldsymbol{I}$$

By substituting above equality into capacity relation , we can further obtain the upper bound of the channel capacity as

$C=K\log_2(SNR/(M+(M^2/T)))+\Sigma E[\log_2\chi_{2i}^2]$ =K1-K+1

The equality holds if and only if the training sequence satisfies, which implies that the OSTBC codes or Walsh codes are the optimal training sequences. In the sufficient high-SNR region, by neglecting the last constant sum-term in, we can get

$$C_{est\_err} \le K\log_2 \frac{\mathrm{SNR}}{M} - K\log_2\left(\frac{M}{T}+1\right)$$

Which we can use for deriving optimal training length in the next section

6.0 Derivation of Optimum Training length

The corresponding capacity relation indicate that training length T is included in a logarithm function, if we want to get a bit more capacity, much larger T is needed. Hence, a large T will not bring evident performance improvement, a suitable training length should be chosen in practice. Capacity could also become a benchmark for us to determine how long the training sequence is necessary in the design of a MIMO system. If we need $C_{est} > \gamma C$(γ is percentage), the following relation should be satisfied.

$$\mathbf{K}\log_2\frac{\mathbf{SNR}}{\mathbf{M}} - \mathbf{K}\log_2\left(\frac{\mathbf{M}}{\mathbf{T}}+1\right) \ge \gamma\,\mathbf{K}\log_2\frac{\mathbf{SNR}}{\mathbf{M}}$$

As K is common on both sides cancel it, we get

$$\log_2\frac{\mathbf{SNR}}{\mathbf{M}} - \log_2\left(\frac{\mathbf{M}}{\mathbf{T}}+1\right) \ge \gamma\log_2\frac{\mathbf{SNR}}{\mathbf{M}}$$

As we know that a log b= log b^a

The above equation can be re written as

$$\log_2\frac{\mathbf{SNR}}{\mathbf{M}} - \log_2\left(\frac{\mathbf{M}}{\mathbf{T}}+1\right) \ge \log_2\left(\frac{\mathbf{SNR}}{\mathbf{M}}\right)^{\gamma}$$

As we know that log a – log b = $\log\left(\frac{a}{b}\right)$

The above Equation can be written as

$$\log_2\left(\frac{\frac{\mathbf{SNR}}{\mathbf{M}}}{\frac{\mathbf{M}}{\mathbf{T}}+1}\right) \ge \log_2\left(\frac{\mathbf{SNR}}{\mathbf{M}}\right)^{\gamma}$$

As we have logarithm to the base to on both sides we can remove it and can be written as

$$\left(\frac{\frac{\mathbf{SNR}}{\mathbf{M}}}{\frac{\mathbf{M}}{\mathbf{T}}+1}\right) \ge \left(\frac{\mathbf{SNR}}{\mathbf{M}}\right)^{\gamma}$$

Interchanging the terms $\left(\frac{\mathbf{SNR}}{\mathbf{M}}\right)^{\gamma}$ and $\left(\frac{\mathbf{M}}{\mathbf{T}}+1\right)$ we get

$$\frac{\left(\frac{\mathbf{SNR}}{\mathbf{M}}\right)}{\left(\frac{\mathbf{SNR}}{\mathbf{M}}\right)^{\gamma}} \ge \left(\frac{\mathbf{M}}{\mathbf{T}}+1\right)$$

The above equation can be modified and written as

$$\left(\frac{\mathbf{SNR}}{\mathbf{M}}\right)^{1-\gamma} \ge \left(\frac{\mathbf{M}}{\mathbf{T}}+1\right)$$

By brining the 1 on right hand side to left hand side , the equation is modified as

$$\left(\frac{\mathbf{SNR}}{\mathbf{M}}\right)^{\mathbf{1-\gamma}} - 1 \geq \left(\frac{\mathbf{M}}{\mathbf{T}}\right)$$

Now interchange the terms $\left(\frac{\mathbf{SNR}}{\mathbf{M}}\right)^{\mathbf{1-\gamma}}$ - 1 and T We get,

$$T \geq \frac{M}{\left(\frac{SNR}{M}\right)^{1-\gamma} - 1}$$

For example if we want to get at least 80 percent of the ideal capacity in a (4,4) MIMO system at SNR=10dB, the training length must be at least

$$T \geq \frac{\mathbf{M}}{\left(\frac{\mathbf{SNR}}{\mathbf{M}}\right)^{\mathbf{1-\gamma}} - \mathbf{1}} = \frac{\mathbf{4}}{\left(\frac{\mathbf{10}}{\mathbf{4}}\right)^{\mathbf{0.2}} - \mathbf{1}} \approx 20$$

And if we want 90 or 95 percent, the corresponding results are 42 and 84 respectively

7.0 RESULTS AND CONCLUSIONS

The result illustrates curves from figures 6-8 are channel capacity versus SNR for different training sequences and training sequence length in the case of (M=8, N=8). It is noted that the random training sequence might degrade the capacity seriously, while the optimal training sequences can achieve substantial performance improvement over the random training ones.

And we can see in figure 7 SNR=5dB, the optimal training sequence in the (M=16, N=8, T=16) case even outperforms the optimal one in the (M=8, N=8, T=8) in figure 6 case. From all the results, it was shown that the optimal training sequence is crucial in the design of a MIMO system and longer training length is needed with the increasing the numbers of transmit and receive antennas.

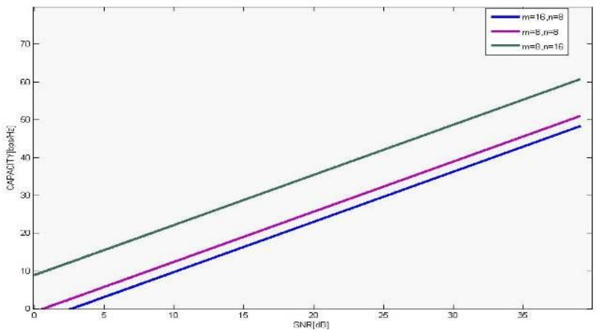

Figure 6 SNR Vs Capacity (constant training)

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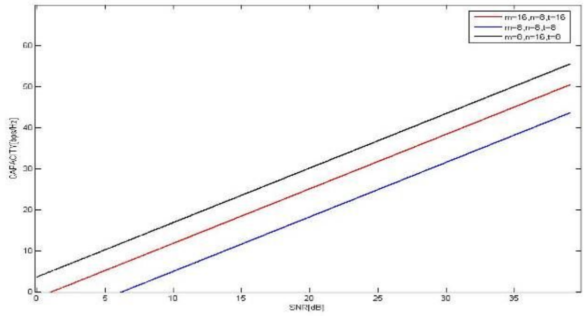

Figure 7 Capacity vs. SNR for varied training

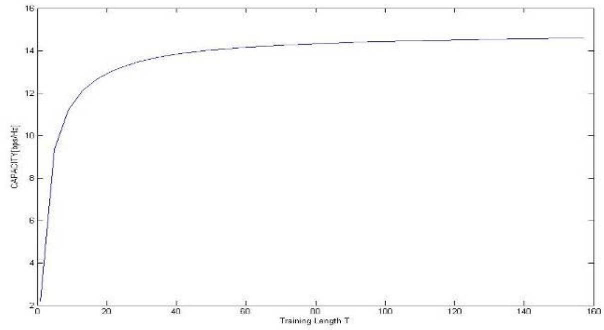

Figure 8 Training Length Vs Capacity

The result in figure 8 shows curves of the channel capacity versus the training length for different SNRs in the (M=4, N=4) case. It is not surprising that the estimation error-involved capacity is approaching the ideal capacity with increasing training length T and the corresponding indicate that training length T is included in a logarithm function, if we want to get a bit more capacity, much larger T is needed. Hence, a large T will not bring evident performance improvement, a suitable training length should be chosen in practice could also become a benchmark for us to determine how long the training sequence is necessary in the design of a MIMO system.

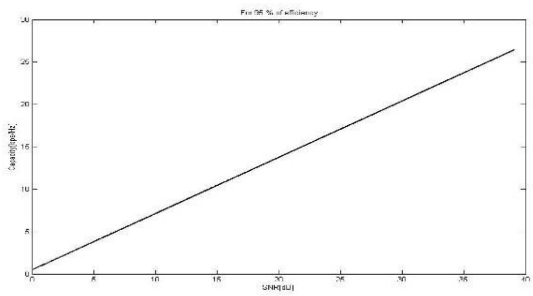

Figure 9 Graph showing the capacity vs. SNR characteristics for 95% efficiency.

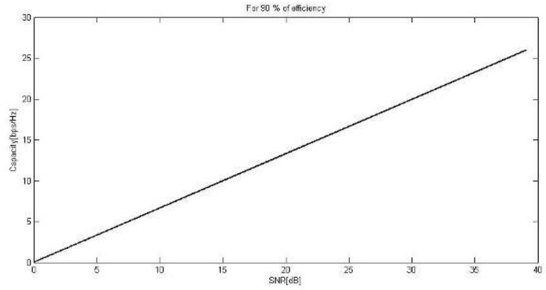

Figure 10 Graph showing the capacity vs. SNR characteristics for 90% efficiency

Figure 9 and 10 shows the graph for capacity vs SNR characteristics for the expected values of 90 and 95 percentages. Here it was shown that "error involved" capacity is closely related to SNR and also Training sequence, by graphical analysis this was justified in this paper. Also it was proven that the long training length will not bring evident improvement on the capacity, only an optimum length of training would allow error involved capacity and actual capacity to go in parallel.

ACKNOWLEDGEMENTS

I would like to thank the faculty members in Department of ECE, JNTU, Hyderabad and the management and higher officials of K L College of Engineering for extending their cooperation in continuing the research.

REFERENCES

[1] aaI. E. Telatar, _Capacity of multi-antenna Gaussian channels,_ *Europ. Trans. Telecomm.*, vol.10, pp. 585-595, Nov./Dec. 1999.

[2] Decision Weighted Adaptive Algorithms with Applications to Wireless Channel Estimation by Shane Martin Haas B.S. Math The University of Kansas, Lawrence, 1998 B.S.E.E. The University of Kansas, Lawrence, 1998

[3] Implementation of Channel Estimation and Multiuser Detection Algorithms for W-CDMA on Digital Signal Processors Sridhar Rajagopal, Gang Xu and Joseph R. Cavallaro Center for Multimedia Communication Department of Electrical and Computer Engineering Rice University, Houston, TX 77251-1892.

[4] The Impact of Training Sequence on the Capacity of MIMO System Li Jiandong, Lu Zhuo, Pang Jiyong, Chen Liang Broadband Wireless Communications Laboratory, Information Science Institute State Key Laboratory of Integrated Service Networks, Xidian University, Xi'an, China

[5] Y. Liang and V. Veeravalli, "Capacity of non coherent time-selective Rayleigh-fading channels," *IEEE Trans. Inform. Theory*, vol. 50, pp. 3095-3110, Dec.2004.

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