

# Analysis of Output Shifted Coding Modulation for Decoding $\pi/4$ Shift DQPSK Signal

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## Summary

This paper proposes a new method to decode  $\pi/4$  shift DQPSK signal named as Output Shifted Coding Modulation (OSCM). OSCM is a combination of channel coding and modulation, also known as coded modulation. Traditionally, receive signal from  $\pi/4$ -shift DQPSK demodulator is converted to binary signal for hard decision decoding or quantized signal for soft decision decoding. The decoding methods that usually used are Viterbi decoder (VD) and Turbo Code. Although turbo code are powerful coding, the implementation are complicated compared to VD. For this new method, received signal from demodulator is directly decoded by OSCM decoder using modified trellis diagram. OSCM introduce two types of decision methods for error correction calculation which are multilevel decision and soft decision. The result from simulation and calculation shows that BER for OSCM are improving compared to high constraint length, k hard decision Viterbi decoder.

## Key words:

Channel coding, Convolutional codes, Viterbi decoding, Trellis codes.

## 1. Introduction

The goal of any forward error correcting codes is to reduce the probability of bit error, or to reduce the signal to noise ratio ( $E_s/N_0$ ), at the cost of expanding more bandwidth than would otherwise be necessary [1]. The channel encoder adds code bits to the transmission bit stream, based on the data bits at its input. These extra bits are used by the channel decoder at the receiver to correct errors introduced into the transmission stream by a noisy or fading channel. The disadvantages of forward error correcting codes are two-fold. Firstly, the injection of extra bits into the transmission stream has the effect that, if the original rate of transmission of 'useful' data bits is to remain the same, the symbol rate over the channel must be increased, thus increasing the bandwidth needed to transmit the signal [2]. Frequently, bandwidth expansion is not an option in modern wireless communication systems, where frequency spectrum is often highly regulated and bandwidth is costly. If the bandwidth is strictly limited,

then the effective data rate has to be decreased by the inclusion of an error correcting code.

The second disadvantage of forward correcting codes is that they add complexity to the design of a communications system. The decoder complexity can affect the development time, physical size, power consumption, memory requirements and total delay of a receiver [3]. Those problems are particularly troublesome at a time when new technologies are evolving, such as 3G, Digital Audio and Video Broadcasting, Wireless Local Loop and Bluetooth, which propose high-speed data transmission over wireless channels.

To address these problems, we proposed in this paper a coded modulation technique which is combination of forward error correcting codes (convolutional and Viterbi code) [4] and  $\pi/4$ -shift DQPSK. The proposed coded modulation is named as Output Shifted Coding Modulation (OSCM). It will be shown later that by using low constraint length convolutional component codes, OSCM able to outperform the convolutional codes using Viterbi decoders with much higher constraint lengths. It is capable in reducing the bandwidth expansion by using  $\pi/4$  DQPSK. This is achieved through fine tuning the set of rules for the mapping of coded bits and also with modifying error detection and correction techniques in trellis diagram.

The rest of the paper is organized as follows: Section 2 and 3 describe both the encoder and decoder of the proposed Output Shifted Coding Modulation Codes. Section 5 illustrates the decoder behaviors using the modified Trellis diagram, while the next section discussed the performance of the OSCM. Finally we conclude the paper in Section 6.

## 2. OSCM Encoder

### 2.1 OSCM Encoder

OSCM encoder consist of modified convolutional encoder (CE) and  $\pi/4$  DQPSK modulator. The example of OSCM encoder is shown in Fig. 1. This encoder

consists of G (7, 5), k=3 CE, which located in the left of the OSCM encoder, while in the right side consist of  $\pi/4$  DQPSK modulator with look up table systems. OSCM encoder started with CE operation. The information sequence  $b_k = (b_0, b_1, b_2, \dots, b_n)$  enters the encoder one bit at a time. Since the encoder is a linear, two encoder output sequences  $c_k^{(1)} = (c_0^{(1)}, c_1^{(1)}, c_2^{(1)}, \dots)$  and  $c_k^{(2)} = (c_0^{(2)}, c_1^{(2)}, c_2^{(2)}, \dots)$  can be obtained as the convolution of the input sequence  $b_k$  with the two encoder "impulse responses". The impulse responses are obtained by letting  $b_k = (1 \ 0 \ 0 \ \dots)$  and observing the two output sequences.

Let say we have input bit is defined as

$$b(D) = b_0, b_1, b_2, \dots, b_n \quad (1)$$

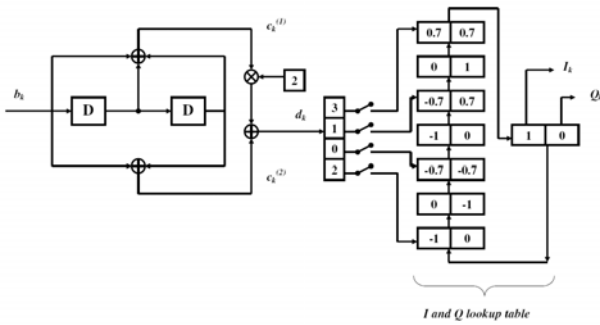


Figure 1.0: Output shifted coding modulation encoder

Table I Next state lookup table for G(7,5)

Current State	Next State, if	
	Input = 0:	Input = 1:
00	00	10
01	00	10
10	01	11
11	01	11

Table II Convolutional encoder output table

Current State	Output Symbol, if	
	Input = 0:	Input = 1:
00	00	11
01	11	00
10	10	01
11	01	10

From trellis diagram for G (7, 5), two lookup table are developed which are next state lookup table (Table

I) and output lookup table (Table II). From Table I, the possible transition of state can be written as:

$$\text{current\_state, } s_k = [b_{k-1} b_{k-2}]_{10} = s_{k-1} \quad (2)$$

$$\text{next\_state, } s_{k+1} = \text{Next\_state\_table}[s_k][b_k] \quad (3)$$

then, from (2) the possible output of convolutional encoder shown can be defined as (4) and (5) using Table II.

$$c_k^{(1)} c_k^{(2)} = \text{Output\_table}[s_k][b_k] \quad (4)$$

$$c^{(1)}(D) c^{(2)}(D) = c_0^{(1)}, c_0^{(2)}, c_1^{(1)}, c_1^{(2)}, c_2^{(1)}, c_2^{(2)}, \dots, c_{k+n}^{(1)}, c_{k+n}^{(2)} \quad (5)$$

Table III Displacement value lookup table

A	B	2A+B	$\Delta\theta$	Displacement
0	0	0	$5\pi/4$	5
0	1	1	$3\pi/4$	3
1	0	2	$7\pi/4$	7
1	1	3	$\pi/4$	1

Table IV I and Q lookup table

Current Level, $L_k$	I	Q
1	1	0
2	0.7	0.7
3	0	1
4	-0.7	0.7
5	-1	0
6	-0.7	-0.7
7	0	-1
8	0.7	-0.7

From the output lookup table, the relationship between the CE output and modulator is developed. This is achieved by considering the  $\pi/4$  DQPSK constellation and searching for a relationship between the input signals  $A_k B_k$  and the value of the absolute phase,  $\theta$ . The relationship between  $A_k B_k$  with Gray coding and  $\theta$  is depicted in Table III. For example if  $[A_k, B_k] = [0, 1]$ , displacement value for this code is 3. Displacement value is calculated through (6). These value is implied to form eight points of the  $\pi/4$  D-QPSK constellation signal, as shown in Table IV.

$$\text{Displacement} = 2A + B \quad (6)$$

As an example, if the present constellation point is  $[I_k, Q_k] = [-1, 0]$  which is in row 5 of the Table IV and the input data is  $[A_k, B_k] = [0, 1]$ , a displacement of three rows is implied in the Table IV. Three row

displacements are made and the data in row eight is recovered:  $[I_{k+1}, Q_{k+1}] = [0.7, -0.7]$ .

From the connection between  $[A_k, B_k]$  and displacement value, we can develop a bit mapper for the convolutional encoder output. The  $A_k B_k$  can be written with relation of convolutional encoder output as in (5).

$$A_k B_k = c_k^{(1)}, c_k^{(2)} \quad (7)$$

The displacement value as in (6) is derived as the  $c_k^{(1)}$  multiplied with decimal value of two and the result will be added with  $c_k^{(2)}$ . This operation could be defined as below.

$$d_k = 2 \cdot c_k^{(1)} + c_k^{(2)} \quad (8)$$

From (8),  $d_k$  can be defined as

$$d_k = d_0, d_1, d_2, \dots, d_n \quad (9)$$

Then, we can directly calculate  $I_k$  and  $Q_k$  value for  $\pi/4$ -DQPSK signal. The algorithm begins with set up the current level or the starting point as referred in Table IV.

$$\text{Current\_level}, L_k = [I_{k-1}][Q_{k-1}] = m_{k-1} \quad (10)$$

From (8) and (10), the mover or displacement value is calculated by adding  $L_k$  and  $d_k$

$$\text{mover}, m_k = L_k + d_k \quad (11)$$

and using rules in (11), we derived

$$\text{mover}, m_k = \begin{cases} L_k + d_k & \text{for } L_k + d_k \leq 8 \\ L_k + d_k - 8 & \text{for } L_k + d_k > 8 \end{cases} \quad (12)$$

finally, the  $I_k$  and  $Q_k$  value are determined as follows by using look up table in Table 4:

$$t_{Ik} = I_k = \text{IQ\_Table}[m_k][0] \quad (13)$$

$$t_{Qk} = Q_k = \text{IQ\_Table}[m_k][1] \quad (14)$$

which, 0 means column I, and 1 means column Q. Therefore, the OSCM encoder value is given by using (13) and (14).

$$t_k = t_{Ik}, t_{Qk}, t_{Ik+1}, t_{Qk+1}, t_{Ik+2}, t_{Qk+2}, \dots \quad (15)$$

### 2.2 Trellis Diagram for OSCM

Based on the connection of CE and  $\pi/4$  D-QPSK modulator, the trellis diagram for OSCM encoder can be drawn as shown in Figure 2. In this trellis, initial value of I and Q is  $[1,0]$ . The trellis begins at  $t_0=0$  or  $t_0$ , to  $t_1$ . The four possible states are depicted as four rows of horizontal

nodes. At  $t=1$ , the only possible channel symbol pairs are 00 and 11 and these values are converted using OSCM encoder becomes  $[-0.7, -0.7]$  and  $[0.7, 0.7]$ . Then, the trellis moves from  $t_1$  to  $t_2$ . The initial values for respectively node are  $[-0.7, -0.7]$  and  $[0.7, 0.7]$ .

The next possible output is repeated using OSCM encoder for next possible channel symbol pairs. At  $t=2$ , for initial value,  $I_1 Q_1 = [-0.7, -0.7]$ , the possible output for node a is  $[0,1]$  and  $[0,-1]$  for node c, while for initial value  $I_1 Q_1 = [0.7, 0.7]$  at node c, the possible output is  $[1,0]$  at node b and  $[-1,0]$  at node d. For  $t=3$ , each node will give two possible output. As example in figure 2, at node a, the two possible output is  $[0.7, -0.7]$  and  $[0.7, 0.7]$ . To choose the right possible output, the OSCM decoder is used.

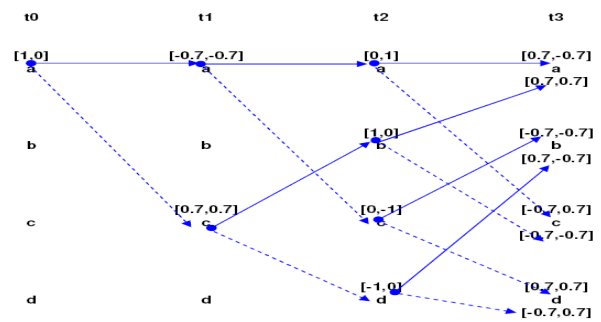


Figure 2.0 Trellis diagram for OSCM encoder

### 3. OSCM Decoder

A modified method to decode convolutional codes and  $\pi/4$  DQPSK signals is introduced which is the combination of Viterbi algorithm and  $\pi/4$  DQPSK demodulator. The output of OSCM encoder can be written as

$$t_k = t_1, t_2, t_3, t_4, t_5, t_6, \dots \quad (16)$$

where the odd and even  $t$  represent I and Q signal respectively. This can be written as

$$t_k = t_{Ik}, t_{Qk}, t_{Ik+1}, t_{Qk+1}, t_{Ik+2}, t_{Qk+2}, \dots \quad (17)$$

and the received signal written as

$$r_k = r_{Ik}, r_{Qk}, r_{Ik+1}, r_{Qk+1}, r_{Ik+2}, r_{Qk+2}, \dots \quad (18)$$

We modified two decisions decoding same as VA. The different between OSCM and VA are its hard decision method and trellis diagram. In VA, we used hamming distance to calculate BM while in OSCM, the branch metric (BM) is calculated based on the difference between received demodulator signal and possible OSCM output. This method is called multilevel decision method. Another difference between VA and OSCM is the output of trellis diagram for OSCM codes is depends on the past I and Q while for VA, its output is fixed.

#### 3.1 Multilevel Decision Input Method

In order to calculate the BM using multilevel decision,

trellis diagram in Figure 2.0 is analyzed. It is calculated based on the difference between  $r_k$  and  $t_k$ .

$$d^H(r_k, t_k) = |r_k - t_k| = z_k \tag{19}$$

where

$$z_k = z_{1k} + z_{Qk} \tag{20}$$

$$z_{1k} = |r_{1k} - t_{1k}| \tag{21}$$

$$z_{Qk} = |r_{Qk} - t_{Qk}| \tag{22}$$

### 3.2 Soft Decision Input Method

Assume that the quantize received signal are

$$r_k = r_{1k}, r_{2k}, r_{3k}, r_{4k}, r_{5k}, \dots \tag{23}$$

BM is calculated based on the Euclidian distance for soft decision method. Therefore by using (23) and using (16), the BM is

$$d^H(r_k, t_k) = |r_k - t_k|^2 = z_k \tag{24}$$

### 3.3 Modified Viterbi Algorithm

From  $z_k$  value which is calculated using (20) or (24), then the path metric can be written as

$$\Gamma_k = z_k \tag{25}$$

the following recursive equation holds:

$$\Gamma_k = \Gamma_{k-1} + z_k \tag{26}$$

When the BM is calculated, by using VA with modified technique, we change the rules of VA by satisfying

1. the path metric, or cost function, defined as:

$$\Gamma(s_k = \sigma_j) = \min_{s_0, s_1, \dots, s_k = \sigma_j} \Gamma_k \tag{27}$$

2. the survivor output, defined as the output of symbols that ends in that state and determines  $\Gamma(s_k = \sigma_j)$ :

$$L(s_k = \sigma_j) = t_k \tag{28}$$

3. the survivor sequence, defined as the sequence of symbols that ends in that state and determines  $\Gamma(s_k = \sigma_j)$ :

$$L(s_k = \sigma_j) = (s_0, s_1, \dots, s_k = \sigma_j) = (aL_1, \dots, a_{k+L_1}) \tag{29}$$

## 4. Traceback using Trellis Diagram of OSCM

In this paper, we focused on the transmit signal corrupted only by AWGN. Let say the transmitted signal using OSCM codes are

$[0.7, 0.7, -1, 0, 0.7, -0.7, 1, 0, 0.7, 0.7, -1, 0, 0.7, -0.7, 1, 0]$  and the corrupted signal are  $[0.7, 0.7, -1, 0, 0.7, \mathbf{0.7}, 1, 0, \mathbf{-0.7}, 0.7, -1, 0, 0.7, -0.7, 1, 0]$  with the bold number indicates the errors. The received signal value is assumed to be near with  $\pi/4$  D-QPSK constellation to make an easy explanation and calculation. In real case, the received signal value is a random value. As we seen in hard decision VD, when this signal is change to binary number, the error increased. In order to solve this problem, the OSCM codes are introduced.

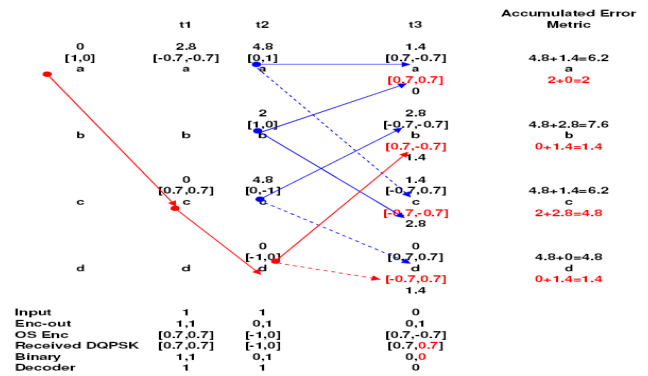


Figure 3.0 Trellis diagram at t=3 for corrupted signal

Figure 3.0 shows the trellis diagram for transition  $t=3$  which the error signal happened. At this time, the error signal received which is  $[0.7, \mathbf{0.7}]$ . At  $t=2$ , where the current state is d, the next possible state are b and d. The ACM for these both state are the minimum than others. Therefore the SOM are  $[0.7, -0.7]$  and  $[-0.7, 0.7]$  for b and d respectively. The BM and ACM for state b and d are same which is 1.4, since that, the best path cannot be selected. The solution to this problem is by looking to next path. In Figure 4.0, at  $t_4$ , the correct path can be chosen by looking at the minimum ACM which is 1.4. That's mean the survivor path are from d to b for  $t=2$  to  $t=3$  and the SOM is  $[0.7, -0.7]$ .

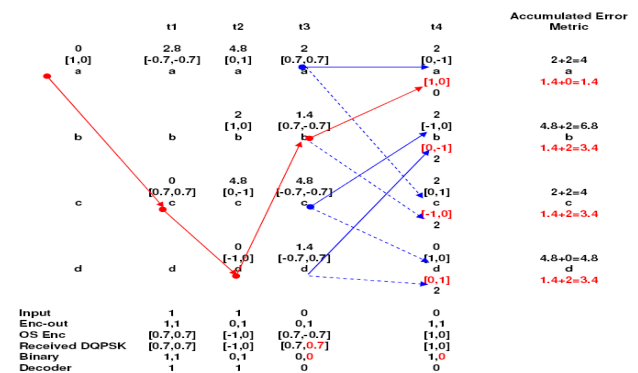


Figure 4.0 Trellis diagram at t=4 for corrupted signal

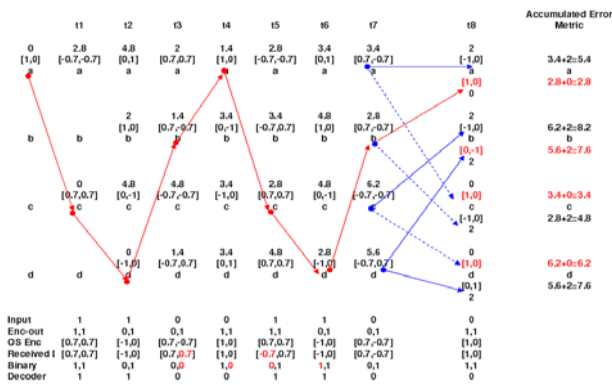


Figure 5.0 Trellis diagram at t=8 for corrupted signal

The complete trace back path is shown in Figure 5.0. All error is fixed by OSCM decoder. From the trellis diagram, the OS decoder output is same with input signal which is 11001100. Table V and VI show the OSCM decoder and Viterbi decoder outputs respectively. It shows that, from received DQPSK, it's better to decode using OSCM decoder than Viterbi decoder because when the DQPSK signal is converted to binary codes, the error increased. Table VII list the total error and BER derived from Table V and VI. It shows OSCM decoder could decode the corrupted signal and gives better BER than Viterbi decoder (VD). For this received signal, OSCM decoder have BER 0, while VD without zero ended have BER 0.625 and VD with zero ended gives reading 0.5.

Table V Result for OSCM Decoder

	t1	t2	t3	t4	t5	t6	t7	t8
Input	1	1	0	0	1	1	0	0
CE	1,1	0,1	0,1	1,1	1,1	0,1	0,1	1,1
OSCM Encoder	0.7,0.7	-1,0	0.7,-0.7	1,0	0.7,0.7	-1,0	0.7,-0.7	1,0
Received DQPSK	0.7,0.7	-1,0	0.7,0.7	1,0	-0.7,0.7	-1,0	0.7,-0.7	1,0
OSCM Decoder	1	1	0	0	1	1	0	0

Note: Red number in table V indicates error signal.

Table VI Result for Viterbi Decoder

	t1	t2	t3	t4	t5	t6	t7	t8
Input	1	1	0	0	1	1	0	0
CE	1,1	0,1	0,1	1,1	1,1	0,1	0,1	1,1
Transmitted DQPSK	0.7, 0.7	-1,0	0.7,-0.7	1,0	0.7,0.7	-1,0	0.7,-0.7	1,0
Received DQPSK	0.7, 0.7	-1,0	0.7,0.7	1,0	-0.7,0.7	-1,0	0.7,-0.7	1,0
Binary Codes	1,1	0,1	0,0	1,0	0,1	0,0	0,1	1,1
Viterbi Decoder	1	1	1	1	0	0	0	1
Viterbi Decoder (zero-ended)	1	1	1	1	0	0	0	0

Note: Red number in table VI indicates error signal.

Table VII Bit error rate analysis for OSCM Decoder and Viterbi Decoder

Coding	Total Error	Bit Error Rate(BER)
OSCM Decoder	0	0
Viterbi Decoder	5	0.625
Viterbi Decoder (Zero-ended)	4	0.5

### 5. Result and Discussion

Figure 6.0 shows the performance between uncoded  $\pi/4$ -DQPSK and multilevel decision OSCM with constraint length, k equal to three. The performance between uncoded  $\pi/4$  DQPSK and multilevel decision OSCM is increased about 6 dB at bit error rate (BER) equal to  $10^{-3}$ . The high performance gained which is obtained by using OSCM, proof that OSCM coding is a potential approach that could fix error for  $\pi/4$  DQPSK signal. We further analyze the results between OSCM and hard decision Viterbi decoder method. As we can see from Figure 7.0, the BER performance is improved along with the increasing of the constraint length. However, the complexity of the codes is higher. As depicted in Fig. 7, it shows that OSCM has a better result compare to hard decision for k equal to 3, 5, 7 and 9. OSCM present at a range of 0.8 to 2.5 dB better than hard decision Viterbi decoder.

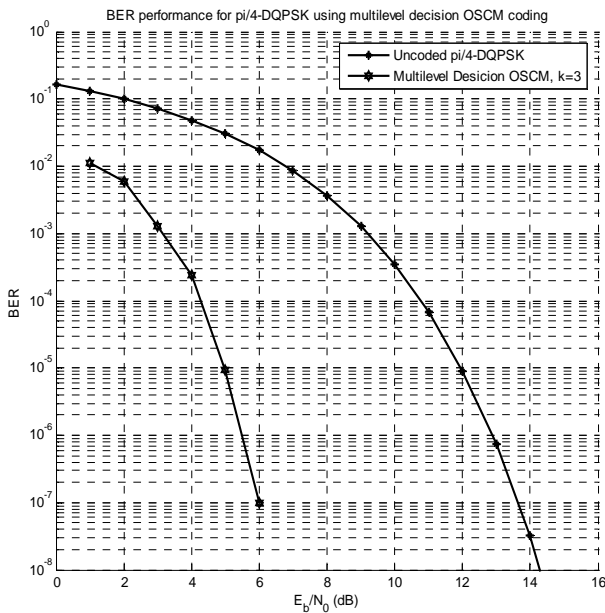


Figure 6.0 BER performance of uncoded  $\pi/4$  DQPSK and OSCM decoder over AWGN channel.

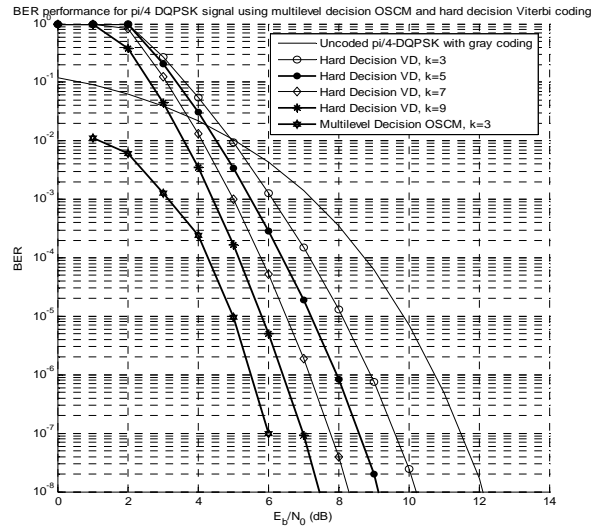


Figure 7.0 BER performance of  $\pi/4$ -DQPSK signals using hard decision Viterbi decoder and multilevel decision OSCM over AWGN channel.

### 6. Conclusion

From the results obtained, it is proven that OSCM is capable to give better performance for  $\pi/4$  DQPSK signal. In fact, the bit error rate is better than Viterbi decoder with k equal to 9. Note that, it is based on multilevel decision method. As the OSCM encoder has a low complexity, the proposed scheme is better than Viterbi Decoder method.

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