Fast Bidimensional Empirical Mode Decomposition Based on an Adaptive Block Partitioning

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Summary
The decomposition of images with great size using the Bidimensional Empirical Mode Decomposition (BEMD) necessitates an important calculation time. To overcome this problem we first proposed in [1] a new approach using Block-based BEMD method (BBEMD) where an input image is subdivided into four subblocks, and then the BEMD is applied on each of the four blocks separately. This method offered a good solution to the calculation time, unfortunately blocking artefact is noticeable in the resulting modes, this is a result of ignoring the interblock correlation during the BEMD process and every block was taken as an independent entity when interpolating the blocks borders in the sifting process. The Lapped Block-based BEMD (LBBEMD) is then proposed to whelm the artefact problem using for every block the neighbouring information by enlarging the block neighbourhood related to the two adjacent blocks inside the image during the blocks decomposition. Enlarging neighbourhood blocks necessitate intensive simulations to have for each block the corresponding enlargement size, thing that affects remarkably on the calculation time. Experimentation shows that the LBBEMD can not handle correctly the artefact problems, because the enlargement size can not be enough "or too big" to surround all the concerned pixels for each block in the image. The problem with the LBBEMD is to exactly define, a priori, for each block the size of the neighbouring enlargement size. In this paper, we will present an adaptive and fast Block Empirical mode decomposition to eliminate definitively the artefacts problems when subdividing an image into lapped blocks and we will prove, by simulations, that our method offers a good trade-off between computational time and decomposition quality.
Key words:
BEMD, fast adaptive lapped block, edge variance information, block-based BEMD method, blocking artefact, lapped block-based BEMD method.

1. Introduction

The concept of the empirical mode decomposition is to expand each signal into a set of functions defined by the signal it self called Intrinsic Mode Functions (IMF), the process is called sifting process. The signal is decomposed into a redundant set of signals denoted IMF and a residue, adding the all the IMF's together with the residue reconstructs the original signal without information loss or distortion. Meanwhile, decomposing images of great size using the BEMD can necessitates an important calculation time. A first idea, to overcome this problem and accelerate the process to minimize the computational time, is to subdivide the image into four overlapped sub-blocks and apply the BEMD on each overlapped block. Block overlapping is done by enlarging each block with a fixed size in the two possible directions to handle the interblock correlation. The question is how we can define for each block the exact enlargement size that can handle correctly the borders artefact. Normally, we use a fixed size of the blocks enlargement for all the blocks in the image, however, for some blocks this enlargement size can be not enough or too big to surround the concerned pixels, that's why we should use a specific enlargement size for each block in the image. In these conditions, finding the exact enlargement size for each block in the image is not a very easy thing and necessitates an intensive simulation which directly influence on the calculation time. That's why we should use an adaptive method to define the size of the enlargement size of neighbourhood block. Here, we propose an adaptive and fast block partitioning method to select the size of the block enlargement using the edge variance information.

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2. The Empirical Mode Decomposition

EMD is an adaptive decomposition of data, introduced by Huang [9] for one dimensional signal and extended to image [3] after that. Researches proves that the EMD is as better quality than Fourier, wavelet and other decomposition techniques in extracting intrinsic components of textures and image compression because of its fully data driven property. This decomposition is also proven as a very powerful tool for multi-scale analysis of non stationary and non linear signals.

The BEMD is a highly adaptive decomposition. It is based on the characterization of the image through its decomposition in intrinsic mode function (IMF) where the image can be decomposed into a redundant set of signals denoted IMF and a residue, adding all the IMF's together with the residue reconstructs the original image without information loss or distortion. An IMF is characterized by some specific properties:

- The number of zero crossing and the number of extrema points is equal or differs only by one,
- It has a zero local mean.

Given a 2D signal m we can describe the principal of the empirical mode decomposition as follows:

1. Initialisation: $r_0 = m$ (the residual) and $j = 1$ (index number of IMF);
2. Extract the jth IMF (Sifting Process):
   a. Initialise $h_0 = r_{j-1}, i = 1$.
   b. Extract local minima and maxima of h_{i-1}.
   c. Compute upper envelope and lower envelope functions x_{i-1} and y_{i-1} by interpolating respectively local minima and local maxima of h_{i-1}.
   d. Compute $m_{i-1} = (x_{i-1} + y_{i-1})/2$, the mean envelope.
   e. Update $h_i = h_{i-1} - m_{i-1}$ and $i = i + 1$.
   f. Calculate stopping criterion
   $$SD = \frac{1}{N}\sum_{k=O}^{K}\left[\frac{\left(h_{i(j-1)}(k) - h_{ij}(k)\right)^2}{h_{i(j-1)}^2(k)}\right]$$
   g. Decision: Repeat steps (b) to (f) until $SD_i \le \xi$, and then put $d_i = h_i$ (j^{ht} IMF).
3. Update residual $r_{ij} = r_{j-1} - m_j$.
4. Repeat steps 1-3 with $j = j + 1$ until the number of extrema in r_j is less than 2.

When the decomposition is achieved, we can write the signal in the following form:

$$m(n) = \sum_{k=1}^{K} d_k(n) + r(n), K \in N^*$$

In Fig. 2 we present the decomposition results of the image, (512×512) pixels, presented in Fig.1 using the BEMD. The decomposition was stopped in two modes and was performed in 74 seconds.

Fig. 1 Image original, (512×512) pixels

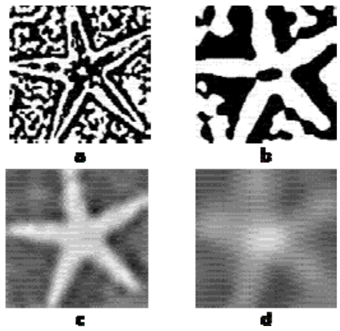


Fig. 2 BEMD in two modes: a) first mode, b) second mode, c) first residue, d) second residue

3. Lapped Block Bidimensional Empirical Mode Decomposition (LBBEMD)

The application of the BEMD for images of great size necessitates an important calculation time (74 seconds to decompose an image (512×512) pixels), to overcome this problem we first proposed a new approach using block-based BEMD method where the input image is subdivided into four blocks and apply the BEMD on each of the four blocks separately. This method presents a compromise between decomposition time and blocks image quality unless in block boundaries blocking artefact are visible. So, we secondly proposed to use for each block the neighbouring information's and apply the BEMD on the lapped blocks.

3.1 Block Bidimensional Empirical Mode Decomposition

Our strategy lies in the utilization of Radial Basis Functions on small blocks; that is, the input image is partitioned into square blocks of pixels of size (k, l) in such a way to decompose each subimages separately.

Let (M, N) be the input image size and let (k, l) represent the block size. By introducing the variables:

$$s_1 = \frac{M}{k} \quad , \quad s_2 = \frac{N}{l}$$

We can deduce the total number of image blocks, which can be set as $Nb = s_1.s_2$.

Given the image space which takes the form:

$$\Omega = \left\{x_i, y_i \mid 1 \le x_i \le M, 1 \le y_j \le N\right\}$$

We define the subset $D^{n_1,n_2} \subset \Omega$ as:

$$D^{n_1,n_2} = \{x_i, y_j \mid n_2 k \le x_i \le (n_2+1)k,\ n_1 l \le y_j \le (n_1+1)l\}$$

It should be noticed that this subspace, which can also be termed image block space, is related to Ω with:

$$\Omega = \bigcup_{n_1=1}^{s_1} \bigcup_{n_2=1}^{s_2} D^{n_1,\,n_2}$$

Then let the image associated to each $D^{n_1,\,n_2}$ subset be defined as follows:

$$I^{n_1,n_2}(x,y) = \left\{ I(x,y) \mid x_i, y_j \in D^{n_1,\,n_2} \right\}$$

This gives:

$$I(x,y) = \bigcup_{n_1=1}^{s_1} \bigcup_{n_2=1}^{s_2} I^{n_1,\,n_2}(x,y)$$

The Bidimennsional empirical mode method by block processing is the same as illustrated in Section 2, except that, in this case, the algorithm will try to use the BEMD on each block separately. The steps of the block-based BEMD are presented in the fig. 3.

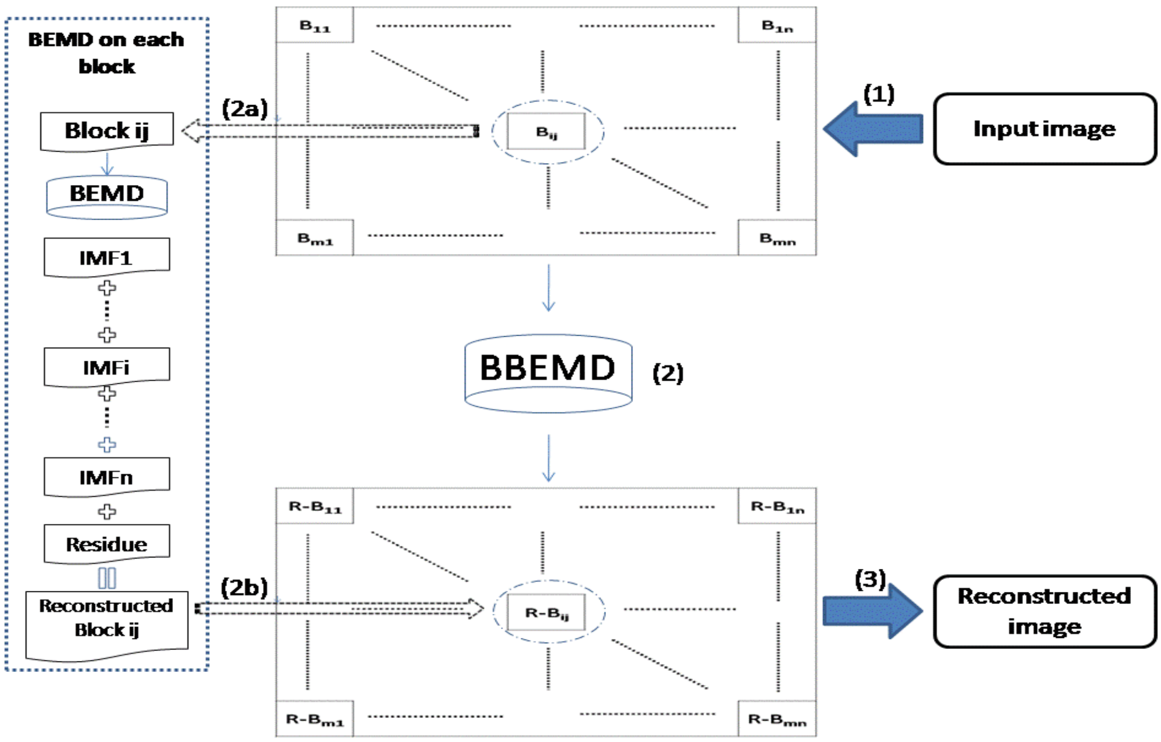


Fig. 3 Illustration of the BBEMD algorithm: (1) Division of the input image into n subimages, (2) BEMD on each Block and merging: (2a) example of the application of the BEMD on the image block ij, (2b) merging the reconstructed block ij, (3) reconstructing the restored BEMD image.

To evaluate experimentally the method, computer simulations has been carried out and applied on the star image (Fig. 1) with as input block sizes (256×256) and compared with the decomposition by the BEMD (Fig. 2). In Fig. 4 we present the decomposition results using BBEMD.

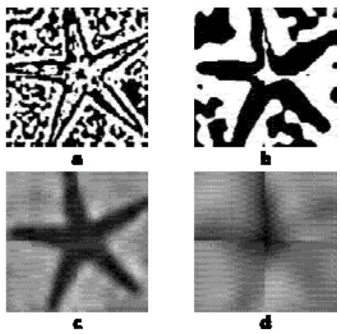


Fig. 4 BBEMD in two modes: a) first mode, b) second mode, c) first residue, d) second residue

The proposed block image empirical mode decomposition offers a good solution for computation time reduction (Table 1) compared with the BEMD. Unfortunately, when adjacent blocks have different BEMD errors, the block boundaries become visible (Fig. 4d). This blocking artefact is therefore more noticeable in the resulting BBEMD modes. These vertical and horizontal lines, caused by this blocking artefact, are generally considered objectionable to human viewers.

3.2 LBBEMD

One technique for mitigating artefacts in block processing involves a pre-processing of the BEMD images. Such techniques allow substantial reduction of the blocking artefact, despite the expense of an increase in the overall mean square BEMD error [2].

It is well known that the blocking effect is a consequence of ignoring the interblock correlation during the BEMD process because every block is taken as an independent entity. Therefore, one of the best ways to minimize the disturbance in the output image is to make use of the interblock correlation. Our method exploits for every block the neighbourhood information related to its adjacent blocks during the computation moment. This approach can achieve a good performance in eliminating the blocking effect and, by the way, avoid other strategies to restore or enhance the image quality by using post processing techniques. Fig. 5 shows the block diagram of the proposed Lapped Block-based BEMD. Note that there are two stages in this block diagram:

1. The algorithm BEMD computation which extracts the block neighbourhood information by proceeding on lapped blocks;
2. The BEMD process which acts on output blocks and merge them into the final image

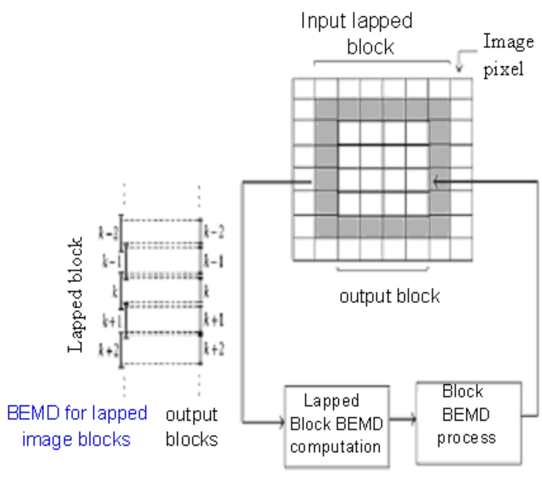


Fig. 5 Illustration of the different stages of the LBBEMD.

3.3 Results

The main problem of the use of the LBBEMD is that we should set in advance the enlargement size for each block. For example if we apply the LBBEMD to an image (512×512) pixels (Fig. 1) and if it is subdivided to four blocks; each block is (256×256) pixels. After intensive simulations and at each time we change the block size enlargement, the best visual result of the implementation of a LBBEMD was obtained with a blocks size enlargements equal to 42 pixels. That means that each block should be (276×276) pixels. In Fig. 6 and Fig. 7 we present the results of the decomposition of the image with respectively blocks size enlargements equal to 20 and 42 pixels.

The results presented in Table 1 (the LBBEMD calculation time is without taking in count the intensive simulations calculation time to get the blocks size enlargement) shows that the technique called LBBEMD involves a reduction of the reconstruction error compared with the BBEMD method. Unfortunately the PSNR value of the LBBEMD indicates that the quality of the reconstructed image is not very good compared with the BEMD. Also, the results of the zooming on the horizontal block borders of the restored image presented in Fig. 7f implies that the artefact problems still subsist on the block borders. All this is due that the block enlargement size, used to resolve the artefact problem, is set manually in advance and it can be, for some blocks, not enough “or too big” to take into account the concerned pixels in the neighbours blocks. So, to deal with such a problem we will propose next an adaptive method to select the enlargement size of each block based on the edge variance information.

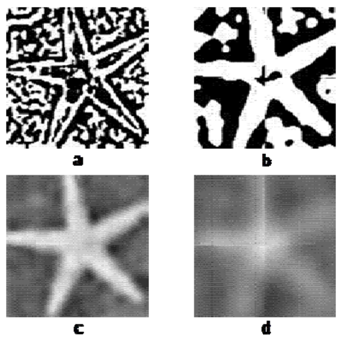


Fig. 6 LBBEMD with block size enlargement equal to 20 pixels in two modes: a) first mode, b) second mode, c) first residue, d) second residue

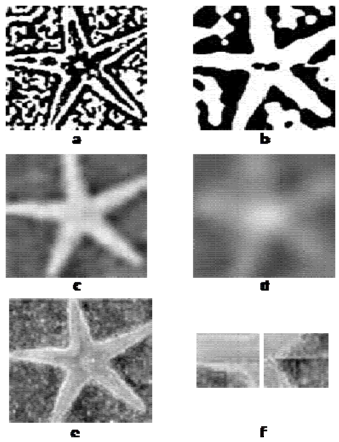


Fig. 7 LBBEMD in two modes with block size enlargement equal to 42 pixels: a) first mode, b) second mode, c) first residue, d) second residue, e) the restored image, f) zooming by 180% on the horizontal block borders of the restored image

Table 1: Values for the PSNR (dB) and the calculation time for the image decomposition using BEMD, BBEMD and LBBEMD (with block size enlargement equal to 42).

	BEMD	BBEMD	LBBEMD
PSNR (dB)	48,3056	22,5656	38,6320
Calculation time (s)	74,0084	28.3544	28.3789

4. BBEMD using adaptive lapped block enlargement (ALBBEMD)

Suppose that $I(x_1, y_1)$ and $I(x_2, y_2)$ are the image values of two pixels that are next to each other in the same row or column but are in different blocks, we assume that the artefact problem of the BBEMD modes is due to the fact that before the sifting process the values of $I(x_1, y_1)$ and $I(x_2, y_2)$ were usually similar, but they have been made more different by interpolation and also every block is taken as an independent entity. One solution is that involved in above where we exploits for every block the neighbourhood information related to its adjacent blocks during the computation moment, but we have to fix a priori for each bloc the size of the enlargement neighbourhood. To surmount the manually size setting, we will use the edge variance information to select adaptively the block enlargement size.

4.1 Block Edge variance

The Edge variance of two pixels that are next to each other and not in the same block can de defined as follow:

$$E = (I(x_1, y_1) - I(x_2, y_2))^2$$

Where $I(x_1, y_1)$ and $I(x_2, y_2)$ are respectively the image values of the two pixels.

Let B_0 be the block to enlarge and (m,n) its size. The horizontal positive and horizontal negative Edge variances at the bloc borders are respectively defined by:

$$E_{h+} = \frac{1}{m^2} \times \sum_{k=1}^{m} (I(x_k, y_n) - I(x_k, y_{n+1}))^2$$

$$E_{h-} = \frac{1}{m^2} \times \sum_{k=1}^{m} (I(x_k, y_n) - I(x_k, y_{n-1}))^2$$

And, the vertical positive and vertical negative Edge variances at the bloc borders are respectively defined by:

$$E_{v+} = \frac{1}{n^2} \times \sum_{l=1}^{n} (I(x_m, y_l) - I(x_{m+1}, y_l))^2$$

$$E_{v-} = \frac{1}{n^2} \times \sum_{l=1}^{n} (I(x_m, y_l) - I(x_{m-1}, y_l))^2$$

The horizontal (resp. vertical) enlargement process for the block B_0 can be done as follow:

Step.1 Initialize the Edge variance in the block borders, E_0,

Step.2 Enlarge the block horizontally (resp. vertically) by a pixel vector and calculate the Edge variance at the block borders, E.

Step.3 Compare the edge variance. If $E \le E_0$ repeat the Step.1 with the block size equal to $(m, n+1)$ (resp. $(m+1, n)$). Else, stop the enlargement.

When subdividing an image into four blocks, the enlargement can be, for each of the four blocks, in two possible directions in such a way that the enlargement should be inside the image.

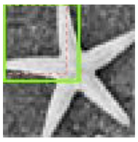
4.2 Adaptive block partitioning

We have tested our algorithm on a collection of images of various natures. Here, we present the results of the decomposition of one of the images using our technique and where the neighbouring enlargement window size by five pixels can not resolve the artefact problem. Our technique is tested on the multi-gray-level image presented in Fig. 1, where initially each block size is (256×256) if subdivided into four blocks.

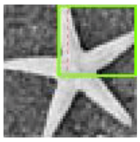
Compared with the LBBEMD, the sizes of the subblock are not the same (Fig. 8). If each block is enlarged with fixed size (42 for example) in the LBBEMD method, each of the four sub-blocks size was (298×298).While, in our method, using the block edge variance, the first block size is (298×310), the second block size is (282×302), the third block size is (322×294) and the fourth block size is (334×302). That's mean that compared with the LBBEMD there were:

- Not enough enlargements in: the horizontal enlargement in the first block, the horizontal enlargement in the second block, the vertical enlargement in the third block and the horizontal and horizontal enlargements in the fourth block.
- Too large enlargements in: the vertical enlargement in the second block and the horizontal enlargement in the third block.

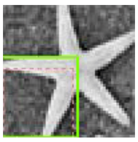
(a)



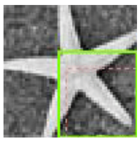
$E.B$ size : (298×310) (b)



$E.B$ size : (282×302) (c)



$E.B$ size : (322×294) (d)



$E.B$ size : (334×302) (e)

Fig. 8 (a) The original image, (b) the first Enlarged Block: the block size is (298×310), (c) the second Enlarged Block: the block size is (282×302), the third Enlarged Block: the block size is (322×294) and (e) the fourth Enlarged Block: the block size is (334×302)

5. Results and discussion

5.1 Experimentation

To evaluate our technique, we will try to decompose the same image used in case of the LBBEMD and compare the PSNR, the calculation time and the zooming on the blocks border.

First, the visual quality of the restored image after decomposition using the ALBBEMD (Fig. 9 e) is very close the original image presented in Fig. 1. Also, in Table 2 the PSNR is nearly equal for the BEMD and the ALBBEMD, and the calculation time is very low compared with the BEMD method. So, we can say that the proposed method (Adaptive Lapped Block Bidimensional Empirical Mode Decomposition) offers a good trade-off between computation time and decomposition quality.

Note that the LBBEMD calculation time presented in Table 2 is without taking in count the intensive simulations calculation time to get the blocks size enlargement. So, compared with our method, if we want to have the same blocks size, for the LBBEMD method, we need to do more than 10189 simulations (if we assume that for each simulation we enlarge each block by one pixel in the two possible directions).

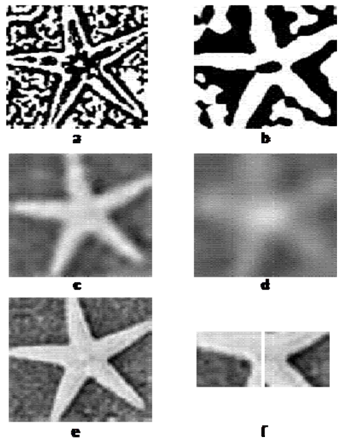


Fig. 9 ALBBEMD in tow modes: a) first mode, b) second mode, c) first residue, d) second residue, e) the restored image, f) zooming by 450% on the horizontal block borders of the restored image

Table 2: Values for the PSNR (dB) and the calculation time for the image decomposition using BEMD, LBBEMD (with block size enlargement equal to 42) and ALBBEMD (LBBEMD with adaptive block size enlargement).

	BEMD	LBBEMD	ALBBEMD
PSNR (dB)	48,3056	38,6320	48,2978
Calculation time (s)	74,0084	28.3789	27.6245

5.2 Discussion

The fixed window enlargement size in the sub-block can not handle the artefact in some images and these artefacts may not been visible in the image normal size, therefore it can be seen when zooming on the block borders (see fig. 9f), also the PSNR values and the calculation time (Table 2) shows that the ALBBEMD is a good quality compared with the LBBEMD in image decomposition/reconstruction and it offers a good solution for the calculation time problem faced with the BEMD. All this is because some pixels that are not in a block and which influences the results in the sifting process and are not taken in account when enlarging the block (size not enough) or when some pixels are taken in account and they should not been added in the sifting process (size too).

6. Conclusion

In this paper, we presented a fast and adaptive solution for the BEMD based on image block partitioning. The simulations results show that the decomposition quality using our method is very similar to the BEMD with a big reduction of computing time. The advantage of the use of an adaptive window when enlarging the block size in ALBBEMD is to take in consideration only the pixels that influence the interpolation result and which are not in the same blocks. So in this way we are able to definitively eliminate the artefact problems when subdividing image to four blocks and to reduce the calculation time.

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