# A New and Efficient method to Evaluate Residual Broadcast Reliability of Fault-tolerant Multistage Interconnection Networks

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**Summary**
This paper defines a new measure of reliability called Residual Broadcast Reliability for fault-tolerant multistage interconnection networks. A new and simple method for evaluating the Residual Broadcast Reliability of multistage interconnection networks (MIN) is proposed. A new algorithm to evaluate the Residual Broadcast Reliability has been proposed. Residual Broadcast Reliability of some important fault-tolerant multistage-interconnection networks have been evaluated and compared by the proposed method.

***Key words:***
Reliability; Probabilistic graph; Interconnection network *;*

## 1. Introduction

In recent years, there has been considerable interest and increased efforts in developing large parallel computing systems. Parallel computers have been applied in real time environments and they are becoming increasingly popular for large commercial applications as well. An important component of parallel computer is its interconnection network. Multistage Interconnection Network (MIN) is a low cost network, which interconnects N inputs with N outputs and has $log_2N$ switching stages. Each stage consists of *2x2* switching elements with or without loop dependent upon the regular or irregular class of network [1]. MIN's have been extensively used in the design of parallel and distributed systems [4] [13].

Reliability evaluation of MIN is important as it determines the usability and efficiency of the network to provide services. There are many reliability measures viz. terminal reliability, network reliability, broadcast reliability etc. The two terminal reliability addresses the probability that a given source-destination pair has at least one fault free path between them. Blake et al. [6] have developed closed form reliability expressions for two selected multistage interconnection networks (MINs). Subsequently, in a later paper, they have proposed a 2-level hierarchical model in which each sub-system is modeled as a markov chain and the system reliability of a MIN is modeled as a series system of 'markov' components. Their methods are good enough for MINs of smaller sizes. Varma et al. [19] considered three redundant–path networks namely Generalized INDRA, Merged Delta and Augmented C-network and derived analytical expressions for them. The method proposed by [7] is not generalized and as such is not applicable to all MINs. The methods [8] [11] [12] compute the bounds on reliability of MINs. In [18], terminal reliability of some specific multistage interconnection networks have been evaluated using graph theoretic techniques. Similarly, different performance measures for multistage interconnection networks can be found in the literature [3] [5] [9] [10] [14] [17].

However, in a parallel computer interconnection system with thousand of processors, the probability that some processors/links become faulty may be quite high and some nodes or links failure may destroy the regularity of the interconnection pattern. Thus, it may also affect the overall functionality of the system. Therefore, for maximum utilization of resources, it is important that all the processors remain connected in spite of some link failures. In this context, a new reliability measure called Residual Broadcast Reliability is defined for multistage interconnection networks. A simple and efficient method to evaluate the residual broadcast reliability of MINs is proposed which is well supported by a polynomial algorithm.

## 2. Proposed Approach (Evaluation of Residual Broadcast Reliability of MINs)

*Definition 1:*
A tree T is said to be a spanning tree of a connected graph G if T is a sub graph of G and T contains all the vertices of G.
*Definition 2:*
Residual broadcast reliability is the probability of existence of minimal good links such that a given source node and all destination nodes remain connected

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The *Residual broadcast reliability* of a graph G is defined as

$R_{Br}$= *Pr{ A given node can be connected to all the nodes in a network through surviving links}*

## 2.1 Mathematical Basis

*Notation:*

| | |
|---|---|
| *G* | *probabilistic graph* |
| *N* | *number of input/output nodes* |
| *E* | *edge set* |
| *S* | *system success expression* |
| *Pr(.)* | *probability* |
| $T_i, T_j$ | *path from the source node to destination node* |
| *e* | *an edge of the graph* |
| *X* | *union of success paths from one source to one destination* |
| *R(t)* | *residual broadcast reliability* |
| $\lambda_l$ | *link failure rate* |
| $\lambda_{SE}$ | *switching element failure rate* |
| $p_l$ | *success probability of a link or edge (e)* |
| $q_l$ | *failure probability of a link or edge* |
| $p_{SE}$ | *success probability of switching elements (SE)* |

At first, a MIN is converted into equivalent probabilistic graph *G(N,E)*. The node set is divided into two categories viz. (i) terminal nodes $(V_t)$ containing the input output nodes of the MIN (ii) intermediary nodes $(V_n)$ containing the switching elements. Any path $T_i$ is the initial path from the source *s* ($s \in V_t$ ) to one destination node *t* ($t \in V_t$).

First, we replace the edge $e_1$ incident upon the terminal node by a parallel edge $e_2$ that provides a path $T_j$ between the source and destination node. Then, we place the complement of the replaced edge in $T_j$. So, the new path $T_j$ generated is disjoint to the existing path $T_i$ . If no such more edge $e_2$ exists, replace the edge $e_3$ that is incident upon the intermediary nodes. Then, we find a new path $T_{j+1}$ from the source to the destination node. Next, we place the complement of $e_3$ in $T_{j+1}$.

This process is repeated till all the disjoint paths between source to destination nodes are generated. So the union of success of paths between the source node to destination node X is expressed as

$$X = T_i \; (T_1 \cup T_2 \cup \cdots)$$
$$X = T_i \bigcup_{j \geq 1} T_j \quad (1)$$

In a similar manner, $X_i$ may be obtained from one source to all destinations.

Thus, the system success is expressed as

$$S = X_1 \cup X_2 \cup \cdots \cup X_m$$
$$S = \bigcup_{i=1}^{m} X_i \quad (2)$$

Finally, the residual broadcast reliability can be computed as

$$R(t) = \{S\}_{dis} \quad \{e, \bar{e}, \cup, \cap \rightarrow p, q, +, \times\} \quad (3)$$

The algorithm presented in the next subsection implements the proposed method.

## 2.1 Algorithm (*MINRBRE*) (Evaluation of Residual Broadcast Reliability For MINs)

*Residual_broadcast_reliability_evaluation (G)*
*{*
*S =* $\phi$ *, X=* $\phi$
*for i=1 to N*
*{*
$T_i$ *= path from source node s to destination nodes* $t_i$*.*
*X=*$T_i$
*for* $T_i$ *generate a disjoint path* $T_j$
*{*
*X=X* $\cup$ $T_j$
*next j*
*}*
*S = S* $\cup$ *(X)*$_{dis}$
*}*

*R = Pr {S}*$_{\{ e,, \bar{e}, \cup, \cap \rightarrow p, q, +, p, q, +, \times \}}$
*}*

**Theorem 1:**

*The running time of the algorithm (MINRBRE) is polynomial in nature with respect to the number of input/output nodes.*

*Proof:*

In the algorithm (MINRBRE), finding the paths from a source to given node requires $O(N^3)$ operations. Since there are N number of destination nodes, finding all paths from one source to all destination nodes requires $O(N \times N^3) = O(N^4)$ operations. So, the running time of the algorithm is $O(N^4)$ which, is polynomial in nature with respect to input/output nodes.

## 3. Analysis of Residual Broadcast Reliability of Some Important Fault-tolerant Multistage Interconnection Networks

The residual broadcast reliability of some important fault-tolerant MINs viz. Extra Stage Cube (ESC) [1], Shuffle exchange network (SEN) [21], Extra stage shuffle exchange network (SEN+) [21], Multi path Chained Baseline Network (MPN) [1] and New four Tree Multistage Interconnection Network (NFT) [22] are evaluated and compared by the proposed algorithm in different environments.

**Calculation of reliability of links/switching element:**

With a given $\lambda_l(t)$, the reliability of links can be computed using the Exponential model i.e.

$$p_l(t) = e^{\left(-\int_0^t [\lambda_l(u)\,du]\right)}$$

$$p_l(t) = e^{-\lambda_l t} \qquad (4)$$

Similarly, the reliability of switching elements can be computed as

$$p_{SE}(t) = e^{\left(-\int_0^t [\lambda_{SE}(u)\;du\;]\right)}$$

$$p_{SE}(t) = e^{-\lambda_{SE} t} \qquad (5)$$

Here, we consider two distinct situations.

**Case 1: When links fail exponentially but switching elements are perfect**

Using equation (4), the reliability of links can be computed with respect to a given time t at different link failure rates. The residual broadcast reliability of different fault-tolerant MINs can be computed by using the proposed method. Table 1, 2 and Fig. 1 present comparative results of residual broadcast reliabilities of five different fault-tolerant MINs. From all these results, it can be concluded that, the MINs can be arranged in the following manner with respect to their residual broadcast reliability values.

NFT > MPN > ESC > SEN+ > SEN (6)

Table 1: Comparison of residual broadcast reliability of MINs when link reliabilities are different

| *Time in Hours* | ***Residual Broadcast Reliability*** | | | | |
|---|---|---|---|---|---|
| | ***ESC*** | ***SEN*** | ***SEN+*** | ***MPN*** | ***NFT*** |
| 100 | 0.803 | 0.705 | 0.756 | 0.830 | 0.912 |
| 200 | 0.676 | 0.593 | 0.623 | 0.710 | 0.792 |
| 300 | 0.533 | 0.432 | 0.489 | 0.590 | 0.665 |
| 400 | 0.456 | 0.304 | 0.354 | 0.472 | 0.523 |
| 500 | 0.340 | 0.170 | 0.218 | 0.395 | 0.400 |
| 600 | 0.205 | 0.016 | 0.108 | 0.260 | 0.276 |
| 700 | 0.108 | 0.015 | 0.097 | 0.136 | 0.154 |
| 800 | 0.086 | 0.013 | 0.081 | 0.102 | 0.139 |
| 900 | 0.070 | 0.012 | 0.070 | 0.081 | 0.101 |
| 1000 | 0.055 | 0.001 | 0.058 | 0.061 | 0.095 |

Table 2: Comparison of residual broadcast reliability of MINs with respect to $\lambda_l$

| *Value of $\lambda_l$* | ***Residual Broadcast Reliability*** | | | | |
|---|---|---|---|---|---|
| | ***ESC*** | ***SEN*** | ***SEN+*** | ***MPN*** | ***NFT*** |
| 0.001 | 0.7408 | 0.6065 | 0.6703 | 0.8187 | 0.9048 |
| 0.002 | 0.5488 | 0.3678 | 0.4493 | 0.6703 | 0.8187 |
| 0.003 | 0.4065 | 0.2231 | 0.3011 | 0.5488 | 0.7408 |
| 0.004 | 0.3011 | 0.1353 | 0.2019 | 0.4493 | 0.6703 |

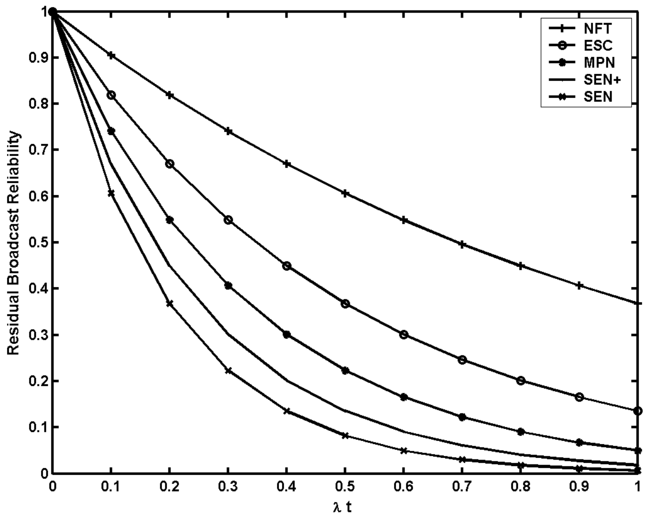


Fig. 1 Residual Broadcast Reliability of MINs with respect $\lambda_l$ t

**Case 2: When both links and switching elements may fail exponentially**

In case 1, it has been assumed that the SEs are perfect, but in actual practice the reliabilities of SEs are less than the reliabilities of links. So, links as well as the SEs may fail exponentially with respect to time t. Further, $\lambda_l \prec \lambda_{SE}$. Using equations (4) and (5), $p_l$ and $p_{SE}$ can be computed and the residual broadcast reliability of said MINs can be evaluated by the proposed method and the results are tabulated and plotted in Table 3 and Fig. 2 respectively. All these results agree with equation (6).

Table 3: Comparison of residual broadcast reliability of MINs when both links and switching elements may fail

| ***Time in Hours*** | ***Residual Broadcast Reliability*** | | | | |
|---|---|---|---|---|---|
| | ***ESC*** | ***SEN*** | ***SEN+*** | ***MPN*** | ***NFT*** |
| 100 | 0.3851 | 0.2765 | 0.3208 | 0.4887 | 0.5962 |
| 200 | 0.2605 | 0.1675 | 0.2173 | 0.3291 | 0.4384 |
| 300 | 0.1737 | 0.1132 | 0.1508 | 0.2168 | 0.2764 |
| 400 | 0.1236 | 0.0934 | 0.1085 | 0.1597 | 0.1862 |
| 500 | 0.0914 | 0.0807 | 0.0996 | 0.1145 | 0.1455 |
| 600 | 0.0828 | 0.0706 | 0.0762 | 0.0938 | 0.1143 |
| 700 | 0.0054 | 0.0039 | 0.0041 | 0.0673 | 0.0983 |
| 800 | 0.0028 | 0.0019 | 0.0023 | 0.0222 | 0.0723 |
| 900 | 0.0015 | 0.0007 | 0.0010 | 0.0161 | 0.0517 |
| 1000 | 0.0009 | 0.0055 | 0.0078 | 0.0090 | 0.0216 |

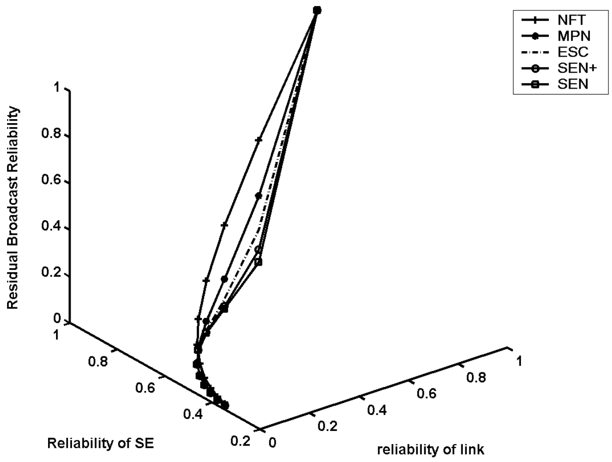


Fig.2 Residual Reliability of Different MINs when both SEs and links fail

## 4. Conclusion:

In this paper, a method for computation of Residual Broadcast reliability of multistage interconnection networks has been proposed. The proposed method has been used to compute the residual reliability of five important fault-tolerant multistage interconnection networks. The result of comparison establishes the New four Tree Multistage Interconnection Network (NFT) MINs to be the best of five MINs from residual broadcast reliability point of view.

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