Robust Legendre Moments Ellipse Fitting from Noisy Image

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Summary
In this paper we present a new efficient method for fitting ellipse to scattered data based on the Legendre moments. The least squares method is the most commonly used technique for fitting an ellipse. However, it has a low breakdown, which means that it performs poorly in the presence of outliers. Our new statistical approach is based on the expansion of the probability density function (p.d.f) in terms of Legendre polynomials which guarantees the extraction of an ellipse even for high rate of outliers and an important level of noise. Any constraint has been required in our approach**;** this leads to be applied for general conic fitting. A comparison is given between our approach and Direct Least Squares fitting of ellipses approach. Several tests demonstrate that it is preferment in terms of accuracy and robustness.

Key words:
Ellipse fitting, Legendre moments, Probability density function, Maximum Entropy Principal, least squares.

1. Introduction

The fitting of the primitive model to image data is a basic task in pattern recognition and computer vision. Detection and recognition of geometric primitives in images are the fundamental tasks of computer vision. A very important primitive is an ellipse which is a perspective projection of a circle that is exploited in many applications of computer vision like 3-D vision and object recognition, medical imaging and industrial inspections [1, 2, 3]. Over the years much attention has been paid to fitting ellipses to data samples, and many variations of the standard method for finding the least squares (LS) solution exist [4,5, 6, 7]. However, computer vision often requires more robust methods that can tolerate large amounts of outliers since there is the likelihood that the data will be substantially corrupted by faulty feature extraction, segmentation errors, etc. While LS is optimal under Gaussian noise it is very sensitive to severe non-Gaussian outliers, and is therefore unsuitable for many vision applications.
Hough transform (HT) provides a popular method for extracting geometric shapes. Primitives on the HT are represented by parametric curves with a number of free parameters [8, 9, 10].

A fair amount of research work has been accomplished in literature on ellipse fitting. The existing ellipse fitting techniques can be categorized in to two types:

- Least square fitting.
- Clustering.

Least squares fitting technique [5, 6] focuses on finding a set of parameters that minimize some distance measure between the data points and the ellipse. These methods are computationally better but are very sensitive to outlines.
Clustering methods focus on mapping sets of points to the parameter space, which are appropriately quantized depending on the application. Hough transform methods are the example of this type of technique [8, 9, 10]. These techniques have some advantages, like high robustness to occlusion and no requirement for pre-segmentation. But they suffer from great shortcomings of high computational complexity and non-uniqueness of solutions, which can render them unsuitable for real applications.
In this paper, a novel ellipse fitting approach is developed using a statistical method based on the estimation of probability density function (pdf) where the ellipse is defined as the local maxima of this pdf. The Main goal of our work is to compute the ellipse directly from a noisy image, without any a priori information and intermediate steps (binarization, filtering).
Our proposed approach is based on the expansion of a multivariate probability density function pdf in terms of Legendre polynomials by means of Legendre moment [14, 15, 16, 17]. For this purpose, the pdf is approximated by a truncated series of polynomials. As the determination of the expansion order is a difficult problem, we propose to estimate the pdf for different orders and to select the optimal one as the one for which the entropy reaches a maximum according to the Maximum Entropy Principal MEP [11, 12, 13].
Having the optimal p.d.f, the detection mode becomes a direct task where the true points of the ellipse are the maxima of the p.d.f, while the points far away from the curve (outliers) will present a low probability density value. Extraction of the maxima of the p.d.f is carried out using a proposed algorithm.
The paper is organized as follows: the next section describes the basis of our statistical model, using Legendre moment. The maximum entropy principal is given in section III. The details of our algorithm are presented in section IV. Section V presents the main results and

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performances of our ellipse fitting method. Finally, sectionVI deals with the summary of important results and conclusion.

2. Statistical model using Legendre Moments

Moment functions have been used as shape descriptors in a variety of applications in image analysis. In this section, Legendre moments are defined and their properties are briefly summarized.

2.1 Legendre Moments

The Legendre moments of order $(p+q)$ is defined for a given real image intensity function $f(x,y)$ as [16]:

$$\lambda_{p,q}=\frac{(2p+1)(2q+1)}{4}\iint_{RR} P_p(x)\ P_q(y)\ f(x,y)dxdy \qquad (1)$$

where $f(x,y)$ is assumed to have bounded support.

The Legendre polynomials $P_p(x)$ are a complete orthogonal basis set on the interval [-1, 1], for an order p, they are defined as [15]:

$$P_p(x)=\frac{1}{2^p\,p!}\frac{d^p}{dx^p}(x^2-1)^p \qquad (2)$$

The orthogonality property is guaranteed by the equality:

$$\int_{-1}^{1} P_p(x)P_q(x)dx=\frac{2}{(2p+1)}\delta_{p,q} \qquad (3)$$

where $\delta_{p,q}$ is the Kronecker function.

The aforementioned properties of the Legendre moments are valid as long as one uses a true analog image function. In practice, the Legendre moments have to be computed from sampled data, i.e., the rectangular sampling of the original image function $f(x,y)$, producing the set of samples $f(x_i,y_j)$ with an (M,N) array of pixels.

The piecewise constant approximation of $f(x,y)$ in (1), proposed recently by Liao and Pawlak [15] yields the following approximation of $\lambda_{p,q}$:

$$\hat{\lambda}_{p,q}=\sum_{i=1}^{M}\sum_{j=1}^{N} H_{p,q}(x_i,y_j)\, f(x_i,y_j) \qquad (4)$$

With the supposition that $f(x,y)$ is piecewise constant over the interval $[x_i-\frac{\Delta x}{2},x_i+\frac{\Delta x}{2}] \times [y_j-\frac{\Delta y}{2},y_j+\frac{\Delta y}{2}]$

where $\Delta x=(x_i-x_{i-1})$ and $\Delta y=(y_j-y_{j-1})$ are sampling intervals in the x and y directions and where

$$H_{p,q}(x_i,y_j)=\frac{(2p+1)(2q+1)}{4}\int_{x_i-\frac{\Delta x}{2}}^{x_i+\frac{\Delta x}{2}}\int_{y_j-\frac{\Delta y}{2}}^{y_j+\frac{\Delta y}{2}} P_p(x)\,P_q(y)dxdy \qquad (5)$$

represents the integration of the polynomial $P_p(x)\,P_q(y)$ around the (x_i,y_j) pixel.

2.2 Estimation of the Probability density function

By taking the orthogonality principle into consideration, the image function $f(x,y)$ can be written as an infinite series expansion in terms of the Legendre polynomials over the square [-1, 1]×[-1,1]:

$$f(x,y)=\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\lambda_{p,q}\ P_p(x)P_q(y) \qquad (6)$$

where the Legendre moments $\lambda_{p,q}$ are computed over the same square.

If only Legendre moments of order $\leq\theta$ are given, the image function reconstructed from $\hat{\lambda}_{p,q}$ can be approximated by a truncated series:

$$\hat{f}_\theta(x_i,y_j)=\sum_{p=0}^{\theta}\sum_{q=0}^{p}\hat{\lambda}_{p-q}\ P_{p-q}(x_i)P_q(y_j) \qquad (7)$$

The estimated probability density function (pdf) for a given order θ denoted $\hat{p}_\theta(x_i,y_j)$ is obtained by normalizing $\hat{f}_\theta(x_i,y_j)$ [17]:

$$\hat{p}_\theta(x_i,y_j)=\frac{\hat{f}_\theta(x_i,y_j)}{\sum_{x_i,y_j\in\Omega}\hat{f}_\theta(x_i,y_j)} \qquad (8)$$

Where:

$\sum_{x_i,y_j\in\Omega}\hat{p}_\theta(x_i,y_j)=1$ and $0\leq\hat{p}_\theta(x_i,y_j)\leq 1$, Ω is the image plane.

The estimated pdf depends only on the expansion order θ, a criterion for choosing this order is explained in the next section according to the maximum entropy principal MEP.

3. Optimal Order Moments Selection using MEP

The determination of the expansion order is a difficult problem and computationally expansive, because we ignore the order of the truncated expansion of *f(x,y)* which gives a good quality of the estimated input image function.
For this purpose, we introduce the maximum entropy principle MEP for the search of this optimal order. This automatic technique can estimate the optimal number of moments directly from the available data and does not require any a priori image information especially for noisy images.
Let G_w be a set of estimated underlying probability density function for various Legendre moment orders θ :

$$G_w = \{\hat{p}_\theta \quad / \quad \theta = 1........\omega\} \tag{9}$$

By applying the maximum entropy principle for noisy images, we deduce that among these estimates of the probability density function, there is one and only one probability density function denoted $\hat{p}^*_\theta(x_i, y_j)$ whose entropy is maximum [12, 13], and which represents the optimal probability density function, and then gives the optimal order of moments.

The Shannon entropy of $\hat{p}^*_\theta(x_i, y_j)$ is defined as:

$$S(\hat{p}_\theta) = -\sum_{x_i, y_j \in \Omega} \hat{p}_\theta(x_i, y_j) \log(\hat{p}_\theta(x_i, y_j)) \tag{10}$$

and the optimal $\hat{p}^*_\theta$ is such that :

$$S(\hat{p}^*_\theta) = MAX\{S(\hat{p}_\theta) \quad / \quad \hat{p}_\theta \in G_W\} \tag{11}$$

The process of determination the optimal order θ consists in estimating the pdf for different orders and selecting the optimal one as the one for which the entropy reaches maximum. The following is basic algorithm which consists in an exhaustive search to determine the optimal order which maximises $S(\hat{p}_\theta)$:

a- Initialize θ

b- Compute the pdf $\hat{p}_\theta$ and its corresponding Shannon entropy $S(\hat{p}_\theta)$

c- If $S(\hat{p}_\theta)$ is maximum, then θ is optimal and $\hat{p}_\theta = \hat{p}^*_\theta$, else $\theta = \theta + 1$ and go to b.

Then, having $\hat{p}^*_\theta$, we assign to each point of the data space, the optimal pdf $\hat{p}^*_\theta(x_i, y_j)$ defined by (8).
In this case, the "good data" are the set of points belonging to the mode of $\hat{p}^*_\theta$. By extracting the local maxima of $\hat{p}^*_\theta$, we can determine the exact points of the ellipse. In the next section, the details of our ellipse extraction algorithm are presented.

4. Ellipse Extraction Algorithm

We define the ellipse as the local maxima of the estimated probability density function selected in the previous section. The extraction of these local maxima allows us to determine the ellipse associated to the shape. The general idea of this algorithm consists of a successive points extraction presenting a local maxima of the selected optimal pdf.
The procedure consists in making a sweep mask of size 3x3 on the image. A comparison of the central pixel of the mask with its close eight neighbours following the eight directions (Figure 1), allows confirming whether this central pixel is a point of the ellipse or not.

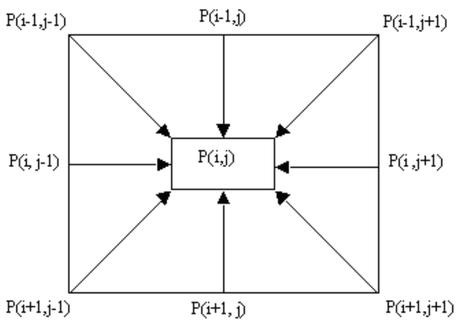


Figure 1: The central pixel P(i,j) with its eight close neighbour.

4.1 Algorithm

let $\hat{p}^*_\theta(x_i, y_j)$ be the optimal pdf :

If $\hat{p}^*_\theta(x_{i-1}, y_{j-1}) < \hat{p}^*_\theta(x_i, y_j)$

and $\hat{p}^*_\theta(x_{i+1}, y_{j+1}) < \hat{p}^*_\theta(x_i, y_j)$

and $\hat{p}^*_\theta(x_{i+1}, y_{j-1}) < \hat{p}^*_\theta(x_i, y_j)$

and $\hat{p}^*_\theta(x_{i-1}, y_{j+1}) < \hat{p}^*_\theta(x_i, y_j)$

and $\hat{p}^*_\theta(x_i, y_{j-1}) < \hat{p}^*_\theta(x_i, y_j)$

and $\hat{p}^*_\theta(x_i, y_{j+1}) < \hat{p}^*_\theta(x_i, y_j)$

and $\hat{p}^*_\theta(x_{i-1}, y_j) < \hat{p}^*_\theta(x_i, y_j)$

and $\hat{p}^*_\theta(x_{i+1}, y_j) < \hat{p}^*_\theta(x_i, y_j)$

Then assign the pixel of coordinate (i,j) to the true ellipse points.

5. Experimental Results

In this section, simulation results are carried out using two test sets. Firstly, our algorithm is applied to different noisy binary data and compared with conventional methods. Secondly, our algorithm is tested for noisy binary images corrupted by a percentage of outliers, where we have addressed the noise sensitivity in the case of Gaussian and impulsive noise.

Noisy binary image:

In this subsection, a comparison study is carried out on simulated and real noisy binary images. The proposed method is compared to Fitzgibbon method [6].

The experiments are performed on a sampled ellipse corrupted with noise from sigma =0,01 to 10 and adding a rate of outliers equal to 2% from data points. The ellipse coefficients (Cx,Cy,Rx,Ry,Theta) were computed from extracted data points which corresponding to local maxima of estimated p.d.f for an optimal order of Legendre moment Figure 4 and compared to the same coefficients given by Fitzgibbon approach in the same conditions.

The extracted ellipses given by the two algorithms, and the estimated p.d.f are depicted in Figure 3a, 3b and Figure 4 for four value of noisy variance.

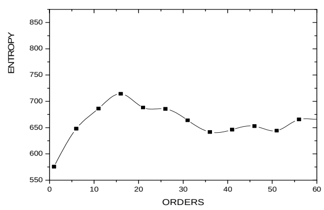


Figure 2: Entropy for different moment order .

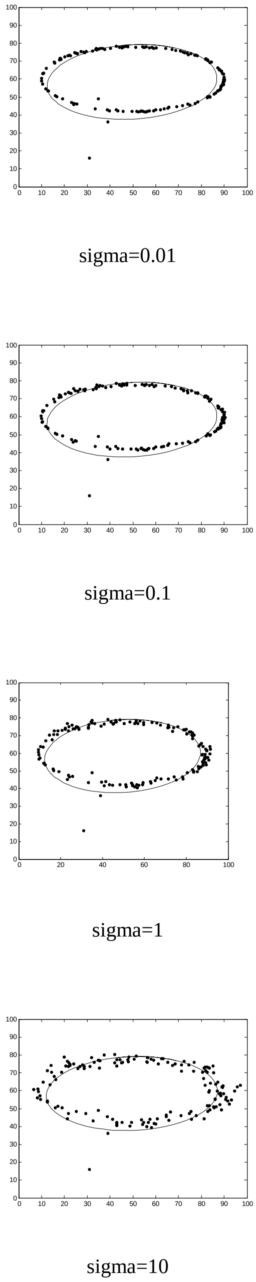


Figure 3(a) Results of the fitting ellipse by Fitzgibbon approach for different noise levels with rate of outliers equal to 2%.

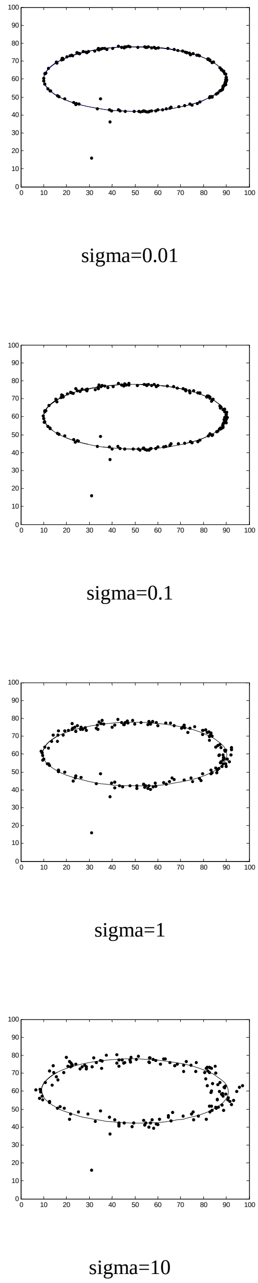
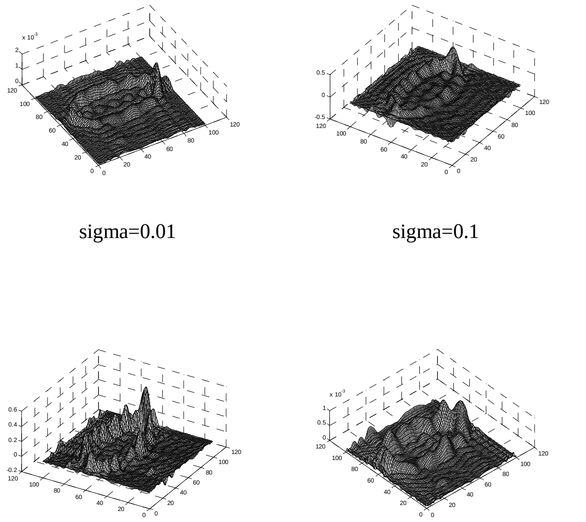


Figure 3(b) Results of the fitting ellipse by our proposed approach for different noise levels with rate of outliers equal to 2%.

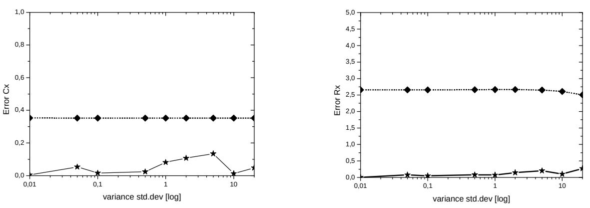


sigma=0.01 sigma=0.1

sigma=1 sigma=10

Figure 4: Optimal Estimated f.d.p for different noise levels with rate of outliers equal to 2%.

The ellipses extracted by our algorithm Figure 3(b) are accuracy and centred as well as original ellipses compared to Fitzgibbon algorithm Figure 3(a).
To compare the performance of the two approaches, we draw in Figure 5 the errors of the parameters of fitted ellipses to original ellipse parameters(ErrorCx, ErrorCy, ErrorRx, ErrorRy, ErrorTheta) for a standard deviation of noise varying from sigma = 0, 01 to 10 for the two algorithms .
Figure 5 shows clearly that the parameters ellipse errors created by our approach are less than those produced by Fitzgibbon method. That is, the parameters of ellipses computation by our algorithm are more accurate by the Fitzgibbon algorithm.



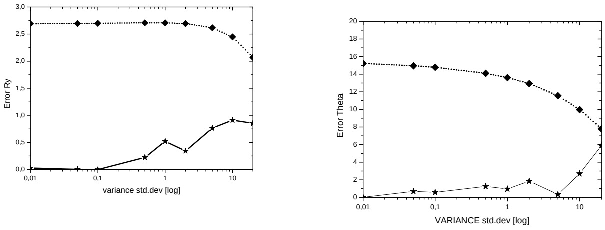
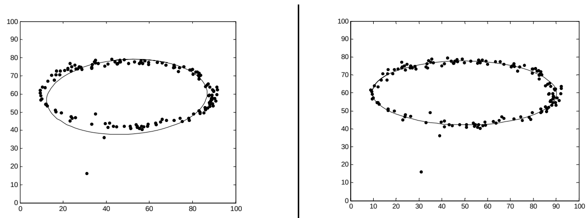


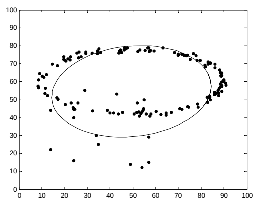
Figure 5 Relative Errors of parameters for ellipse fitted by two approaches for different noise levels with rate of outliers equal to 2% . Dashed curve represent our approach.

5.1 Outliers Sensibility

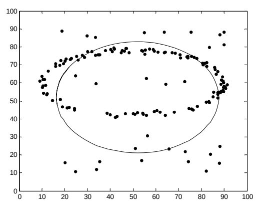
To see the ability of the proposed ellipse fitting approach to noisy data with a different rate of outliers, the proposed method is experimented with a whole ellipse centred at (50, 60) of semi axes (40, 18) and rotated by 0 degrees corrupted by a Gaussian noise with sigma =1and outliers with different rate (2%, 10% 20% and 40%).
The extracted ellipses obtained by Fitzgibbon method are shown in the first column of figure 6. The extracted ellipses generated by our algorithm are shown in the second column of figure 6.
Table 1 shows the calculated error of different parameters Cx, Cy, Rx, Ry and Theta of the ellipse. The error is obtained by taking the difference between the parameter of the original ellipse and the extracted one.
The first, second and third column of Table 2 shows respectively the different rate of Outliers, the error obtained by the Fitzgibbon method and the error obtained by our algorithm.
The obtained results figure 6 illustrate clearly the high insensitivity of the proposed method figure 6b to the Outliers compared to the well known Fitzgibbon method figure 6a.



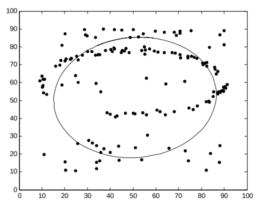
Rate= 2% Rate= 2%



Rate= 2%

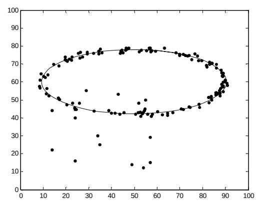


Rate= 2%

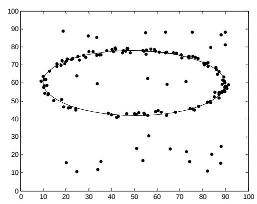


Rate= 2%

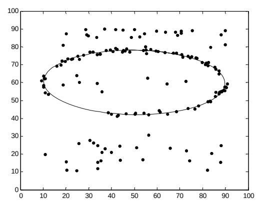
Figure 6 (a) . Original ellipses and extracted ellipses by Fitzgibbon method



Rate= 2%



Rate= 2%



Rate= 2%

Figure 6 (b). Original ellipses and extracted ellipses by our proposed approach

Figure 5. Ellipse fitting of noisy data corrupted by Outliers: The first column shows the noisy images corrupted by different rate of Outliers (respectively 2% 10% 20% and 40%) and the extracted ellipse by the Fitzgibbon method. The second column shows the extracted ellipse obtained using our proposed approach for optimal orders.

Table 2.a : Calculated Error of Cx parameter

(%) Outliers	Fitzgibbon method	Proposed method
2	0,5192	0,1559
10	0,5837	0,1108
20	1,972	0,0766
40	0,8691	0,1607

Table 2.b : Calculated Error of Cy parameter

Outliers rate	Fitzgibbon method	Proposed method
2	1,634	0,0661
10	5,3969	0,0955
20	8,0256	0,0952
40	8,3261	0,308

Table 2.c : Calculated Error of Rx parameter

Outliers rate	Fitzgibbon method	Proposed method
2	2,6646	0,0743
10	4,7959	0,55
20	4,3386	0,348
40	6,6363	0,0527

Table 2.d : Calculated Error of Ry parameter

Outliers rate	Fitzgibbon method	Proposed method
2	2,7081	0,5227
10	7,1137	0,1038
20	12,8888	0,1869
40	18,0147	0,4302

Table 2.d : Calculated Error of Theta parameter

Outliers rate	Fitzgibbon method	Proposed method
2	13,6165	0,966
10	31,9318	0,2156
20	0,4807	1,0028
40	65,8843	4,0705

Table 2: The calculated error of different parameters Cx, Cy, Rx, Ry and Theta (Table 2a, Table 2b, Table 2c and Table 2d) of the ellipse corrupted by 2%, 10%, 20% and 40% rate of Outliers (column 1). The error is obtained by taking the difference between the parameter of the original ellipse and the extracted one by the Fitzgibbon method (column 2) or by the proposed method (column 3).

6. Conclusion

In this paper, we have proposed a novel approach to robust ellipse fitting, based on a statistical method using the Legendre moment theory optimized by Maximum Entropy Principle (MEP). This new concept of ellipse fitting is based on to three steps. In the first one, an estimation of the underlying probability density function (pdf) using Legendre moment is carried out. In the second step, the choose of the optimal pdf is performed using the maximum entropy principal criterion. Finally, the subset of local maxima pixels of the optimal pdf are extracted as the true points of the ellipse. The advantage of our algorithm is that no a priori information and intermediate steps about the original data image are needed. Through a comparative study with other well established algorithms, it performed quite well in experimental tests and demonstrates a great robustness against high noise levels and high rate of Outliers.

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