Robust stability of discrete-time uncertain stochastic BAM neural networks with time-varying delays

Yongming Li [1] **and Qizhan Lu** [2] **and Qiankun Song** [2]

1.Department of Mathematics, Sichuan University of Sience & Engineering, Sichuan 643000, P.R. China
2.Department of Mathematics, Chongqing Jiaotong University, Chongqing, 400074, P.R. China

Summary
In this paper, the global exponential stability is investigated for the discrete-time uncertain stochastic bidirectional associate memory neural networks with time-varying delays. For the neural networks under study, a generalized activation function is considered, and the traditional assumptions on the boundedness, monotony and differentiability of the activation functions are removed. By utilizing suitable Lyapunov–Krasovsky functional and using stochastic analysis theory and inequality technique, several sufficient conditions for checking the global robust exponential stability of the addressed neural networks are obtained in terms of linear matrix inequalities (LMIs), which can be checked numerically using the effective LMI toolbox in MATLAB. An example is given to show the effectiveness and less conservatism of the proposed criteria.

Key words:
BAM neural network; stochastic neural networks; discrete-time; exponential robust stability; time-varying delay.

1. Introduction

The bidirectional associative memory (BAM) neural network was first introduced by Kosko [1]. Recently, the dynamics such as stability and periodicity of BAM neural networks have received much attention due to their potential application in associative memory, parallel computation and optimization problems [2, 3]. In such applications, it is of prime importance to ensure that the designed neural network is stable.

As is well known, in both biological and man-made neural networks, the delays occur due to finite switching speed of the amplifiers and communication time [2]. Time delays may lead to oscillation, divergence, or instability which may be harmful to a system [3]. Therefore, the stability analysis of neural networks with consideration of time delays becomes extremely important to manufacture high quality neural networks. Recently, many criteria on stabilities have been given for various delayed BAM neural networks, for example, see [2]-[11] and references therein.

When modeling real nervous systems, stochastic disturbances and parameters uncertainties are probably two main resources of the performance degradations of the implemented neural networks. The reasons are as follows: 1) the synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes; and 2) the connection weights of the neurons depend on certain resistance and capacitance that include uncertainties. Therefore, the stability analysis for stochastic neural networks with or without parameter uncertainties become increasingly significant, and some results related to this problem have recently been published, for example, see [12]-[24] and references therein.

It is worth noticing that, up to now, most neural networks have been assumed to act in a continuous-time manner. However, when implementing the continuous-time recurrent neural network for computer simulation, for experimental or computational purposes, it is essential to formulate a discrete-time system that is an analogue of the continuous-time recurrent neural network. To some extent, the discrete-time analogue inherits the dynamical characteristics of the continuous-time recurrent neural network under mild or no restriction on the discretization step-size, and also remains functional similarity to the continuous-time recurrent neural network and any physical or biological reality that the continuous-time recurrent neural network has [25]. Unfortunately, as pointed out in [26, 27, 28], the discretization cannot preserve the dynamics of the continuous-time counterpart even for a small sampling period. Therefore, there is a crucial need to study the dynamics of discrete-time neural networks. Very recently, the discrete-time uncertain stochastic neural networks with time delays was considered, the exponential stability problem for a class of discrete-time uncertain stochastic neural networks with time delays was studied [29]. To the best of our knowledge, few authors study the global exponential stability problem of bidirectional associate memory neural networks with discrete-time uncertain stochastic neural networks with time delays.

Motivated by the above discussion, in this paper, we consider the bidirectional associate memory neural networks with discrete-time uncertain stochastic neural networks with time delays and analyze its global exponential robust stability.

Corresponding author. Address: Department of Mathematics, Chongqing Jiaotong University, Chongqing, 400074, P. R. China. E-mail address: qiankunsong@163.com (Q.Song). This work was jointly supported by the National Natural Science Foundation of China under Grant 10772152, and the Scientific Research Fund of Chongqing Municipal Education Commission under Grant KJ070401.

2. Model description and preliminaries

In this paper, we consider the following neural network model:

$$\begin{cases} x(k+1) = \left(C_1 + \Delta C_1\right) x(k) + \left(A_1 + \Delta A_1\right) f_1(y(k)) \\ \quad + \left(B_1 + \Delta B_1\right) g_1(y(k-\sigma(k))) \\ \quad + h_1(k, y(k), y(k-\sigma(k)) w_1(k) \\ y(k+1) = \left(C_2 + \Delta C_2\right) y(k) + \left(A_2 + \Delta A_2\right) f_2(x(k)) \\ \quad + \left(B_2 + \Delta B_2\right) g_2(x(k-\tau(k))) \\ \quad + h_2(k, x(k), x(k-\tau(k)) w_2(k) \end{cases} \quad (1)$$

where $x(k) = (x_1(k), x_2(k), \ldots, x_n(k))^T$, $x_i(k)$ is the state of the i th neuron at time k from the neural field FX ; $y(k) = (y_1(k), y_2(k), \ldots, y_n(k))^T$, $y_i(k)$ is the state of the i th neuron at time k from the neural field FY ; $C_1 = diag\left(c_{11}, c_{21}, ..., c_{n1}\right) > 0$, c_{i1} describes the rate with which the i th neuron will reset its potential to the resting state in isolation when disconnected from the networks and external inputs of the neural field FX ; $C_2 = diag\left(c_{12}, c_{22}, ..., c_{n2}\right)$, c_{i2} describes the rate with which the i th neuron will reset its potential to the resting state in isolation when disconnected from the networks and external inputs of the neural field FY ; $A_1 = (a_{ij})_{n\times n}$ is the connection weight matrix from the neural field FY ; $B_1 = (b_{ij})_{n\times n}$ is the delayed connection weight matrix from the neural field FY ; $A_2 = (a'_{ij})_{n\times n}$ is the connection weight matrix from the neural field FX ; $B_2 = (b'_{ij})_{n\times n}$ is the delayed connection weight matrix from the neural field FX ; $f_1(y(k)) = (f_{11}(y_1(k)), f_{21}(y_2(k)), \ldots, f_{n1}(y_n(k)))^T$, $f_{j1}(y_j(k))$ denotes the activation function of the j th neuron at time k from the neural field FY ; $f_2(x(k)) = (f_{12}(x_1(k)), f_{22}(x_2(k)), \ldots, f_{n2}(x_n(k)))^T$, $f_{j2}(x_j(k))$ denotes the activation function of the j th neuron at time k from the neural field FX ; $g_1\left(x\left(k-\sigma(k)\right)\right) = \left(g_{11}\left(x\left(k-\sigma(k)\right)\right), g_{21}\left(x\left(k-\sigma(k)\right)\right), ..., g_{n1}\left(x\left(k-\sigma(k)\right)\right)\right)^T$, $g_2\left(x\left(k-\tau(k)\right)\right) = \left(g_{12}\left(x\left(k-\tau(k)\right)\right), g_{22}\left(x\left(k-\tau(k)\right)\right), ..., g_{n2}\left(x\left(k-\tau(k)\right)\right)\right)^T$; $\tau(k)$ and $\sigma(k)$ are the transmission delays and satisfy $\tau_m \le \tau(k) \le \tau_M$ and $\sigma_m \le \sigma(k) \le \sigma_M$ ($\tau_m \ge 0$, $\tau_M \ge 0$, $\sigma_m \ge 0$, $\sigma_M \ge 0$ are known positive integers); $\Delta C_1(k), \Delta C_2(k), \Delta A_1(k), \Delta A_2(k), \Delta B_1(k), \Delta B_2(k)$ represent the time-varying parameter uncertainties, and are assumed to satisfy the following admissible conditions:

$$\left[\Delta C_1(k), \Delta A_1(k), \Delta B_1(k)\right] = MF(k)\left[N_{11}, N_{21}, N_{31}\right], \quad (2)$$

$$\left[\Delta C_2(k), \Delta A_2(k), \Delta B_2(k)\right] = MF(k)\left[N_{12}, N_{22}, N_{32}\right], \quad (3)$$

where $M, N_{i1}, N_{i2}\ (i = 1, 2, 3)$ are known real constant matrices, and $F(k)$ is the unknown time-varying matrix-valued functions subject to the following conditions:

$$F^T(k) F(k) \le I, \quad \forall k \in N^+, \quad (4)$$

In model (1), $w_1(k)$ and $w_2(k)$ is a scalar Wiener process (Brownian Motion) with

$$E[w(k)] = 0, \ E\left[w^2(k)\right] = 1, \quad (5)$$

$$E[w(i)w(j)] = 0 (i \ne j), \quad (6)$$

and $h_i(k, x, y) : R \times R^n \times R^n \to R^n$ is the continuous function, and is assumed to satisfy that

$$\begin{aligned} h_1^T(k, x, y) h_1(k, x, y) &\le \rho_1 x^T x + \rho_2 y^T y, \\ h_2^T(k, x, y) h_2(k, x, y) &\le \rho_3 x^T x + \rho_4 y^T y, \end{aligned} \quad (7)$$

where $\rho_1 > 0, \rho_2 > 0$ are known constant scalars.

The initial conditions associated with model (1) are given by

$$\begin{cases} x_i(s) = \phi_i(s), s \in N\left[-\tau_M, 0\right], i = 1, 2, ..., n, \\ y_i(s) = \psi_i(s), s \in N\left[-\sigma_M, 0\right], i = 1, 2, ..., n. \end{cases} \quad (8)$$

Throughout this paper, we make the following assumptions:

Assumption (H1). The activation functions in model (1) satisfy

$$l_{ji}^- \le \frac{f_{ji}(\alpha_1) - f_{ji}(\alpha_2)}{\alpha_1 - \alpha_2} \le l_{ji}^+, \quad (9)$$

$$\upsilon_{ji}^- \le \frac{g_{ji}(\alpha_1) - g_{ji}(\alpha_2)}{\alpha_1 - \alpha_2} \le \upsilon_{ji}^+, \quad (10)$$

for $i = 1, 2; \quad j = 1, 2, \ldots, n$, $\alpha_1 \ne \alpha_2, \alpha_1, \alpha_2 \in R$, where l_{ji}^-, l_{ji}^+, $\upsilon_{ji}^-, \upsilon_{ji}^+$ are some constants.

Assumption (H2).

$$f_1(0) = g_1(0) = 0, \quad (11)$$

$$f_2(0) = g_2(0) = 0.. \quad (12)$$

Definition 1 : The model (1) is said to be robustly exponentially stable in the mean square if there exist constants $\alpha > 0, \mu \in (0,1)$ such that every solution of the model (1) satisfies that

$$E\left\{\left|x(k)\right|^2 + \left|y(k)\right|^2\right\}$$

$$\le \alpha\mu^k[\max_{-\tau_M \le i \le 0} E\left|x(i)\right|^2 + \max_{-\sigma_M \le i \le 0} E\left|y(i)\right|^2]$$

for all positive integers.

To prove our results, the following lemmas are necessary, which can be found in [14] and [15].

Lemma 1. [14]Given constant matrices P, Q and R, where $P^T = P$, $Q^T = Q$, then $\begin{pmatrix} P & R \\ R^T & -Q \end{pmatrix} < 0$ is equivalent to the following conditions

$$Q > 0, \quad P + RQ^{-1}R^T < 0.$$

Lemma 2. [15]Let ϑ, ψ, F be real matrices of appropriate dimensions with F satisfying $F^T F \le I$. Then, for any scalar $\varepsilon > 0$:

$$\vartheta F\psi + (\vartheta F\psi)^T \le \varepsilon^{-1}\vartheta\vartheta^T + \varepsilon\psi^T\psi.$$

3. Main results

In this section, we shall establish our main criteria based on the LMI approach.

For presentation convenience, in the following, we denote

$$C_1(k) = C_1 + \Delta C_1(k), A_1(k) = A_1 + \Delta A_1(k),$$

$$B_1(k) = B_1 + \Delta B_1(k), C_2(k) = C_2 + \Delta C_2(k),$$

$$A_2(k) = A_2 + \Delta A_2(k), B_2(k) = B_2 + \Delta B_2(k),$$

and

$$L_1 = diag\left(l_{11}^- l_{11}^+, l_{21}^- l_{21}^+, \ldots, l_{n1}^- l_{n1}^+\right),$$

$$L_2 = diag\left(\frac{l_{11}^- + l_{11}^+}{2}, \frac{l_{21}^- + l_{21}^+}{2}, \ldots, \frac{l_{n1}^- + l_{n1}^+}{2}\right),$$

$$L_3 = diag\left(l_{12}^- l_{12}^+, l_{22}^- l_{22}^+, \ldots, l_{n2}^- l_{n2}^+\right),$$

$$L_4 = diag\left(\frac{l_{12}^- + l_{12}^+}{2}, \frac{l_{22}^- + l_{22}^+}{2}, \ldots, \frac{l_{n2}^- + l_{n2}^+}{2}\right),$$

$$\gamma_1 = diag\left(\upsilon_{11}^- \upsilon_{11}^+, \upsilon_{21}^- \upsilon_{21}^+, \ldots, \upsilon_{m1}^- \upsilon_{m1}^+\right),$$

$$\gamma_2 = diag\left(\frac{\upsilon_{11}^- + \upsilon_{11}^+}{2}, \frac{\upsilon_{21}^- + \upsilon_{21}^+}{2}, \ldots, \frac{\upsilon_{m1}^- + \upsilon_{m1}^+}{2}\right),$$

$$\gamma_3 = diag\left(\upsilon_{12}^- \upsilon_{12}^+, \upsilon_{22}^- \upsilon_{22}^+, \ldots, \upsilon_{n2}^- \upsilon_{n2}^+\right),$$

$$\gamma_4 = diag\left(\frac{\upsilon_{12}^- + \upsilon_{12}^+}{2}, \frac{\upsilon_{22}^- + \upsilon_{22}^+}{2}, \ldots, \frac{\upsilon_{n2}^- + \upsilon_{n2}^+}{2}\right).$$

Our main results are given in the following theorem.

Theorem 1 Suppose that Assumptions (1) and (2) hold. Then, the model(1) is robustly globally exponentially stable in the mean square, if there exist two positive definite matrices P and Q, two scalars $\lambda^* > 0$, $\varepsilon > 0$, and four diagonal matrices Λ_1, Λ_2, Γ_1 and Γ_2 such that the following two LMIs hold:

$$P \le \lambda^* I, \tag{13}$$

$$\begin{pmatrix} \Psi_1 + \varepsilon\hat{N}^T N & \hat{P}M \\ M^T\hat{P}^T & -\varepsilon I \end{pmatrix} < 0, \tag{14}$$

where

$$\psi_1 = \begin{bmatrix} R_{11} & R_{12} & \overline{R}_{13} \\ R_{12}^T & R_{22} & \overline{R}_{23} \\ \overline{R}_{13}^T & \overline{R}_{23}^T & R_{33} \end{bmatrix}$$

$$R_{11} = \begin{bmatrix} \Pi_1 - \Lambda_2 L_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & -Q + \lambda^*\rho_4 I - \Gamma_2\gamma_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\Lambda_1 & 0 & \Lambda_1 L_2 & 0 \\ 0 & 0 & 0 & -\Gamma_1 & 0 & \Gamma_1\gamma_2 \\ 0 & 0 & \Lambda_1 L_2 & 0 & \Pi_2 - \Lambda_1 L_1 & 0 \\ 0 & 0 & 0 & \Gamma_1\gamma_2 & 0 & -Q + \lambda^*\rho_2 I - \Gamma_1\gamma_1 \end{bmatrix},$$

$$\prod_1 = -P + (\tau_M - \tau_m + 1)Q + \lambda^*\rho_3 I,$$

$$\prod_2 = -P + (\tau_M - \tau_m + 1)Q + \lambda^*\rho_2 I,$$

$$R_{12} = \begin{bmatrix} -\Lambda_2 L_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\Gamma_2\gamma_4 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

$$R_{22} = \begin{bmatrix} -\Lambda_2 & 0 \\ 0 & -\Gamma_2 \end{bmatrix},$$

$$\overline{R}_{13} = \begin{bmatrix} C_1 P & 0 \\ 0 & 0 \\ A_1 P & 0 \\ B_1 P & 0 \end{bmatrix}, \quad \overline{R}_{23} = \begin{bmatrix} 0 & C_2 P \\ 0 & 0 \\ 0 & A_2 P \\ 0 & B_2 P \end{bmatrix},$$

$$R_{33} = \begin{bmatrix} -P & 0 \\ 0 & -P \end{bmatrix},$$

$$\hat{N} = \begin{bmatrix} N_{11} & 0 & N_{21} & N_{31} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & N_{12} & 0 & N_{22} & N_{32} & 0 & 0 \end{bmatrix},$$

$$\hat{P} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P \end{bmatrix}^T .$$

Proof. In order to establish the stability conditions, we introduce the following Lyapunov-Krasovskii functional candidate:

$$V(k) = V_1(k) + V_2(k) + V_3(k), \quad (15)$$

where

$$V_1(k) = x^T(k)Px(k) + y^T(k)Py(k),$$

$$V_2(k) = \sum_{i=k-\tau(k)}^{k-1} x^T(i)Qx(i) + \sum_{i=k-\sigma(k)}^{k-1} y^T(i)Qy(i),$$

$$V_3(k) = \sum_{j=k-\tau_M+1}^{k-\tau_m} \sum_{i=j}^{k-1} x^T(i)Qx(i) + \sum_{j=k-\sigma_M+1}^{k-\sigma_m} \sum_{i=j}^{k-1} y^T(i)Qy(i).$$

Calculating the difference of $V(k)$ along the model (1), and taking the mathematical expectation, we have

$$E\{\Delta V(k)\} = E\{\Delta V_1(k)\} + E\{\Delta V_2(k)\} + E\{\Delta V_3(k)\}, \quad (16)$$

Where:

$$\begin{aligned} E\{\Delta V_1(k)\} &= E\{V_1(k+1) - V_1(k)\} \\ &= E\Big\{\big[C_1(k)x(k) + A_1(k)f_1(y(k)) + B_1(k)g_1(y(k-\sigma(k)))\big]^T P \\ &\quad \times \big[C_1(k)x(k) + A_1(k)f_1(y(k)) + B_1(k)g_1(y(k-\sigma(k)))\big] \\ &\quad + h_1^T(k, y(k), y(k-\tau(k)))Ph_1(k, y(k), y(k-\tau(k))) \\ &\quad + \big[C_2(k)y(k) + A_2(k)f_2(x(k)) + B_2(k)g_2(x(k-\tau(k)))\big]^T P \\ &\quad \times \big[C_2(k)y(k) + A_2(k)f_2(x(k)) + B_2(k)g_2(x(k-\tau(k)))\big] \\ &\quad + h_2^T(k, y(k), y(k-\tau(k)))Ph_2(k, y(k), y(k-\tau(k))) \\ &\quad - x^T(k)Px(k) - y^T(k)Py(k)\Big\}, \end{aligned} \quad (17)$$

$$\begin{aligned} E\{\Delta V_2(k)\} &= E\{V_2(k+1) - V_2(k)\} \\ &= E\Bigg\{ \sum_{i=k+1-\tau(k+1)}^{k} x^T(i)Qx(i) - \sum_{i=k-\tau(k)}^{k-1} x^T(i)Qx(i) \\ &\quad + \sum_{i=k+1-\tau(k+1)}^{k} y^T(i)Qy(i) - \sum_{i=k-\tau(k)}^{k-1} y^T(i)Qy(i) \Bigg\} \\ &= E\Big\{x^T(k)Qx(k) - x^T(k-\tau(k))Qx(k-\tau(k)) \\ &\quad + \sum_{i=k+1-\tau(k+1)}^{k-1} x^T(i)Qx(i) - \sum_{i=k-\tau(k)+1}^{k-1} x^T(i)Qx(i) \\ &\quad + y^T(k)Qy(k) - y^T(k-\sigma(k))Qy(k-\sigma(k)) \\ &\quad + \sum_{i=k+1-\sigma(k+1)}^{k-1} y^T(i)Qy(i) - \sum_{i=k-\sigma(k)+1}^{k-1} y^T(i)Qy(i) \Big\} \\ &= E\Big\{x^T(k)Qx(k) - x^T(k-\tau(k))Qx(k-\tau(k)) \\ &\quad + \sum_{i=k+1-\tau_m}^{k-1} x^T(i)Qx(i) + \sum_{i=k+1-\tau(k+1)}^{k-\tau_m} x^T(i)Qx(i) - \sum_{i=k-\tau(k)+1}^{k-1} x^T(i)Qx(i) \\ &\quad + y^T(k)Qy(k) - y^T(k-\sigma(k))Qy(k-\sigma(k)) \\ &\quad + \sum_{i=k+1-\sigma_m}^{k-1} y^T(i)Qy(i) + \sum_{i=k+1-\sigma(k+1)}^{k-\sigma_m} y^T(i)Qy(i) - \sum_{i=k-\sigma(k)+1}^{k-1} y^T(i)Qy(i) \\ &\le E\Big\{x^T(k)Qx(k) - x^T(k-\tau(k))Qx(k-\tau(k)) \\ &\quad + \sum_{i=k+1-\tau_M}^{k-\tau_m} x^T(i)Qx(i) \\ &\quad + y^T(k)Qy(k) - y^T(k-\sigma(k))Qy(k-\sigma(k)) \\ &\quad + \sum_{i=k+1-\sigma_M}^{k-\sigma_m} y^T(i)Qy(i) \Big\}, \end{aligned} \quad (18)$$

$$\begin{aligned} E\{\Delta V_3(k)\} &= E\{V_3(k+1) - V_3(k)\} \\ &= E\Bigg\{ \sum_{j=k-\tau_M+2}^{k+1-\tau_m} \sum_{i=j}^{k} x^T(i)Qx(i) - \sum_{j=k-\tau_M+1}^{k-\tau_m} \sum_{i=j}^{k-1} x^T(i)Qx(i) \\ &\quad + \sum_{j=k-\sigma_M+2}^{k+1-\sigma_m} \sum_{i=j}^{k} y^T(i)Qy(i) - \sum_{j=k-\sigma_M+1}^{k-\sigma_m} \sum_{i=j}^{k-1} y^T(i)Qy(i) \Bigg\} \\ &= E\Bigg\{ \sum_{j=k-\tau_M+1}^{k-\tau_m} \left(x^T(k)Qx(k) - x^T(j)Qx(j)\right) \\ &\quad + \sum_{j=k-\sigma_M+1}^{k-\sigma_m} \left(y^T(k)Qy(k) - y^T(j)Qy(j)\right) \Bigg\} \\ &= E\Bigg\{ (\tau_M - \tau_m)x^T(k)Qx(k) - \sum_{i=k-\tau_M+1}^{k-\tau_m} x^T(i)Qx(i) \\ &\quad + (\sigma_M - \sigma_m)y^T(k)Qy(k) - \sum_{i=k-\sigma_M+1}^{k-\sigma_m} y^T(i)Qy(i) \Bigg\}. \end{aligned} \quad (19)$$

Notice that from (7) and (13), it is easy to see that

$$\begin{aligned} & h_1^T(k, y(k), y(k-\sigma(k)))Ph_1(k, y(k), y(k-\sigma(k))) \\ & \le \lambda_{\max}(P)h_1^T(k, y(k), y(k-\sigma(k)))h_1(k, y(k), y(k-\sigma(k))) \\ & \le \lambda^*\left(\rho_1 y^T(k)y(k) + \rho_2 y^T(k-\sigma(k))y(k-\sigma(k))\right), \end{aligned} \quad (20)$$

$$\begin{aligned} & h_2^T(k, x(k), x(k-\tau(k)))Ph_2(k, x(k), x(k-\tau(k))) \\ & \le \lambda_{\max}(P)h_2^T(k, x(k), x(k-\tau(k)))h_2(k, x(k), x(k-\tau(k))) \\ & \le \lambda^*\left(\rho_3 x^T(k)x(k) + \rho_4 x^T(k-\tau(k))x(k-\tau(k))\right). \end{aligned} \quad (21)$$

Substituting (17)-(21) into (16) yields

$$E\{\Delta V(k)\}$$
$$\le E\{[C_1(k)x(k)+A_1(k)f_1(y(k))+B_1(k)g_1(y(k-\sigma(k)))]^T$$
$$\times P[C_1(k)x(k)+A_1(k)f_1(y(k))+B_1(k)g_1(y(k-\sigma(k)))]$$
$$+[C_2(k)y(k)+A_2(k)f_2(x(k))+B_2(k)g_2(x(k-\tau(k)))]^T P$$
$$\times[C_2(k)y(k)+A_2(k)f_2(x(k))+B_2(k)g_2(x(k-\tau(k)))]$$
$$+x^T(k)\left[-P+(\tau_M-\tau_m+1)Q+\lambda^*\rho_3 I\right]x(k)$$
$$+x^T(k-\tau(k))\left[-Q+\lambda^*\rho_4 I\right]x(k-\tau(k))$$
$$+y^T(k)\left[-P+(\sigma_M-\sigma_m+1)Q+\lambda^*\rho_1 I\right]y(k)$$
$$+y^T(k-\tau(k))\left[-Q+\lambda_1^*\rho_2 I\right]y(k-\tau(k))\}$$
$$=E\{\xi^T(k)\Phi_1\xi(k)+\xi^T(k)\eta_1(k)P\eta_1^T(k)\xi(k)$$
$$+\xi^T(k)\eta_2(k)P\eta_2^T(k)\xi(k)\} \tag{22}$$

where

$$\xi(k)=(x^T(k),x^T(k-\tau(k)),f_1^T(y(k)),g_1^T(y(k-\sigma(k))),$$
$$y^T(k),y^T(k-\sigma(k)),f_2^T(x(k)),g_2^T(x(k-\tau(k))))^T,$$

$$\eta_1(k)=(C_1^T(k),0,A_1^T(k),B_1^T(k),0,0,0,0)^T,$$
$$\eta_2(k)=(0,0,0,0,C_2^T(k),0,A_2^T(k),B_2^T(k))^T,$$

$$\Phi_1=\begin{bmatrix}\Pi_1&0&0&0&0&0&0&0\\0&-Q+\lambda^*\rho_4 I&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&\Pi_2&0&0&0\\0&0&0&0&0&-Q+\lambda^*\rho_2 I&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\end{bmatrix}$$

With

$$\Pi_1=-P+(\tau_M-\tau_m+1)Q+\lambda^*\rho_3 I,$$
$$\Pi_2=-P+(\sigma_M-\sigma_m+1)Q+\lambda^*\rho_1 I.$$

From (9) and (10), we can get that [29]

$$\begin{bmatrix}y(k)\\f_1(y(k))\end{bmatrix}^T\begin{bmatrix}L_1\Lambda_1&-L_2\Lambda_1\\-L_2\Lambda_1&\Lambda_1\end{bmatrix}\begin{bmatrix}y(k)\\f_1(y(k))\end{bmatrix}\le 0, \tag{23}$$

$$\begin{bmatrix}x(k)\\f_2(x(k))\end{bmatrix}^T\begin{bmatrix}L_3\Lambda_2&-L_4\Lambda_2\\-L_4\Lambda_2&\Lambda_2\end{bmatrix}\begin{bmatrix}x(k)\\f_2(x(k))\end{bmatrix}\le 0, \tag{24}$$

$$\begin{bmatrix}y(k-\sigma(k))\\g_1(y(k-\sigma(k)))\end{bmatrix}^T\begin{bmatrix}\gamma_1\Gamma_1&-\gamma_2\Gamma_1\\-\gamma_2\Gamma_1&\Gamma_1\end{bmatrix}\begin{bmatrix}y(k-\sigma(k))\\g_1(y(k-\sigma(k)))\end{bmatrix}\le 0, \tag{25}$$

$$\begin{bmatrix}x(k-\tau(k))\\g_2(x(k-\tau(k)))\end{bmatrix}^T\begin{bmatrix}\gamma_3\Gamma_2&-\gamma_4\Gamma_2\\-\gamma_4\Gamma_2&\Gamma_2\end{bmatrix}\begin{bmatrix}x(k)\\g_2(x(k-\tau(k)))\end{bmatrix}\le 0, \tag{26}$$

It follows from (22)-(26) that

$$E\{\Delta V(k)\}$$
$$\le E\{\xi^T(k)\Phi_1\xi(k)+\xi^T(k)\zeta_1^T(k)P\zeta_1(k)\xi(k)$$
$$+\xi^T(k)\zeta_2^T(k)P\zeta_2(k)\xi(k)\}$$
$$-\begin{bmatrix}y(k)\\f_1(y(k))\end{bmatrix}^T\begin{bmatrix}L_1\Lambda_1&-L_2\Lambda_1\\-L_2\Lambda_1&\Lambda_1\end{bmatrix}\begin{bmatrix}y(k)\\f_1(y(k))\end{bmatrix}$$
$$-\begin{bmatrix}x(k)\\f_2(x(k))\end{bmatrix}^T\begin{bmatrix}L_3\Lambda_2&-L_4\Lambda_2\\-L_4\Lambda_2&\Lambda_2\end{bmatrix}\begin{bmatrix}x(k)\\f_2(x(k))\end{bmatrix}$$
$$-\begin{bmatrix}y(k-\sigma(k))\\f_1(y(k-\sigma(k)))\end{bmatrix}^T\begin{bmatrix}\gamma_1\Gamma_1&-\gamma_2\Gamma_1\\-\gamma_2\Gamma_1&\Gamma_1\end{bmatrix}\begin{bmatrix}y(k-\sigma(k))\\f_1(y(k-\sigma(k)))\end{bmatrix}$$
$$-\begin{bmatrix}x(k-\tau(k))\\g_2(x(k-\tau(k)))\end{bmatrix}^T\begin{bmatrix}\gamma_3\Gamma_2&-\gamma_4\Gamma_2\\-\gamma_4\Gamma_2&\Gamma_2\end{bmatrix}\begin{bmatrix}x(k)\\g_2(x(k-\tau(k)))\end{bmatrix}$$
$$=E\{\xi^T(k)R_1\xi(k)+\xi^T(k)\eta(k)\Xi\eta^T(k)\xi(k), \tag{27}$$

where $\eta(k)=(\eta_1(k),\eta_2(k))$,

$$\Xi=\begin{pmatrix}P&0\\0&P\end{pmatrix},\ R_1=\begin{pmatrix}R_{11}&R_{12}\\R_{12}^T&R_{22}\end{pmatrix}.$$

From Lemma 2, we know that

$$R_1+\eta(k)\Xi\eta^T(k)<0 \tag{28}$$

is equivalent to

$$\psi_1(k)=\begin{bmatrix}R_{11}&R_{12}&R_{13}\\R_{12}^T&R_{22}&R_{23}\\R_{13}^T&R_{23}^T&R_{33}\end{bmatrix}<0, \tag{29}$$

where

$$R_{13}=\begin{bmatrix}C_1(k)P&0\\0&0\\A_1(k)P&0\\B_1(k)P&0\end{bmatrix},\quad R_{23}=\begin{bmatrix}0&C_2(k)P\\0&0\\0&A_2(k)P\\0&B_2(k)P\end{bmatrix},$$

$$R_{33}=\begin{bmatrix}-P&0\\0&-P\end{bmatrix}.$$

Notice that R_{13} and R_{23} can be decomposed as follows:

$R_{13} = \bar{R}_{13} + \Delta R_{13}$, $R_{23} = \bar{R}_{23} + \Delta R_{23}$

where

$$\bar{R}_{13} = \begin{bmatrix} C_1 P & 0 \\ 0 & 0 \\ A_1 P & 0 \\ B_1 P & 0 \end{bmatrix}, \ \Delta R_{13} = \begin{bmatrix} (\Delta C_1)^T P & 0 \\ 0 & 0 \\ (\Delta A_1)^T P & 0 \\ (\Delta B_1)^T P & 0 \end{bmatrix},$$

$$\bar{R}_{23} = \begin{bmatrix} 0 & C_2 P \\ 0 & 0 \\ 0 & A_2 P \\ 0 & B_2 P \end{bmatrix}, \Delta R_{23} = \begin{bmatrix} 0 & (\Delta C_2)^T P \\ 0 & 0 \\ 0 & (\Delta A_2)^T P \\ 0 & (\Delta B_2)^T P \end{bmatrix}.$$

Let

$$\psi_1 = \begin{bmatrix} R_{11} & R_{12} & \bar{R}_{13} \\ R_{12}^T & R_{22} & \bar{R}_{23} \\ \bar{R}_{13}^T & \bar{R}_{23}^T & R_{33} \end{bmatrix},$$

$$\Delta \psi_1(k) = \begin{bmatrix} 0 & 0 & \Delta R_{13} \\ 0 & 0 & \Delta R_{23} \\ (\Delta R_{13})^T & (\Delta R_{23})^T & 0 \end{bmatrix},$$

Then

$$\psi_1(k) = \psi_1 + \Delta \psi_1(k). \qquad (30)$$

Let

$$\hat{P} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P \end{bmatrix}^T,$$

$$\hat{\zeta}(k) = \begin{bmatrix} \Delta C_1(k) & 0 & \Delta A_1(k) & \Delta B_1(k) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Delta C_2(k) & 0 & \Delta A_2(k) & \Delta B_2(k) & 0 & 0 \end{bmatrix},$$

$$\hat{N} = \begin{bmatrix} N_{11} & 0 & N_{21} & N_{31} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & N_{12} & 0 & N_{22} & N_{32} & 0 & 0 \end{bmatrix}.$$

By Lemma 1, it is not difficult to verify that

$$\begin{aligned} \Delta \psi_1(k) &= \hat{P}\hat{\zeta} + \hat{\zeta}^T \hat{P}^T \\ &= \hat{P}MF(k)\hat{N} + \hat{N}^T F^T(k) M^T \hat{P}^T \\ &\le \varepsilon \hat{N}^T \hat{N} + \varepsilon^{-1} \hat{P} M M^T \hat{P}^T. \end{aligned} \qquad (31)$$

From (30) and (31), we can get that

$$\Psi_1(k) \le \Psi_1 + \varepsilon \hat{N}^T \hat{N} + \varepsilon^{-1} \hat{P} M M^T \hat{P}^T \qquad (32)$$

By Lemma 1, we know that (14) is equivalent to

$$\Psi_1 + \varepsilon \hat{N}^T \hat{N} + \varepsilon^{-1} \hat{P} M M^T \hat{P}^T < 0. \qquad (33)$$

Along the similar line of the proof of Theorem 1 in [29], we can prove that model (1) is global exponentially stable in the mean square. The proof is completed.

4 Example

Consider a neural network (1), where

$$C_1 = \begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.4 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0.3 & -0.1 & 0.2 \\ 0 & -0.3 & 0.2 \\ -0.1 & -0.1 & -0.2 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.2 & 0.1 & 0.1 \\ -0.2 & 0.3 & 0.1 \\ 0.1 & -0.2 & 0.3 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.4 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.3 & -0.1 & 0.2 \\ 0 & -0.3 & 0.2 \\ -0.1 & -0.1 & -0.2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.2 & 0.1 & 0.1 \\ -0.2 & 0.3 & 0.1 \\ 0.1 & -0.2 & 0.3 \end{bmatrix},$$

$M = [0.1, 0.1, 0.1]^T$,

$N_{11} = N_{21} = N_{31} = N_{12} = N_{22} = N_{32} = [0.1 \quad 0.1 \quad 0.1]$,

$\tau_M = 5$, $\tau_m = 3$, $\rho_1 = \rho_2 = \rho_3 = \rho_4 = 0.2$

Take the activation functions as follows

$f_{11}(s) = \tanh(0.6s) - 0.2\sin s$, $f_{21}(s) = \tanh(-0.4s)$, $f_{31}(s) = \tanh(-0.2s)$, $f_{12}(s) = \tanh(0.6s) - 0.2\sin s$, $f_{22}(s) = \tanh(-0.4s)$, $f_{32}(s) = \tanh(-0.2s)$,

$g_{11}(s) = \tanh(-0.4s) + 0.2\sin s$, $g_{21}(s) = \tanh(0.2s)$, $g_{31} = \tanh(0.4s)$, $g_{12} = \tanh(-0.4s) + 0.2\sin s$, $g_{22} = \tanh(0.2s)$, $g_{32} = \tanh(0.4s)$.

From the above parameters, it can be verified that：

$$L_1 = \begin{bmatrix} -0.16 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ L_2 = \begin{bmatrix} -0.3 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} -0.16 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ L_4 = \begin{bmatrix} -0.3 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$$

$$\gamma_1=\begin{bmatrix}-0.12 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},\ \gamma_2=\begin{bmatrix}0.2 & 0 & 0\\ 0 & -0.1 & 0\\ 0 & 0 & -0.2\end{bmatrix}$$

$$\gamma_3=\begin{bmatrix}-0.12 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},\ \gamma_4=\begin{bmatrix}0.2 & 0 & 0\\ 0 & -0.1 & 0\\ 0 & 0 & -0.2\end{bmatrix}.$$

By using the Matlab LMI Toolbox, we solve LMIs (13) and (14), and obtain the feasible solutions as follows:

$$P=\begin{bmatrix}4.9188 & -0.3387 & 0.1408\\ -0.3387 & 3.3838 & 0.4585\\ 0.1408 & 0.4585 & 4.3376\end{bmatrix},$$

$$Q=\begin{bmatrix}0.6892 & -0.0758 & 0.0128\\ -0.0758 & 0.4075 & 0.0849\\ 0.0128 & 0.0849 & 0.7707\end{bmatrix},$$

$$\Lambda_1=\begin{bmatrix}2.3582 & 0 & 0\\ 0 & 3.4050 & 0\\ 0 & 0 & 3.2779\end{bmatrix},$$

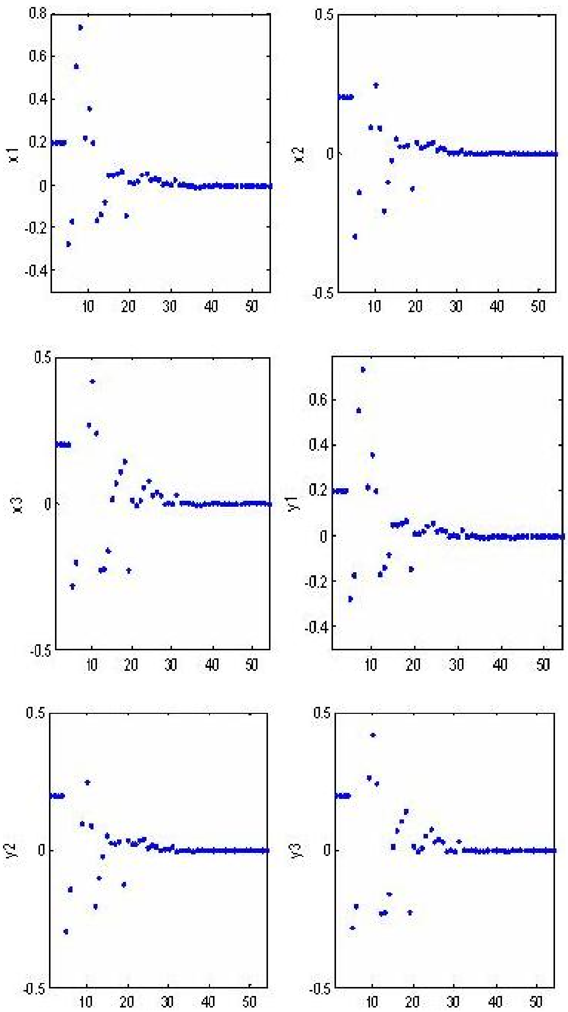
$$\Lambda_2=\begin{bmatrix}2.3995 & 0 & 0\\ 0 & 3.4266 & 0\\ 0 & 0 & 3.3604\end{bmatrix},$$

$$\Gamma_1=\begin{bmatrix}2.3122 & 0 & 0\\ 0 & 3.2173 & 0\\ 0 & 0 & 3.4451\end{bmatrix},$$

$$\Gamma_2=\begin{bmatrix}2.3250 & 0 & 0\\ 0 & 3.2840 & 0\\ 0 & 0 & 3.4659\end{bmatrix},$$

$$\varepsilon=2.1953,\qquad \lambda^*=0.1854.$$

By Theorem 1, we know that the considered neural network is robustly globally exponentially stable in the mean square. A numerical simulation of the network is shown in the following Figures, and it verifies the convergence of the neural network state.



5 Conclusions

In this paper, the global exponential stability has been investigated for the discrete-time uncertain stochastic bidirectional associate memory neural networks with time-varying delays and generalized activation function. A sufficient condition for checking the global robust exponential stability of the addressed neural networks has been obtained in terms of linear matrix inequalities (LMIs), which can be checked numerically using the effective LMI toolbox in MATLAB. An example is given to show the effectiveness and less conservatism of the proposed criteria.

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Yongming Li was born in 1965. He received the B.S. degree in Mathematics in 1986 from Sichuan Normal University, Chengdu, China. His current research interests include neural networks, chaos synchronization and stability theory.

Qizhan Lu was born in 1985. He received the B.S. degree in information and computing science in 2008 from Chongqing Jiaotong University, Chongqing, China. His current research interests include chaos synchronization and stability theory.

Qiankun Song was born in 1964. He received the B.S. degree in Mathematics in 1986 from Sichuan Normal University, Chengdu, China, and the M.S. degree in Applied Mathematics in 1996 from Northwestern Polytechnical University, Xi'an, China. He was a student at refresher class in the Department of Mathematics, Sichuan University, Chengdu, China, from September 1989 to July 1990. From July 1986 to December 2000, he was with Department of Mathematics , Sichuan University of Science and Engineering, Sichuan, China. From January 2001 to June 2006, he was with the Department of Mathematics, Huzhou University, Zhejiang, China. In July 2006, he moved to the Department of Mathematics, Chongqing Jiaotong University, Chongqing, China. He is currently a Professor at Chongqing Jiaotong University. He is the author or coauthor of more than 40 journal papers and one edited book. His current research interests include neural networks, chaos synchronization and stability theory.