An Efficient Model for Vibration Control by Piezoelectric Smart Structure Using Finite Element Method

Premjyoti G.Patil
Assistant Professor, Dept. of IS & Engg
Nagarjuna College of Engg & Technology
Devanahalli, Bangalore, India

Dr.Y.S.Kumara Swamy
Sr.Prof &HOD, Dept. of MCA (VTU)
Dayananda Sagar Institutions
Bangalore, India

Summary
Aerospace, one of the emerging fields is affecting greatly because of some unpreventable vibrations. Crude vibrations get generated at the time of craft operations. The beams which are used in the craft vessels in more numbers are the frequent victims of these vibrations. These vibrations tend to make the beam deformation – very risky for lives if they are left as simple. Although a lot of suggestions came for handling the problem, their approach in modeling for beam deformation remains somewhat inefficient either as per the computational or as per implementation concerns. The paper proposed here provides a mathematical model for the deformation of cantilever beam using Finite Element Method that makes the approach so efficient. The mathematical model formulated here will lay a strong foundation to wipe out the menacing effects of such beam deformation due to the vibrations without any computational as well as implementation complexities. In addition, a theoretical analysis is also done to find out the dominating frequencies of vibration that plays a major rule in beam deformation.

Keywords:
Aerospace, Smart structure, cantilever beam, Finite Element Analysis, vibration.

1. Introduction

Light-weight structures operating at high speeds may suffer significant vibration problems, thus degrading positioning accuracy and requiring larger settling times [1]. Using large, complex and light weight space structures to attain augmented functionality at a condensed launch cost is the recent trend of spacecraft design. In these space structures, the mixture of a large and light weight design results are being exceptionally flexible and having low fundamental vibration modes. Hence, such structures (host structures) become the most frequent sufferers of vibrations. Therefore, the structures will lose their life time. This results in damage of such structures. Preventing such deformation of the materials from this crude vibration is very essential. But the prediction of the vibrations and its effects are complex. Some kinds of vibration slam the target in different modes. So, the complexity again increases. As they claim life threatening effects, the vibrations can not left as a simple task.

In the field of aerospace, vibration control is a typical job that should be performed carefully with the consideration of a lot of physical constraints such as stiffness, elasticity and so on. It is impossible to mitigate the cause of vibration but it is possible to countervail against the undesired effects caused by vibration.

1.1. Cantilever Beam

Beam is one of the primary elements of an engineering structure, which is employed in wide-ranging structural applications. In addition a beam-like slender member can be employed in the modeling of structures similar to the helicopter rotor blades, spacecraft antennae, flexible satellites, airplane wings, gun barrels, robot arms, high-rise buildings, long-span bridges, and subsystems of more complex structures.

Despite the existence of different kinds of beams, the versatile applications of the cantilever type devices have engrossed huge attention. The cantilever type devices can be employed in the form of transducers for the conversion of the quantities for instance mass, temperature, inertia and magnetic fields into mechanical deformation. Numerous industries employ them in applications such as: accelerometers in the automobile industry; filters, inductors, resonators for telecommunications; and as atomic force microscopes in the field of science. These devices are possibly subjected to relatively large impact forces which can be outside their designed specifications when incorporated into vehicles and portable products. Due to the generation of relatively large forces at points of contact for relatively short periods of time, the impact is characterized under shock and vibration [3].

1.2. Review of Beam Vibration

The elastic properties precisely the stiffness and mass distribution of a vibrating body was established by Barcilon employing the measurement data [13]. For illustration a discretised beam was analyzed, in which one end was free and the other was either free or supported or clamped or constrained in an unusual way (this end is termed as "constrained" end).An impulse force was

Manuscript received November 5, 2008
Manuscript revised November 20, 2008

applied to the free end of the stationary beam, and the resultant deformation and slope of the constrained end is calculated. In order to deduce the elastic properties of the beam it is essential to identify the three sets of natural frequencies which are equivalent to the measured deformations and slopes. These trios of spectra are referred by Barcilon as the “sympathetic” spectra. The solution of the inverse problem, if it exists is unique, when these three sympathetic spectra are identified. Later on when Barcilon examined an apparent paradox between this uniqueness result and a paper by Boley and Golub , a multiplicity of solutions was identified when constructing a symmetric pentadiagonal matrix from its spectra.

According to Barcilon the pentadiagonal systems is the outcome of the beam vibration problems recast as finite difference problems, and therefore the apparent paradox. The illustration of Barcilon that Boley and Golub selected their three spectra without regard for “sympathy” between the spectra lead to the solution of the apparent paradox.

A beam was modeled by Gladwell employing rigid rods joined together by rotational springs, with lumped masses at the joints, in which one end was clamped and the other was either free or pinned or clamped or sliding. Essential and adequate conditions for the existence of a discrete model having a given spectrum were established and a procedure to identify the model was setup. Later on Gladwell evaluated the literature for solutions to inverse vibration problems. Basically this analysis considers the problem of determining the system’s properties (such as, mass and stiffness) from vibration measurements. The problem of system identification was examined by Berman employing the data obtained from dynamic tests of the structure. The linear mass, damping and stiffness matrix was employed to model the structure. In conclusion Berman states that the usage of the test data to minimally modify a realistic analytic model (depending on a set of physical constraints) is the most promising approach to modeling.

The external and internal forces were reconstructed based on the measured structural responses presuming a priori knowledge of the mass distribution and dynamic behavior of the system, and a linear elastic system with proportional viscous damping by Ory, Glaser, and Holzdeppe. The number of dynamic response measuring locations must be higher than the number of significant modes. A discretised cantilever beam that had several measurement locations along the beam was provided as an instance by them. One of the above authors extended the work later on.

A few of the simulations were done with the intention of controlling the residual vibrations of the cantilever beams [5]. Certain works emphasized on the analysis of free flexural vibrations on specially manufactured beams such as anisotropic laminated composite beam [6]. A study was also done on the stabilization of the vibrations of cantilever type in nanometer scale [7].

The well-posed ness of a Timoshenko beam1 with axially varying physical properties and sliding ends was established by Arosio, Panizzi, and Paoli. According to them, an iteration of the Fourier series cannot be employed to investigate the equation and thereby a variational approach developed by Washizu was employed as the alternate method.

Gopalakrishnan, Martin, and Doyle recast the dynamics of the Timoshenko beam in order that the description only necessitates information at the end points. The resulting dynamic stiffness relations were assembled (similar to finite elements) permitting exact frequency dependent response for the Timoshenko beam irrespective of element length.

Tanaka and Bercin constructed the solution for the free vibration of a Timoshenko beam employing the boundary integral equation. A general Timonshenko beam of open cross-section with non-coincident shear centre and centroid was modeled. They illustrated that the simpler Bernoulli-Euler beam theory model generates unacceptably large errors (particularly in case of higher order modes).

An approximate solution for the transverse vibration of a non-uniform Bernoulli-Euler beam with time-dependent elastic boundary conditions was developed by Lee and Lin. Furthermore, a numerical solution is also determined for the frequency equation of the transverse vibration of a simple beam [4].

1.3. Aerospace - Smart Structure

A structure that can sense an external riot [14] and react with active control in real time in upholding the mission necessities is known as a Smart Structure. A host structure, integrated with sensors and actuators synchronized by a controller, has been classically consisted in a Smart Structure. The sensors are in sundry types, yet, the piezoelectric sensors grab most of its applications due to its thriving features. Some studies were also done on the piezo-electric transducers and their advancements in control of vibrations [9]. As it has the ability to carry out self diagnosis and acclimatize to environmental change, this integrated structured system is called Smart Structure.

Spillman, Sirkis, and Gardiner defined a smart structure with the aid of excerpts from diverse sources which is read as follows:

“a smart structure is a non-biological structure having the following attributes:

1) a definitive purpose, 2) means and imperative to achieve that purpose, and 3) a biological pattern of functioning”[10][12].

In aerospace, the usage of such smart structures in cantilever beams, for example helicopter blades, will avoid the beam deformation due to vibrations. However, it needs an efficient model to identify the vibration, which mostly disturbs the host structure.

1.4. Finite Element Method

For discovering inexact solutions of partial differential equations (PDE) and of integral equations, the finite element method (FEM) (at times referred to as finite element analysis) is a numerical technique. The solution approach is based either on eradicating the differential equation entirely (steady state problems), or rendering the PDE into an approximating system of ordinary differential equations, which are subsequently solved by means of standard techniques for instance Euler's method, Runge-Kutta, etc.[2].

For the analysis of piezoelectric structural elements, numerous finite element models have been proposed since the early 70s. Till the early 90s, they were chiefly dedicated to the design of ultrasonic transducers. Interests have been directed towards applications in smart materials and structures by the late 80s. Quite a few review papers and bibliographies have emerged in the open literature on the finite element technology and modeling of structural elements throughout the most recent two decades.

A profound survey was held on finite element modeling and the advancements in its formulations and applications for the finite element modeling of adaptive structural elements namely, solids, shells, plates and beams [28]. Moreover, the model was also applied for the optimal design of piezoelectric actuators [11]

2. Proposed Methodology

In the proposed Methodology, a new mathematical model for the beam deformation and the frequency of its corresponding vibration is presented. Among a sundry of beams deployed in Air Craft materials, cantilever beams are going to be used for the model. Most of the equipments such as wings, blades used in aircraft coincide the shape of the cantilever beam. So, it is effective in showing the interest on considering cantilever beams for modeling.

An arrangement of the mathematical formulations used in the modeling of beam deformation is given sequentially as follows

Determination of Elasticity Matrix

The Elasticity Matrix D can be calculated for the beam both for plane Stress and Plane Strain. In the case of Plane Stress, the Elasticity matrix is given by,

$$D = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \quad (1)$$

Similarly for Plane Strain, the Elasticity matrix is

$$D = \frac{E}{(1-\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \quad (2)$$

where,
E – Young’s Modulus
ν - Poisson’s ratio

Strain Matrix

The general format derived for the formation of strain matrix is as follows,

$$B = \partial N \quad (3)$$

Hence,

$$b_{x1} = \frac{\partial N_1(x,y)}{\partial y}$$

$$b_{y1} = \frac{\partial N_1(x,y)}{\partial y}$$

$$b_{x2} = \frac{\partial N_2(x,y)}{\partial x}$$

$$b_{y2} = \frac{\partial N_2(x,y)}{\partial y}$$

$$\vdots$$

$$b_{xL} = \frac{\partial N_L(x,y)}{\partial x}$$

$$b_{yL} = \frac{\partial N_L(x,y)}{\partial y}$$

Applying it in the general matrix format, the resulting matrix is as follows

$$[B] = \begin{bmatrix} b_{x1} & b_{x2} & \cdots & b_{xL} & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & b_{y1} & b_{y2} & \cdots & b_{yL} \\ b_{y1} & b_{y2} & \cdots & b_{yL} & b_{x1} & b_{x2} & \cdots & b_{xL} \end{bmatrix} \quad (4)$$

Determination of Finite Element operators

Stiffness matrix and the force vector are the finite element operators used in the linear static modeling of the beam. For any element, stiffness matrix is formed using the relation,

$$k = \int_{A^e} t[B]^T [D][B] \; dA \quad (5)$$

where t is the thickness of the beam. This equation can be further simplified for the convenience of application by using the area – volume relation. Hence the equation is

$$k = \int_{V^e} [B]^T [D][B] \; dV \quad (6)$$

By applying the elasticity matrix and strain matrix in (6) the stiffness matrix can be calculated.

The external force vector applied in the beam element is given by

$$f^e = \int_A N \; dA \quad (7)$$

The use of quadrature rule reduces the complexity in using the integrals in determining the equation. The rule is given as follows

$$\int_{x=-a}^{x=a} f(x)dx = a\sum_q f(ax_q)W_q \quad (8)$$

where

W_q = quadrature weights

Thus the using the quadrature rule makes the way to solution more convenient.

Displacement vector

The finite dimensional function is given by the following equation

$$u^h = \sum_{j=1}^{L} N_j(x) d_j \quad (9)$$

where

L - number of nodes

$N_j(x)$ - finite Element shape functions

d_i - nodal unknowns for the node j

In case of the scalar fields the location of the nodal unknowns in d is most apparently given as:

$$d_j = d[j] \quad (10)$$

However there is some uncertainty in the case of vector fields, in identifying the location of the nodal unknown d_{ji}, where I refers to the node number and i refers to the component of the vector nodal unknown d_j .There is a necessity to define a mapping from the node number and vector component to the index of the nodal unknown vector d. This mapping can be represented as

$$f : (j,i) \to \mathrm{n} \quad (11)$$

Where j is the node number, f is the mapping, i is the component and n is the index

Thus the location of unknown u_{ji} in d is defined as follows

$$u_{ji} = d_f(j,i) \quad (12)$$

The i component of the displacement at node j is located as follows in u

$$u_{ji} = u(n) \quad (13)$$

By the arrangement of alternation between each spatial component, the displacement vector will take the form as follows,

$$u = \begin{pmatrix} u_{1x} \\ u_{1y} \\ \vdots \\ u_{2x} \\ u_{2y} \\ \vdots \\ \mathrm{u}_{\mathrm{Nx}} \\ u_{Ny} \\ \vdots \end{pmatrix} \quad (14)$$

By this mapping the displacement at node j is located in u by using the following equation in (13)

$$n = D(j-1) + i \quad (15)$$

where D represents the number of dimensions of the beam

By another option of grouping all the like components, the displacement vector will be formed as

$$u = \begin{pmatrix} u_{1x} \\ u_{2x} \\ \vdots \\ \mathrm{u}_{nx} \\ \mathrm{u}_{1y} \\ \mathrm{u}_{2y} \\ \vdots \\ \mathrm{u}_{ny} \end{pmatrix} \qquad (16)$$

For this form, the displacement at node j is located at in u with the application of the following in (13)

$$n = L(i-1) + j \qquad (17)$$

Linear Algebraic System

Potential energy of element

$$\Pi_{elt} = \frac{1}{2}\int_{V^{elt}} \sigma^T \varepsilon dV - \int_{V^{elt}} u^T \times dV - \int_{S_T^{elt}} u^T T_S dS \qquad (18)$$

Total potential energy is the sum of potential energies of the elements and it is given by

$$\Pi = \sum_{elt} \Pi_{elt} \qquad (19)$$

The major parameters play a key role in beam deformation such as displacement, stress and strain needs approximation.

$$u = Nd \qquad (20)$$

$$\varepsilon = Bd \qquad (21)$$

$$\sigma = DBd \qquad (22)$$

By applying all the approximated values (20), (21) and (22) in the equation (18)

$$\Pi_{elt}(d) = \frac{1}{2} d^T K d - d^T f \qquad (23)$$

Using the principle of minimum potential energy

$$\frac{\partial \Pi_{elt}(d)}{\partial d} = Kd - f = 0 \qquad (24)$$

Thus,

$$Kd = f \qquad (25)$$

Eventually, the equation (25) will possess an equivalent as follows

$$Ku = f \qquad (26)$$

The element operator should be scattered into the global operator after it is calculated.

These tasks of scattering the element stiffness matrix into global stiffness matrix could be done through the Matlab and so that time taken for performing such operation is reduced.

Taking the length of the Beam as *l*, the second polar moment of inertia *I*, half the distance of the outer fiber of the beam *c* and peak magnitude M_p the solutions behind the beam bending problem are as follows

$$f_{qx} = \frac{-M_p y(l-x)}{I} \qquad (27)$$

$$f_{qy} = \frac{M_p (c^2 - y^2)}{2I} \qquad (28)$$

$$u_1 = \frac{-M_p \{3(l^2 - (l-x)^2) + (2+v)(y^2 - c^2)\} y}{6EI} \qquad (29)$$

$$u_2 = \frac{M_p \{3((l-x)^3 - l^3) - [(4+5v)c^2 + 3l^2]x + 3v(l-x)y^2\} y}{6EI} \qquad (30)$$

The solution is taking the *x* and *y* coordinates as well. The final section of solving linear algebraic system is applying the boundary conditions which finally give the net solution for all the problems. The most important factor to be considered while in applying the boundary condition is to maintain the symmetric behavior of the stiffness matrix which is the most important property of the linear algebraic system. In addition a weighting factor is also used to maintain the conditioning of the stiffness matrix.

Frequency of different modes

The frequency at can be determined by using the mathematical formula as given as follows

$$f(n) = \frac{1}{2\pi}\sqrt{\frac{EI}{\rho A_c}}\left(\frac{k(n)}{l}\right)^2$$

n -no. of modes

$k(n)$ -model coefficient

ρ -density in kg/m^3

A_c -Cross-sectional area

Not all the modes of frequency affect the beam, only the lower order modes dominates in the beam deformation. Since the higher order modes have very less magnitude, it is reasonable in concentrating in the lower especially in first three to six.

3. Implementation and Results

The mathematical model suggested here is experimentally verified by simulating a cantilever beam in MATLAB that undergoes deformation.

The beam, initially, will be simulated as a mesh of nodes or as a mesh of finite element, because the finite element analysis is taking the problem. The initial mesh of beam is shown in figure 1.

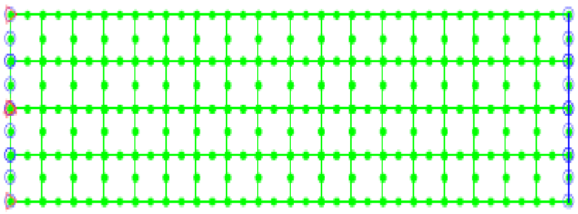

Fig1. Cantilever beam in mesh like structure

The implementation of the mathematical model results in the simulation that plots the deformation of displacement in the beam and also the deformation of stress in the beam in figure 2 and figure 3 respectively.

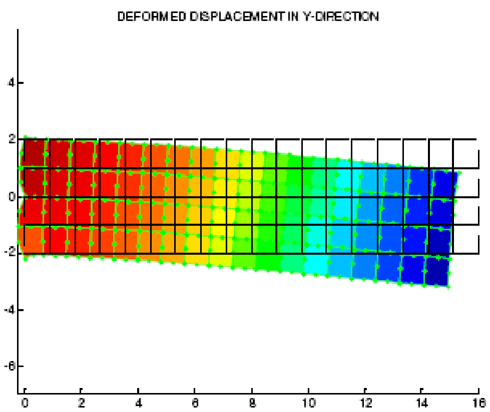

Fig2. Displacement deformation in cantilever beam

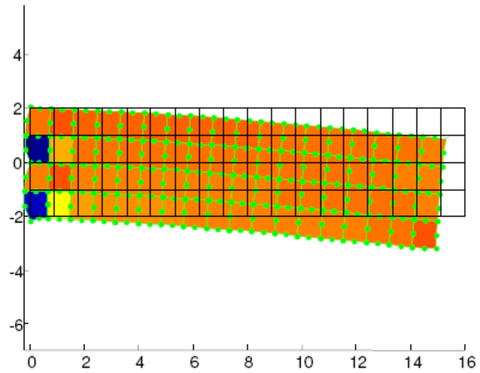

Fig3. Stress deformation in cantilever beam

More over, a calculation of the frequency of vibration is also coded that displays clearly the dominating modes of frequency of vibration. So, the simulation verifies the mathematical formulation implicated in the proposed model.

4. Conclusion

In this methodology the cruel effects of vibrations are concerned and so the mathematical model using the Finite Element method is formulated for cantilever beams in order to estimate the beam deformation. Using the mathematical model, the beam deformation is plotted using MATLAB which shows reduction in computational complexity. Along with them, the most dominating modes of frequencies of vibrations are also calculated hypothetically as well. By deep use of the formulation suggested in this model it is very easy to root out the beam deformation and so the counteracting steps to avoid that become somewhat effective.

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Mrs. Premjyoti Patil working presently as Assistant Professor in Department of Information Science & Engineering at Nagarjuna College of Engineering & Technology, Devanahalli, Bangalore. She has presented more than 6 papers in National & International conferences. Her areas of interest are control systems, power electronics & biomedical instrumentation.

Dr.Y.S.Kumara Swamy, working presently as Sr.Prof and Head in Department of MCA (VTU) at Dayananda Sagar Institutions, Bangalore. He has published more than 150 research papers and guiding 38 Ph.D students in the area of Computer Science/ Mathematics and also selection committee member for ISRO/UGC/DSI profiles.