# **Design of Decentralized Fuzzy Controllers for Quadruple tank Process**

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#### Summary

The quadruple-tank process has been widely used in control literature to illustrate many concepts in multivariable control, particularly, performance limitations due to multivariable right half-plane zeros. The main feature of the quadruple-tank process is the flexibility in positioning one of its multivariable zeros on either half of the 's' plane. The objective of the current study is to design a Decentralized fuzzy controller for a multivariable laboratory four-tank process. Simulation results confirm the effectiveness of the proposed control methodology

*Key words:* : *Fuzzy control, Quadruple tank process, Decentralized control, non-minimum phase system* 

# 1. Introduction

Chemical plants are tightly integrated process, that exhibit non-linear behavior and complex dynamic properties. Many industrial controlled problems have multiple manipulated and controlled variables. It is common for models of industrial processes to have significant uncertainties, strong interaction, and/or nonminimum phase behavior(i.e., right-half-plane transmission zeros). The multi variable four-tank system exhibits characteristics of interest in both control and research education.

The linearized dynamics of the quadruple-tank system has a multivariable zero that is possible to move along the real axis either in left half-plane or right halfplane by simply changing the valves. [1]. It exhibits elegantly complex dynamics ,which includes interactions ,transmission zero, and non-minimum phase characteristics that emerge from a simple cascade of tanks. To control a quadruple tank system ,one essential problem is how to handle the interactions among two loops .An effective approach is to apply the so called decentralized control strategy: each loop is controlled by

one controller independently based on local information and local actions.

Fuzzy control has found promising applications for a wide variety of industrial systems Based on the universal approximation capability ,many effective fuzzy control schemes have been developed to incorporate with human experts knowledge and information in a systematic way, which can also guarantee various stability and performance criteria, not only for SISO nonlinear systems but also for MIMO nonlinear systems. The main advantages of these fuzzy-logic-based control schemes lies in the fact that the developed controllers can deal with increasingly complex systems and to implement controllers without precise knowledge of the model structure of underlying dynamic systems.

This paper presents a fuzzy decentralized control methodology for a quadruple tank process.

The rest of the paper is structured as follows: Section 2 describes the Plant description of Quadruple tank process. The design of controllers for quadruple tank process is explained in Section 3 followed by Results and discussions in Section 4.The conclusion is explained in section 5.

# 2. Quadruple Tank Process

This is a new laboratory process, which was designed to illustrate performance limitations due to zero location in multivariable control systems. The process is called quadruple-tank process [1] and consists of four interconnected water tanks and two pumps. The system is shown in Figure.1. Its manipulated variables are voltages to the pumps and the controlled variables are the water levels in the two lower tanks. The quadruple-tank process can easily be built by using two double-tank processes. The output of each pump is split into two using a threeway valve. Thus each pump output goes to two tanks, one lower and another upper, diagonally opposite and the ratio of the split up is controlled by the position of the valve. With the change in position of the two valves, the system can be appropriately placed either [1] in the minimumphase or in the non-minimum phase. The physical parameters of the process given by Johansson [1] are given in Table 1. The material balance for the quadruple-

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tank process is given by the Equations 1 to 4. Note that  $\gamma_1$ and  $\gamma_2$  are the ratios in which the outputs of the two pumps get divided. The sampling time of the process is assumed as 1 second.

If  $\gamma_1$  is the ratio of flow to the first tank, then 1 - $\gamma_1$  will be the flow to the fourth tank. As the inputs to the pumps are the voltages  $v_1$  and  $v_2$ ,  $k_1$  and  $k_2$  are conversion factors, expressed in flow per unit voltage input to the pumps. The outputs are  $y_1$  and  $y_2$  (voltages from the level measurement devices).

$$\frac{dh_1}{dt} = \frac{-a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1k_1}{A_1}v_1 \qquad (1)$$
$$\frac{dh_2}{dt} = \frac{-a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2k_2}{A_2}v_2 \qquad (2)$$

$$\frac{dh_3}{dt} = \frac{-a_3}{A_3} \sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}v_2$$
(3)

$$\frac{dh_4}{dt} = \frac{-a_4}{A_4} \sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}v_1$$
(4)

Where,  $A_i$  -Cross sectional area of the tank (m<sup>2</sup>)

- $a_i$  -Cross sectional area of the outlet hole (m<sup>2</sup>)
- h<sub>i</sub>- Water level (m)



Fig. 1. Quadruple-tank process The parameter values of the quadruple tank process are represented in the following table.

Table 1: Physical parameters of the tanks

Sl.no	Description	Value
1	Area of the tanks $A_1 A_3$	28 cm <sup>2</sup>
2	Area of the tanks $A_2 A_4$	$32 \text{ cm}^2$
3	Area of outlet pipes a <sub>1</sub> a <sub>3</sub>	0.071 cm <sup>2</sup>
4	Area of outlet pipes a2 a4	$0.057 \text{ cm}^2$
5	Constant k	0.50 V/cm
6	Gravitational constant g	981 cm/s <sup>2</sup>

The model and control of the process are studied at two operating points;  $P_{-}$  at which the system exhibits minimum-phase characteristics and  $P_{\perp}$  at which the system exhibits non-minimum characteristics. The chosen operating points correspond to the values given in the Table 2.

Table 2: Operating parameters of minimum-phase and non-minimumphase system

Parameters	Minimum Phase	Non Minimum Phase
$h_1^0, h_2^0$	12.4 , 12.7	12.6 , 13.0
$h_3^0, h_4^0$	1.8 , 1.4	4.8,4.9
$v_1^0, v_2^0$	3.00, 3.00	3.15, 3.15
k <sub>1</sub> ,k <sub>2</sub>	3.33, 3.35	3.14 , 3.29
$\gamma_1, \gamma_2$	0.70,0.60	0.43, 0.34

The open loop response of non-minimum phase and minimum phase system are shown in Fig.2 and Fig.3



Fig .2 Non-.Minimum phase Open loop Response



Fig.3 Minimum phase open loop Response

The transfer function matrices for both the phases of the system are

$$G_{-}(s) = \begin{bmatrix} \frac{2.6}{1+62s} & \frac{1.5}{(1+23s)(1+62s)} \\ \frac{1.4}{(1+30s)(1+90s)} & \frac{2.8}{1+90s} \end{bmatrix}$$
  
and  $G_{+}(s) = \begin{bmatrix} \frac{1.5}{1+63s} & \frac{2.5}{(1+39s)(1+63s)} \\ \frac{2.5}{(1+56s)(1+91s)} & \frac{1.6}{1+91s} \end{bmatrix}$  (5)

The dominant time constants are similar for both the operating conditions. For the first transfer matrix, these are found at -0.060 and -0.018. Both the zeros lie on the left half of the s plane, and hence, the system is in the minimum-phase. For the second case, the zeros are located at -0.057 and 0.013. Since one of the zeros is in the right half of the s plane, the system is in the non-minimum phase.

#### **3.Design Of Controllers**

#### 3.1 Decentralized PI Controller

The structure shown in Fig. 4 has individual PI controller [5] for individual loops. The manipulated variables are the function of error of that particular loop. The pairing of the loops is decided by the Relative Gain Array (RGA) analysis [2]. For minimum-phase settings,  $\lambda$  is 1.40. So, u<sub>1</sub> must be paired with y<sub>1</sub> and u<sub>2</sub> must be paired with y<sub>2</sub> for better performance. For non-minimum-phase system,  $\lambda$  is -0.64 [3]. So, u1 must be pared with y2 and u2 must be paired with y1 to achieve good control performance.



Fig.4. Multi-loop control structure for minimum-phase system with two PI controllers C1 and C2.

The PI controller equation is given by

 $Y_j(s)=K_j (1+1/T_{ij} s)$ , j=1,2. (6) The controller parameters are obtained by the direct synthesis method both for the minimum phase system and non-minimum phase system.

For the minimum-phase the controller settings  $\{K_1,Ti_1\}=\{2.39,62\}$  and  $\{K_2,Ti_2\}=\{3.21,90\}$  gives better performance. The non-minimumphase controller settings are  $\{K_1,Ti_1\}=\{1.36,102\}$  and  $K_2,Ti_2\}=\{0.24,147\}$ .

# 3.2 Decentralized Fuzzy Logic Controller Design

Fuzzy logic control is derived from fuzzy set theory introduced by Zadeh in 1965 .In fuzzy set theory, the transition between membership and non-membership can be gradual .Therefore boundaries of fuzzy sets can be vague and ambiguous, making it useful for approximate system. Combining multi valued logic, probability theory, and knowledge base, FLC is a digital methodology that simulates human thinking by incorporating the imprecision inherent in all physical systems. Fuzzy logic controller is an attractive choice when precise mathematical formulations are not possible[6,7]. The decentralized fuzzy control structure includes two fuzzy SISO controllers. In the proposed control method for the quadruple tank process, two fuzzy logic controllers used separately for controlling the level outputs. The structure is shown in Fig.5.



Fig.5. Decentralized control structure for minimum-phase system with two Fuzzy controllers

Basic configuration of FLC comprises of three principal components: fuzzification interface, , decision making logic and a defuzzification interface, as presented at the end.

#### Fuzzification

Fuzzy logic uses linguistic variables instead of numerical variables. The process of converting a numerical variable in to a linguistic variable is called fuzzification. In the present work the error and change in error of level outputs (h1 and h2) are taken as inputs and the pump voltages(v1,v2) are the controller outputs. The error and change in error is converted into seven linguistic values namely NB,NM,NS,ZR,PS,PM and PB. Similarly controller output is converted into seven linguistic values namely NB,NM,NS,ZR,PS,PM and PB. Triangular membership function is selected and the elements of each of the term sets are mapped on to the domain of corresponding linguistic variables

#### Decision Logic stage

Basically, the decision logic stage is similar to a rule base consisting of fuzzy control rules to decide how FLC works. This stage is constructed by expert knowledge and experiences. The rules are generated heuristically from the response of the conventional controller .49 rules are derived for each fuzzy controller from careful analysis of trend obtained from the simulation of conventional controller and known process knowledge. The rules are enumerated in Table 4.The decision stage processes the input data and computes the controller outputs. *Defuzzification* 

The output of the rule base is converted into crisp value, this task is done by defuzzification module. Centroid method of defuzzification is considered for this application. The parameters of FLC designed, are presented in Table 3.

Parameter	Value
No. of input variables	2
No. of output variables	1
No. of linguistic variables	7
No. of rules	49
Membership function	Triangular
Defuzzification	Centre of gravity method

Table .4:Rule Table of Fuzzy Logic controller for loop 1

Ν	NB	NM	NS	ZR	PS	PM	PB
de							
e 🔪							
ci 🔪							
NB	NB	NB	NB	NM	NS	NS	ZR
NM	NB	NB	NM	NS	NS	ZR	PM
NS	NB	NM	NS	NS	ZR	PS	PM
ZR	NM	NM	NS	ZR	PS	PM	PB
PS	NM	NS	ZR	PS	PS	PM	PB
PM	NS	ZR	PS	PS	PM	PB	PB
PB	ZR	PS	PS	PM	PB	PB	PB

#### 4. Results And Discussions

This section discusses the simulation studies carried out on the quadruple-tank process. To validate the performance of the designed controllers, closed-loop simulations were conducted. For the purpose of simulation, the 'real' process is simulated by the nonlinear state space model. Separate simulations were performed for both minimum-phase and non-minimum phase systems. The performances were compared in terms of integral error values.

#### 4.1 Decentralized PI controller

The controller parameters are evaluated by using direct synthesis method. The set point for tank 1 is changed to 13.4cm initially and h1 reaches the set point almost in 350 seconds for minimum Phase and 400 seconds for non minimum phase. The set point for tank 2 is changed to 13.4cm at the beginning and h2 reaches the set point almost 550 seconds for minimum phase. The closed loop responses with Decentralized PI controller are shown in Fig.6 and Fig 7.





#### 4.2 Decentralized Fuzzy controller

The controller parameters are designed. The set point for tank 1 is altered to 13.4cm in the beginning and h1 reaches the set point almost in 50 seconds for minimum Phase and 100 seconds for non minimum phase. The set point for tank 2 is changed to 13.4cm at the beginning and h2 reaches the set point just about 150 seconds. for minimum phase and 250 seconds for non minimum phase.. The closed loop responses with Decentralized Fuzzy controller are shown in Fig.8 and Fig 9.

The performance of the two controllers is evaluated using performance indices namely Integral Square error(ISE) and Integral Absolute Error(IAE). A control system is considered optimal when it minimizes the above integrals.Table.5 summarizes the integral error values for the two control schemes. Decentralized Fuzzy controller has the least ISE, and IAE values.



Fig.8: Closed loop response with Decentralized fuzzy controller (Minimum phase)



Table 5. Quantitative comparison of performance indices

Controller	Perfor- mance	Minimum Phase		Non-minimum phase	
	indices	Loop 1	Loop 2	Loop1	Loop 2
Decentralized PI controller	IAE	7.1	5.9	33.4	81.8
Decentralized Fuzzy controller	IAE	2.6	1.9	12.3	45.2
Decentralized PI controller	ISE	2.6	1.3	12.7	18.4
Decentralized Fuzzy controller	ISE	2.5	0.9	10.9	11.3

# **5.**Conclusion

This work clearly shows the potential advantages of using decentralized fuzzy controller for a quadruple tank process. The control algorithm has a good set point tracking without any offset with reasonable settling time. The comparison of the present two controllers, reveals that decentralized fuzzy controller is superior resulting in smoother controller output without oscillations which would increase the actuator life.

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